Essays on Behavioral Portfolio Management: Theory and Application

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1 Introduction

> *"Zudem gilt, dass Wirtschaft zu 50 Prozent aus Psychologie besteht."*
>
> *Angela Merkel, 2009*[1]

Anecdote 1: In mid 2009, a school friend asked me what to do with antique gold he recently inherited from his elderly aunt. At the time, the price of gold had been steadily moving upward for more than six month and had reached the all time high of USD 1,000 per ounce. My friend noted that the increasing gold price had made him think about selling it. Anecdote 2: For his graduate studies, my brother received BAföG, an interest-free student loan from the German administration that has to be repaid usually starting no later than five years after graduation. He graduated one year ago and has been working since that time, making good money. Several weeks ago, he stated that he wanted to repay the loan earlier. I asked him why, it is interest-free. He responded: "I just want to get rid of it." Anecdote 3: Some weeks ago, I refueled my car. Next to me was a customer who put exactly EUR 20.01 of fuel in his car. I asked him why he did not fill his car up. He responded: "That's enough to get along the next two days. Prices will surely drop."

The preceding anecdotes all illustrate human inconsistencies in financial decision making. My school friend's tendency to short his gold position is part of the *disposition effect*, a behavioral anomaly in which investors tend to sell assets whose prices have increased, while keeping assets whose prices have decreased (Shefrin and Statman, 1985). Gold price continued to increase to USD 1,500 in April 2011. Meanwhile, although the BAföG is an interest-free loan, my brother would like to settle his loan before the five-year period is over. Most people, including my brother, do not manage their investments

[1]Translation: 50 percent of economics is psychology. Interview in the Frankfurter Allgemeine Zeitung on November 14, 2009.

as a whole. Instead, they separate choices into *mental accounts* (Thaler, 1999). My brother manages his student loan in a separate account and is therefore prone to close this negative account. Finally, the customer at the gasoline station was confident that prices would drop and therefore was willing to pay extra transaction costs. The effect in which someone's subjective confidence in their judgments is reliably greater than their objective accuracy is called *overconfidence* (Barber and Odean, 2001).

Other behavioral anomalies in financial decision making in general and portfolio formation in particular include (i) *lacking diversification*, individual investors hold only a few assets (Barber and Odean, 2000); (ii) *naive diversification*, the tendency to split money evenly across available assets (Benartzi and Thaler, 2001); (iii) *home bias*, the tendency to prefer domestic over foreign assets (French and Poterba, 1991); (iv) *lacking self-control*, people's difficulty controlling their emotions (Mitchell and Utkus, 2006); (v) *availability*, people's tendency to predict the likelihood of an outcome based on how many outcomes of each type come readily to mind (Tversky and Kahneman, 1973); (vi) *emotions and cognition*, errors that stem from the way that people think (Wright and Bower, 1992); and (vii) *herding*, describing how individuals in a group act together without planned direction (Shiller, 2005, pp. 157-172).

Traditional economic theory is unable to explain the anecdotes and other well-documented anomalies, such as those mentioned above, because it assumes that individuals have complete information and are able to process this information, that individuals are rational decision makers, and that individuals' preferences are well-defined and constant over time. It further posits that people make decisions by maximizing a utility function in which all relevant constraints and preferences are included and weighed appropriately. Based on these assumptions, the path-breaking work by Markowitz (1952b) prescribes that portfolio formation is solely based on expected portfolio return and portfolio variance, and that decision makers maximize quadratic utility. However, mean-variance theory has not become part of the accepted wisdom among individual and institutional investors. Instead, a sequence of violations of mean-variance behavior has been documented in the literature including a lack of diversification, the tendency to diversify naively, and ignorance of correlations between assets associated with a rejection of the portfolio variance as an adequate measure of risk (Kroll et al., 1988a,b). In contrast, behavioral portfolio theory, which takes violations of rational economic behavior into account, describes rather than prescribes investors behavior. Behavioral portfolio theory is a subdiscipline of behavioral finance, which Shefrin (2002, p. 3) defines briefly "... as the application of psychology to financial behavior".

The aim of this dissertation is to investigate new theoretical and empirical issues related to behavioral portfolio theory, including aspects like the lack of diversification (chapters 2, 5), home bias (chapter 3), lack of self-control (chapters 2, 5), availability heuristic (chapter 2), emotion and cognition (chapters 2, 5), mental accounting (chapters

4, 5), herding (chapter 3), ignorance of correlations (chapters 2, 3, 4), and overconfidence (chapter 5). Each of the following four chapters is part of an independent research paper and can thus be studied separately. The relevant literature is also presented separately and is therefore omitted at this point.

Chapter two investigates the role of behavioral portfolio management in old-age provision in Germany. I implement a general version of Shefrin and Statman's (2000) single mental account model, which combines SP/A theory, the underlying decision framework, and Telser's (1955) safety-first rule. Assuming empirically distributed returns, the model is then transformed in its deterministic equivalent, which is solved numerically using mixed-integer linear programming. Using return data of the seven most suitable German retirement investments, I simulate security-minded, potential-minded, cautiously hopeful and mean-variance portfolios, where the latter serves as a comparison model.

An important feature of behavioral portfolio theory is the aspiration level or threshold return that is independently chosen by the investor. The threshold level is commonly assumed to be fixed. However, in chapter three, I present several reasons for a random threshold level and suggest an answer to the open question when the random threshold is preferred over the fixed and vice versa.

Whereas chapters two and three deal with a single mental account, chapter four studies behavioral portfolio theory with multiple mental accounts. Based on a recent paper by Das et al. (2010), I analyze a model in which goal-specific asset selection is allowed, namely, the investor is allowed to select assets that meet the goal of, for instance, a retirement account. In this case, subportfolios - a solution to a mental account - induce a mean-variance efficient frontier on which the aggregate portfolio can be found. When goal-specific asset selection is not allowed, I present a closed-form solution and a utility function consistent with Friedman and Savage's (1948) solution to the well-known insurance lottery puzzle.

Based on a joint research paper with Kathrin Johansen, chapter five analyzes individual investors' gambling behavior in general and the relation between lottery tickets and common stocks in particular. Using behavioral theoretic arguments, we hypothesize that both assets act as substitutes. We test this hypothesis using the Einkommens- und Verbrauchsstichprobe (German survey of household income and expenditure), a representative dataset for the German population for a sample of more than 40,000 households.

2 A Behavioral Portfolio Analysis of Retirement Portfolios in Germany

This chapter is based on Singer (2009).

"Der Unterschied zwischen Wunsch und Wirklichkeit bei der privaten Altersvorsorge hat ein Rekordniveau erreicht."

Michael Meyer, 2010[1]

2.1 Introduction

To most individuals saving for retirement is the number one financial goal. Due to demographic changes, tight public budgets, and reduced generosity of occupational pension plans never in the post-World War Two era has been more reason to encourage workers to provide for their own retirement. Many recent studies, among them Oehler and Werner (2008), Bateman et al. (2010), Bridges et al. (2010), Knoll (2010) and Mitchell (2010), point to the shift of responsibility for an adequate old age provision toward individuals' shoulders. In a 2009 OECD study, Antolin and Whitehouse document a pension gap for 11 of 30 OECD countries, among them the United States, the United Kingdom, Germany and France. They define the pension gap as the difference between the replacement rate - the relationship between income in retirement and earnings when working - from the mandatory pension system and the OECD average. For the 11 countries they calculate a pension gap of 18.2% on average. To close this gap, however, reveals a complex task to most people since it requires accurate estimates of uncertain future processes including lifetime earnings, asset returns, tax rates, family and health status, and

[1] Translation: The divergence between individuals' desire and their actual behavior in provision for old age has reached its highest level. In a special supplement on saving for retirement to the Frankfurter Allgemeine Zeitung on November 24, 2010. Michael Meyer is chief customer officer at Deutsche Postbank AG in Bonn.

longevity. As a consequence, people reveal several behavioral problems when confronted with this decision problem, which can not be explained by traditional economic models, such as mean-variance theory (Markowitz, 1952b). For an overview of deviations from traditional economic theory in the retirement planning context see for example Mitchell and Utkus (2006), Oehler and Werner (2008), and Knoll (2010).

The aim of this chapter is therefore to investigate the role of behavioral portfolio selection in provision for old age. The model used here is a general version of behavioral portfolio theory by Shefrin and Statman (2000) and is employed to return data of retirement investments considered as most suitable by German households. To show the difference to traditional portfolio theory, I also compute mean-variance portfolios and find that they are not able to describe "real" investors behavior. Behavioral portfolios, in contrast, exhibit a large difference to mean-variance portfolios in terms of level of diversification, an impact of emotions since the security-minded (potential-minded) is the most conservative (aggressive) portfolio, and concentrated portfolios with a large proportion in only one secure asset and a small proportion in risky assets. I conclude that behavioral portfolio theory has remarkably power in understanding, describing and selecting retirement portfolios.

This chapter contributes to the existing literature in at least three ways: One, it provides a numerical rather than a probabilistic version of behavioral portfolio theory, that can be applied to a large amount of data and is therefore well-suited for financial planners and financial software. Two, simulated portfolios indicate that behavioral portfolio theory performs better in analyzing retirement investments than traditional theory. Three, related to Hoffmann et al. (2010), this chapter successfully demonstrates how behavioral portfolio theory can be applied to real financial problems.

The rest is organized as follows: Section 2.2 contains the theoretical background including the underlying decision framework, Shefrin and Statman's (2000) portfolio model, the deterministic equivalent version and a brief description of mean-variance theory. Section 2.3 descriptively analyzes responses of more than 10,000 German households on the question of suitability and ownership of retirement investments. Based on the results, I collect return data of the seven most suitable retirement investments. Section 2.4 presents optimal behavioral and mean-variance portfolios. Section 2.5 discusses the findings in the light of related studies, provides several conclusions and recommendations for financial planning.

2.2 Theoretical background

This section is divided into five subsections and starts in 2.2.1 with a brief introduction to rank-dependent utility theory. Based on this, subsection 2.2.2 presents SP/A theory, the underlying decision model. Subsection 2.2.3 shows how Shefrin and Statman (2000) combine SP/A theory and behavioral portfolio selection. Subsection 2.2.3 ends with providing a different formulation of Shefrin and Statman's (2000) single mental accounting behavioral portfolio theory. As this model is probabilistic, subsection 2.2.4 provides a deterministic equivalent version that can be treated numerically by mixed-integer linear programming. Finally, to relate the results obtained with behavioral portfolio theory to textbook theory, subsection 2.2.5 gives a brief introduction to the well-known mean-variance portfolio model.

2.2.1 Rank-dependent utility

The best way to understand rank-dependent utility, which was independently proposed by Quiggin (1982), Yaari (1987), Allais (1988) and Quiggin (1993), is to start with expected utility

$$EU(R_i) = \sum_{j=1}^{m} p_j u(R_{ij}) \,, \tag{2.1}$$

in which $u(R_{ij})$ is the utility of outcome j for lottery R_i and the p_j's are the outcomes' associated probabilities. As the primary goal of this dissertation is to study a portfolio selection problem, R_{ij} is interpreted as return of asset i at time j with $i = 1, \ldots, n$. Rank-dependent utility assumes that returns are ordered from lowest to highest, i.e. $R_{i,1} \leq \ldots \leq R_{i,m}$, and substitutes decision weights, $w(p)$, for probabilities,

$$RDU(R_i) = \sum_{j=1}^{m} w(p_j) u(R_{ij}) \,, \tag{2.2}$$

where

$$w(p_j) = h\left(\sum_{k=1}^{j} p_k\right) - h\left(\sum_{k=1}^{j-1} p_k\right) \tag{2.3}$$

and the function h transforms decumulative probabilities into the range $[0,1]$. Let $D_{ij} = prob(R_i \geq R_{ij})$ denote the decumulative probability distribution function and assuming utility is linear,[2] equation (2.2) can be equivalently written as

$$RDU(R_i) = \sum_{j=1}^{m} h(D_{ij})(R_{ij} - R_{i,j-1}) \tag{2.4}$$

with $R_{i,0} = 0$ for all i. In the separate appendix I show that equation (2.2) and (2.4) are formally the same. Note, in the special case where h is the identity function, rank-dependent utility collapses to expected utility. In expected utility, risk aversion is equivalent to concave utility, whereas in rank-dependent utility with linear utility function, however, risk aversion is equivalent to a convex probability transformation function, h (see Quiggin, 1993).

[2] The special case of linear utility is what Yaari (1987) assumes in his "dual theory of choice under risk".

2.2.2 SP/A Theory

SP/A theory developed by Lopes (1987) and Lopes and Oden (1999) is a dual criterion theory of choice under uncertainty in which the process of choosing between alternatives entails integrating two logically and psychologically separate criteria: The SP-criterion in which S stands for security and P stands for potential captures individuals' desire for both risk aversion (security) and risk loving (potential) and is closely related to Friedman and Savage's (1948) observation that individuals who buy insurance policies often buy lottery tickets at the same time. The A-criterion stands for aspiration and operates on a principle of stochastic control (Lopes and Oden, 1999, p. 291), that is individuals are assumed to assess the attractiveness of a lottery by the probability that the lottery fails to achieve an aspiration level.

In the SP/A framework, two emotions operate on the willingness to take risk: fear and hope[3]. Lopes and Oden (1999) model these emotions using rank-dependent utility. Specifically, the SP-criterion is modeled by equation (2.4) in which the probability transformation function has the following shape: Fear is what Lopes (1987) and Lopes and Oden (1999) call security-mindedness and leads individuals to overweight probabilities attached to unfavorable outcomes, that is modeled by[4]

$$h_S(D) = D^{1+q_S}\,, \quad q_S > 0\,. \tag{2.5}$$

Hope is what Lopes (1987) and Lopes and Oden (1999) call potential-mindedness and leads individuals to overweight probabilities attached to favorable outcomes, that is modeled by

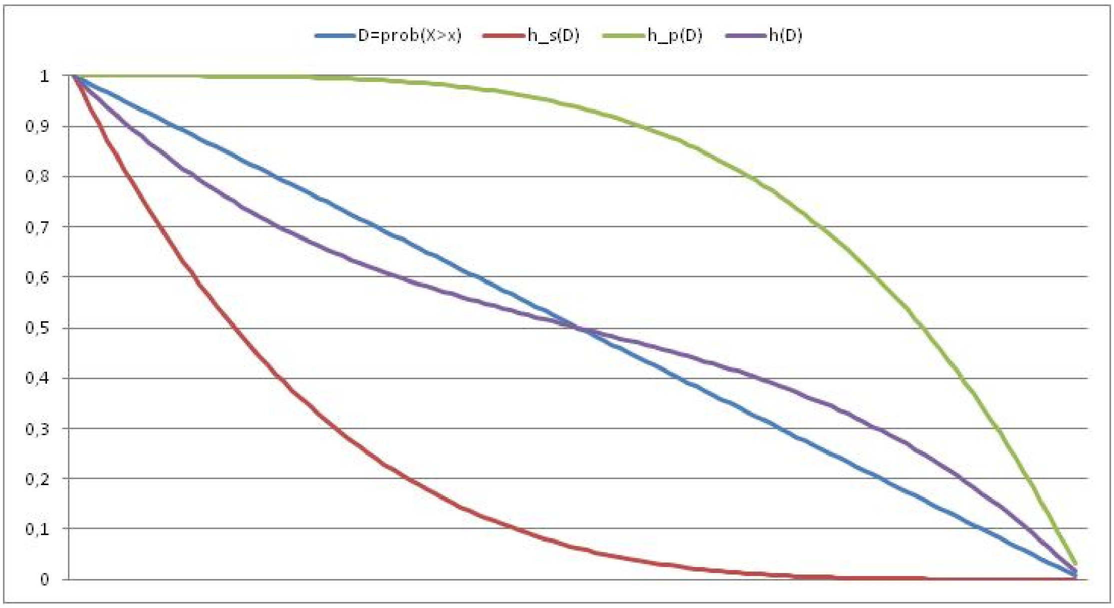
$$h_P(D) = 1 - (1-D)^{1+q_P}\,, \quad q_P > 0\,. \tag{2.6}$$

As we have learned from Friedman and Savage (1948) that fear and hope reside within individuals simultaneously, the final shape of the probability transformation function is a convex combination of h_S and h_P:

$$h(D) = \lambda h_S(D) + (1-\lambda) h_P(D)\,, \quad \lambda \in [0,1]\,. \tag{2.7}$$

If $\lambda = 1$, the decision maker is strictly security-minded. If $\lambda = 0$ the investor is strictly potential-minded. If $0 < \lambda < 1$, the decision maker is both with the magnitude of fear and hope depending on λ. Lopes and Oden (1999) call this behavior cautiously hopeful and also distinguish between λ for gains and for losses. The probability transformation function plays a key role in descriptive decision models such as SP/A theory and Cumulative Prospect Theory (Tversky and Kahneman, 1992). For a detailed discussion of the probability weighting function see for example Prelec (1998). Figure 2.1 shows decumulative probability distribution functions for security-minded, potential-minded and cautiously hopeful behavior.

[3]In related literature the terms pessimism and optimism are sometimes used instead.
[4]For sake of simplicity, subscripts are omitted at this point.



Note: The straight line represents the decumulative probability distribution function. $h_S(D)$ shows strictly security-minded behavior with parameters $q_S = 3$ and $\lambda = 1$, while $h_P(D)$ shows strictly potential-mindedness with parameters $q_P = 3$ and $\lambda = 0$. $h(D)$ illustrates cautiously hopeful behavior with parameters $q_P = q_S = 3$ and $\lambda = 0.5$. Outcomes are not specified, but rank from lowest to highest.

Figure 2.1: Transformed decumulative probability distribution functions

2.2.3 A stochastic behavioral portfolio model

In their behavioral portfolio theory, Shefrin and Statman (2000) develop a single and a multiple mental account version. As this chapter focuses on the specific goal of a secure retirement, that is organized in one mental account (see for example Das et al., 2010), the single mental account behavioral portfolio model is of particular interest. In the model, Shefrin and Statman (2000) combine SP/A theory and Telser's (1955) safety-first rule, that is, investors select portfolios which maximize expected return subject to the constraint that the probability of not achieving a threshold return is bounded. In this context the threshold return is interpreted as aspiration level. In contrast to Shefrin and Statman (2000) I provide a different definition of the single account version:

Problem 1.

$$\max_x x^T RDU(R) \quad s.t.$$

$$prob(x^T R < A) \quad \leq \quad \alpha\,, \tag{2.8}$$

$$\mathbf{1}^T x \quad = \quad 1\,, \tag{2.9}$$

$$x \quad \geq \quad 0\,, \tag{2.10}$$

where $\mathbf{1} = (1, \ldots, 1)^T \in R^n$, $x = (x_1, \ldots, x_n)^T$ being the vector of portfolio weights of n assets with $RDU(R) \in R^n$ being the behavioral mean return vector, A is the aspiration level of portfolio return and the maximum probability of the portfolio failing to reach A is α. The SP-criterion is captured in the objective of problem 1 by the vector of rank-dependent utilities of all n assets, i.e. equation (2.4) is applied to each of the n assets. The aspiration-criterion is captured in the safety-first constraint (2.8) by the parameter pair (A, α), where α denotes the probability of not achieving aspiration level A. Note, the feasible domain of problem 1 is only determined by the aspiration-criterion and not by the SP-criterion. Equation (2.9) is the fully invested or budget constraint and (2.10) is the short sale constraint, which was not explicitly imposed by Shefrin and Statman (2000), but, to my belief, appears essential in the context of retirement planning. Shefrin and Statman (2000, pp. 133) provide a solution to a simplified version of problem 1 under empirical distributed returns. Under the same assumption, the subsequent subsection provides a more general deterministic version of problem 1, which can be solved numerically by mixed-integer linear programming.

2.2.4 An equivalent deterministic behavioral portfolio model

Many papers dealing with safety-first constraints such as (2.8) assume normal distributed asset returns (Leibowitz and Henriksson, 1989; Leibowitz and Kogelman, 1991; Albrecht, 1993; Shefrin and Statman, 2000; Das et al., 2010; Singer, 2010), although the normal assumption is mainly rejected in empirical finance. Others employ probability inequalities such as the Chebyshev inequality to get and upper bound of the safety-first constraint (Roy, 1952; Telser, 1955; Kall and Mayer, 2005; Singer, 2010), although Chebyshev's inequality is a rather crude approximation. In this chapter, however, I assume the general case of empirical distributed returns, which is closely related to Shefrin and Statman (2000, Theorem 1, pp. 133). This assumption is particularly driven by the fact that SP/A theory is a discrete choice model. Using techniques from stochastic linear programming (Prékopa, 1995; Birge and Louveaux, 1997; Uryasev and Pardalos, 2001; Ruszczyński and Shapiro, 2003; Kall and Mayer, 2005), especially the transformation technique by Raike (1970), problem 1 can be restated as

Problem 2.

$$\begin{aligned}
\max_x x^T RDU(R) \quad & s.t. \\
x^T R_j + M(1-d_j) \quad & \geq \quad A\,, \quad j = 1,\ldots,m\,, && (2.11)\\
\sum_{j=1}^{m} p_j d_j \quad & \geq \quad 1-\alpha\,, && (2.12)\\
d_j \quad & \in \quad \{0,1\}\,, \quad j=1,\ldots,m\,, && (2.13)\\
\mathbf{1}^T x \quad & = \quad 1\,, && (2.14)\\
x \quad & \geq \quad 0\,, && (2.15)
\end{aligned}$$

where R_j is the realized return vector of all assets at time j, d_j's are binary auxiliary variables, p_j's are the probabilities from equation (2.1) and M is a large number (The separate appendix provides a precise definition of M.). Problem 2 is the deterministic equivalent of problem 1 and can be treated numerically. As problem 2 has n real $(x_1, \ldots, x_n)$ and m binary $(d_1, \ldots, d_m)$ decision weights it belongs to the class of mixed-integer linear optimization problems. To solve this problem, I use SCIP (**S**olving **C**onstraint **I**nteger **P**rograms) developed by Achterberg (2007), currently the fastest non-commercial mixed-integer programming solver.

2.2.5 Mean-variance portfolio model

In mean-variance portfolio theory (Markowitz, 1952b), investors select efficient portfolios on the basis of portfolio variance, σ^2, and expected return, μ. The model with short sale constraint states as follows

Problem 3.

$$\min_x \sigma^2(x^T R) \quad s.t.$$

$$x^T \mu \geq A\,, \tag{2.16}$$

$$\mathbf{1}^T x = 1\,, \tag{2.17}$$

$$x \geq 0\,, \tag{2.18}$$

in which $\sigma^2(x^T R)$ is the variance of the portfolio return and A is the minimum desired return and has a similar meaning as the aspiration level in behavioral portfolio theory. The optimal solution to problem 3 minimizes the portfolio variance and achieves at least return A. If short sales are allowed, i.e. constraint (2.18) is omitted, problem 3 can be solved analytically (see for example Huang and Litzenberger, 1993), otherwise by quadratic programming. For a detailed introduction to mean-variance theory see for example Markowitz (1970) or Elton et al. (2007).

2.3 Data

To get a better understanding of retirement portfolios in Germany I first analyze survey data of the Spiegel-Verlag survey “Soll und Haben” (Debit and Credit) 2004 in which 10,100 individuals in Germany were asked about their attitudes on the subject of investments. Among other things surveyed, subjects were asked about the suitability and their actual ownership of investments as provision for old age. Table 2.1 shows responses to both questions. The first two rows of table 2.1 reveal that the most popular investments as provision for old age are the endowment insurance and property used by owners. Concerning actual ownership, savings such as savings accounts, savings contracts/plans and savings accounts with special interest rates rank third. Hence, retirement portfolios are

	Suitability of investments as provisions for old age		Ownership of investments as provisions for old age	
	# persons	%	# persons	%
Endowment insurance	3630	34.2	2291	18.0
Property used by owners	3562	33.0	1873	16.7
Savings account	733	8.7	1391	16.2
Savings contract/ savings plan with regular deposits/ agreed term	1537	15.2	867	7.4
Savings account with special interest rate	971	10.6	664	7.1
Stocks and equity funds	1173	8.3	896	6.0
Building society savings contract	986	9.2	532	4.4
Private pension insurance	2172	18.6	618	4.1
Fixed-interests securities	1391	11.7	575	4.0
Occupational pension	1305	11.0	506	3.4
Property let to tenants	1843	16.3	396	2.5
Gold and other luxury goods	360	3.8	98	0.8

Note: Multiple responses were allowed. Investments are sorted in descending order by ownership.

Table 2.1: Suitability and ownership of investments for old age provision

dominated by relatively safe assets, which has also been documented by Börsch-Supan and Eymann (2000) who analyzed German household portfolios in the 1980s and 1990s. Only 6% and 4% hold equity and bonds, respectively. Notably, only 4.1% own a private pension insurance, although it ranks third in terms of suitability (18.6%). Moreover, table 2.1 documents the "self-control problem" (Mitchell and Utkus, 2006; Oehler and Werner, 2008), which supports the view of a wide divergence between individuals' desire and their actual behaviors. Among all assets, the percentages of suitability are higher than those of ownership. One reason for the divergence is that subjects are attracted by more assets as they actual own, which implies a lack of diversification in realized retirement portfolios. Clearly, I find that only 9.3% hold three or more assets and only 4% hold four or more assets.

Based on the above findings, I collect return data of investments suited for old-age provision:

1. Endowment insurance: To approximate returns of the endowment insurance, I use net returns of investments published by the German Insurance Association (GDV).

2. Property: Property used by owners and property let to tenants are approximated as one time series by the index for housing calculated by the Bundesbank, based on data provided by BulwienGesa AG.

3. Savings accounts and other investments with banks: Savings accounts, savings contracts/savings plans with regular deposit/agreed terms are approximated as one time series following Westerheide's (2005) methodology. I extract following time series: The interest rates for savings deposits are composed of deposit rates of banks in Germany with minimum rates of return and with agreed notice of three months. The average rates are calculated as unweighted arithmetic means from the interest rates reported to be within the spread. The spread is ascertained by eliminating the reports in the top 5% and the bottom 5% of the interest rate range. Including rates for savings deposits, I consider the difference between the Bundesbank's interest rate statistics and the new European Central Bank's statistics, with the latter started at January 2003. Moreover, I consider savings bonds with fixed maturity of four years, overnight money, savings with/without contract period for varying investment volumes, and fixed-term deposits. All time series are extracted from Deutsche Bundesbank. A representative time series is obtained by calculating the arithmetic mean of all time series mentioned above.

4. Stocks: Investments in stocks, equity funds and other investment funds are represented by the returns of the DAX performance index.

5. Building society savings contract: To approximate returns of the building society savings contract I follow Statistisches Bundesamt Deutschland (Federal Office of Statistics) and use yields on debt securities outstanding issued by residents, also published by Deutsche Bundesbank.

6. Bonds: Fixed-interests securities, private and occupational pensions are approximated as one time series by the returns of the REX performance index, which measures the performance of German government bonds.

7. Gold and other luxury goods: Returns of gold and other luxury goods are approximated by the returns of gold traded at Frankfurt stock exchange.

In total, I collect monthly return data for all assets from January, 1998 to December, 2007, which comprises ten years. The returns of the endowment insurance and property, for which only yearly data is available, are divided equally into monthly returns. Table 2.3 shows summary statistics of the dataset.

	Min	Max	Mean	S.D.	Skewness	Kurtosis
Endowment insurance	0.3875	0.6317	0.4839	0.1011	0.6132	-1.4415
Property	-0.0930	0.1479	0.0330	0.0684	-0.1608	-0.6400
Savings accounts and other investments with banks	0.1557	0.3548	0.2423	0.0550	0.3222	-0.9532
Stocks	-29.3327	19.3738	0.5341	6.8776	-0.9845	3.2687
Building society savings contract	0.2417	0.4667	0.3522	0.0579	0.1425	-0.9108
Bonds	-1.9184	2.4964	0.3889	0.8708	-0.1866	-0.4473
Gold and other luxury goods	-11.8484	14.7314	0.6150	3.4697	0.2290	2.5230

Table 2.2: Summary statistics of monthly return distributions of retirement assets

2.4 Results

Using the ten year dataset described in the previous section, problem 2 is solved by mixed-integer linear programming. The SP-criterion is determined by the parameters q_S that measures the strength of security-mindedness, q_P that measures the strength of potential-mindedness, and λ that determines the strength of security relative to potential. Note, if q_S and q_P are set to zero, problem 1 and 2 collapses to Telser's (1955) model which selects portfolios that maximize expected return subject to the safety-first constraint (2.8). The aspiration-criterion is determined by the parameters A and α, where α denotes the probability of not achieving aspiration level (threshold return) A. As the SP-criterion only impacts the objective function, the feasible domain of problem 1 and 2, respectively, is solely determined by the aspiration-criterion, i.e. fear and hope have no impact on the feasibility of problem 1 and 2, respectively. Figure 2.2 shows feasible (A, α) combinations for the ten year dataset in a reasonable range. Figure 2.2 documents a positive relation between A and α in the sense that when α decreases A must also decrease to preserve feasibility.

The remainder of this section documents optimal portfolio weights of four different scenarios:

1. Security-minded behavior: In this scenario, the investor is assumed to be strictly security-minded with parameters $q_S = 0.05$ and $\lambda = 1$. To the best of my knowledge it exists no estimate for q_S based on real data for this scenario.

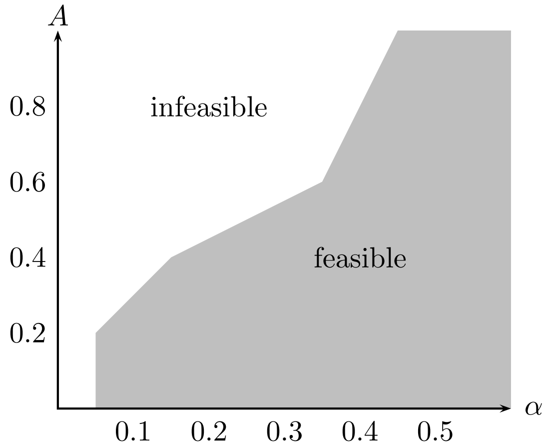


Figure 2.2: Feasible (A, α)-combinations

2. Potential-minded behavior: In this scenario, the investor is assumed to be strictly potential-minded with parameters $q_P = 0.05$ and $\lambda = 0$. To the best of my knowledge it exists no estimate for q_P based on real data for this scenario.

3. Cautiously-hopeful behavior: In this scenario, I use the parameter estimates by Lopes and Oden (1999), which are $q_S = q_P = 1.053$ and $\lambda_{gains} = 0.505$ for gains and $\lambda_{losses} = 0.488$ for losses. Undocumented results indicate that cautiously hopeful behavior modeled by the probability weighting function of Cumulative Prospect Theory (Tversky and Kahneman, 1992) does not significantly differ from that modeled by SP/A theory. As the primary goal of this chapter is not the probability weighting function, I do not document these results, but they are available upon request.

4. Mean-variance behavior: In order to compare results from behavioral portfolio theory with those of classic portfolio theory, I also calculate optimal portfolios using problem 3. The aspiration level, A, is the same in both problem 2 and 3.

Figure 2.3 contains optimal portfolios for all scenarios with $A = 0.2$ and various α. Note, as the parameter α is not part of the mean-variance model, all panels of figure 2.3 show the same mean-variance portfolio.[5] Among all behavioral portfolios, a large concentration of wealth in the endowment insurance can be observed; in the security-minded scenario it reaches even 100%. All other secure investments (property, savings and building society savings contract) are uninvested by all behavioral portfolios. The reason for

[5] The global minimum variance portfolio is $(\mu, \sigma) = (0.185, 0.050)$. Thus, choosing $\mu = A = 0.2$ delivers the optimal portfolio lying on the upper branch of the mean-variance frontier and is therefore efficient in mean-variance sense. Any μ or A below the global minimum variance portfolio delivers optimal but inefficient portfolios.

this result is that the return of the endowment insurance is the highest among all secure investments that meet the aspiration level. This pattern of behavioral portfolios is the same as that documented by Shefrin and Statman (2000), that is a combination of risk-free return (endowment insurance) and a lottery ticket (the proportion invested in risky assets). Mean-variance portfolios, in contrast, invest in all secure assets, that is because the key element of mean-variance theory is exploiting correlations between assets. The structural difference between behavioral and mean-variance portfolio theory leads also to the result that mean-variance portfolios are more diversified than behavioral portfolios, a pattern which I observe for all documented and undocumented results.

Among all behavioral portfolios the security-minded is the most conservative portfolio as it invests 100% in the endowment insurance, whereas cautiously hopeful and potential-minded portfolios invest a small fraction in risky assets. As can be seen from panel 2.3(b) and 2.3(c) the potential-minded portfolio is the most aggressive behavioral portfolio as it invests the largest fraction in risky assets. The reason why the mean-variance portfolio has its largest position in savings, is that the mean return of savings, which is 0.2423, meets the aspiration level of $A = 0.2$, and the standard deviation of savings is the lowest among all assets. As the objective in mean-variance problem 3 is to minimize portfolio variance, savings take the largest fraction. Note further, as α increases the proportion invested in safe assets decreases while the proportion in risky assets increases.

Figure 2.4 contains optimal portfolios for all scenarios for $A = 0.4$ and $\alpha = 0.2$ (panel 2.4(a)) and $\alpha = 0.3$ (panel 2.4(b)). Note, it exists no feasible solution for $\alpha = 0.1$, which can also be seen from figure 2.2. For all behavioral portfolios, the same pattern as for $A = 0.2$ can be observed. But, the mean-variance portfolio is not invested in property and savings which is due to the fact that the mean return of property (0.0330) and savings (0.2423) does not achieve the aspiration level (threshold return) of $A = 0.4$, respectively.

2.5 Discussion

This chapter investigates the role of behavioral portfolio theory in provision for old age in Germany. The behavioral portfolio model implemented here is a general version of Shefrin and Statman's (2000) single mental account model which combines SP/A theory and Telser's (1955) safety-first rule. Assuming empirical distributed returns the model is then transformed in its deterministic equivalent, which is solved numerically by mixed-integer linear programming. Using return data of the seven most suitable German retirement investments, I simulate portfolios for security-mindedness, potential-mindedness, cautiously hopeful and mean-variance behavior, where the latter serves by way of comparison.

The main findings indicate (i) an impact of emotions on behavioral portfolios, since the security-minded (potential-minded) is the most conservative (aggressive) portfolio;

(ii) concentrated behavioral portfolios with a large proportion in only one secure asset (endowment insurance) and a small proportion in risky assets; and (iii) mean-variance portfolios which are more diversified than behavioral portfolios. To relate the results to others, first, I observe the same pattern of behavioral portfolios as documented by Shefrin and Statman (2000), that is a combination of risk-free return (endowment insurance) and a lottery ticket (the proportion invested in risky assets). This pattern is also similar to that in Shiller's (2005) life-cycle portfolios which allocate account balances between stocks and bonds. Second, the large concentration in the endowment insurance is consistent with my own empirical findings in section 2.3 and those by Börsch-Supan and Eymann (2000) for German household portfolios in the 80s and 90s. Third, the lack of diversification in simulated behavioral portfolios has been widely shown in empirical studies on private portfolios, among them Blume and Friend (1975), Kelly (1995), Tyynelä and Perttunen (2002) and Goetzmann and Kumar (2008). Fourth, since mean-variance theory exploits the correlation structure of all assets, it is theoretically not surprising that mean-variance portfolios invest in more than one "riskless" asset. But, allowing for the fact that individual investors ignore correlations (Kroll et al., 1988b; Siebenmorgen and Weber, 2003), this result may practically appear puzzling to most individual investors. Taken as a whole, I conclude that behavioral portfolio theory has remarkably power in understanding, describing and selecting retirement portfolios.

Nevertheless, I am completely aware of at least four shortcomings. One, the fact that I was not able to obtain precise return data for some retirement investments - predominantly secure investments - may induce an approximation bias. Two, as I assume empirical distributed returns, the deterministic model is of mixed-integer type, for which computing time increases disproportional as input size increases. Three, retirement planning involves long time horizons. Thus, one could argue that dynamic models are better capable in the domain of life cycle saving. Four, according to the SP-criterion of SP/A theory, probability distributions of returns are only transformed in the objective function, whereas they are treated as raw probabilities in the safety-first constraint (2.8). Consequently, return distributions in the constraints should also be transformed.

These shortcomings leave much room for future research. Another extension involves the aspiration level, A, which is assumed to be fixed. However, in long time decisions investors' aspiration may vary or they wish a benchmark return such as the return of the S&P500 as aspiration level. In such cases, the aspiration level can be modeled as a random variable (see for example Singer (2010) and chapter three of this dissertation). Instead of modeling emotions such as hope and fear, rank-dependent utility can also model the availability heuristic (Tversky and Kahneman, 1973) in which people predict the likelihood of an outcome based on how many outcomes of each type (return increase or return decrease) come readily to mind. As recent outcomes are more available than those many years ago, the availability heuristic is modeled by rank-dependent utility with outcomes sorted by date rather than from lowest to highest.

The results presented here have several implications for professional advisers and for financial planning software. As "‘[f]inancial planners have a responsibility to guide clients in a manner consistent with how those clients are likely to behave" (Mitchell, 2010, p. 5), this model helps them to understand how their clients behave and guide them on how they should behave, i.e. give a suitable advice. Since this model can simulate several behavioral scenarios it can be used as a powerful tool in institutional and private portfolio management software. It can also be used as an "auto-pilot" that Mitchell and Utkus (2006, p.92) suggest as a default plan to prevent people from inertia and encourages them to more retirement saving.

2.A Appendix

2.A.1 Proof of equation (2.4)

Proposition. Let $R_1 \leq \ldots \leq R_m$ be ordered outcomes of the random variable R, p_j's are the outcomes' associated probabilities and $D_j = prob(R \geq R_j)$ denotes the decumulative probability distribution function. Then the expected value, $E(R)$, can be written as

$$E(R) = \sum_{j=1}^{m} D_j (R_j - R_{j-1})\,, \tag{2.19}$$

with $R_0 = 0$.

Proof: As outcomes are sorted from lowest to highest, the decumulative probability distribution function can be restated as $D_j = \sum_{i=j}^{m} p_i$. Then, equation (2.19) can be written as

$$\sum_{j=1}^{m}\sum_{i=j}^{m} p_i (R_j - R_{j-1}) = \sum_{j=1}^{m} R_j \sum_{i=j}^{m} p_i - \sum_{j=1}^{m} R_{j-1} \sum_{i=j}^{m} p_i\,.$$

Extracting the term for $j = m$ from the first addend and rearranging indices in the second addend yields

$$R_m p_m + \sum_{j=1}^{m-1} R_j \sum_{i=j}^{m} p_i - \sum_{j=1}^{m-1} R_j \sum_{i=j+1}^{m} p_i\,,$$

which can be written as

$$R_m p_m + \sum_{j=1}^{m-1} R_j \underbrace{\left(\sum_{i=j}^{m} p_i - \sum_{i=j+1}^{m} p_i \right)}_{=p_j} = \sum_{j=1}^{m} p_j R_j\,.$$

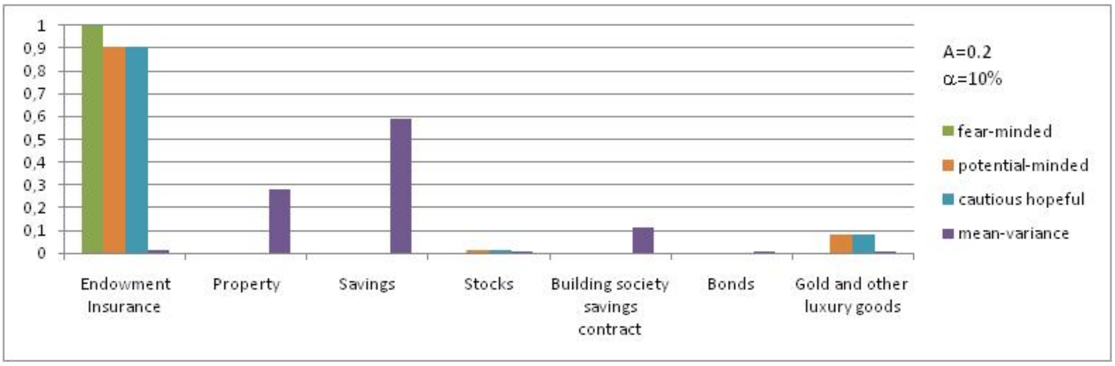
$\square$

2.A.2 A precise definition for M in problem 2

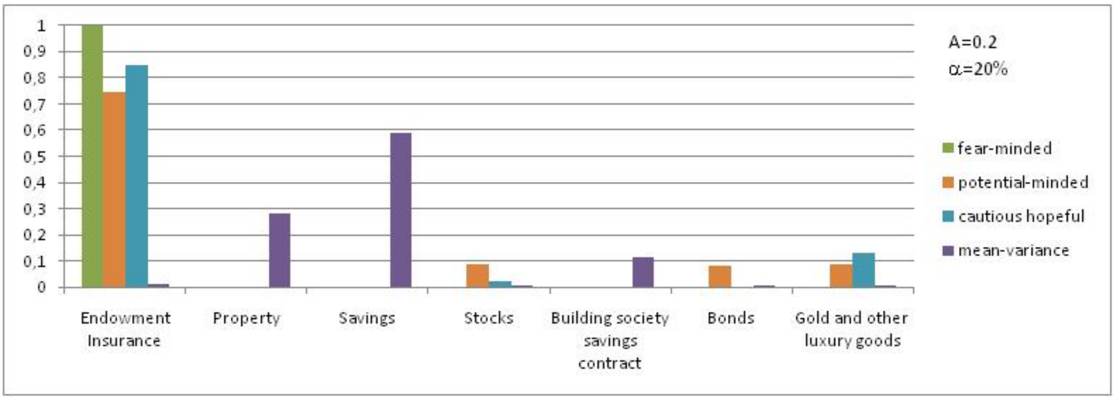
If $d_i = 1$, M drops out. If $d_i = 0$, M has to be chosen such that constraint (2.11) always holds. The worst can happen is when x takes one for the lowest outcome

$$R_{\min} := \min_{\substack{j=1,\dots,m \\ i=1,\dots,n}} R_{i,j} \,.$$

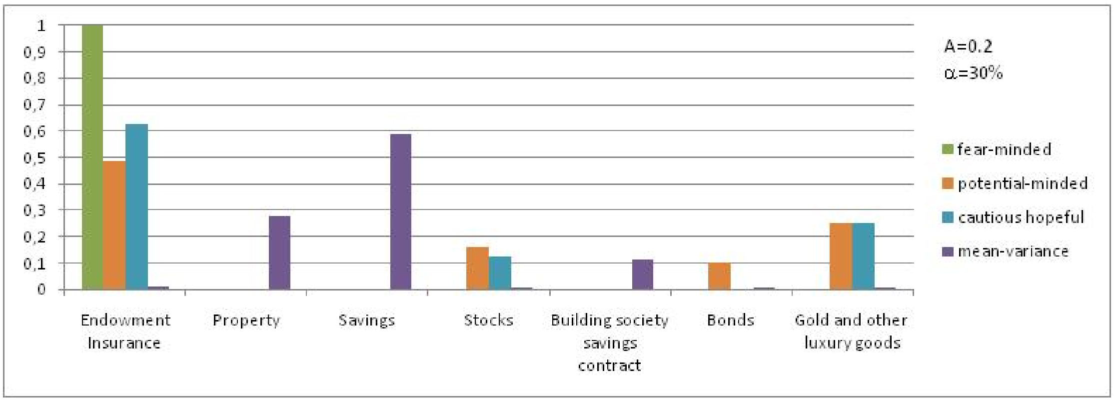
Thus, the left-hand side of (2.11) takes the value $R_{\min} + M$. To ensure feasibility, M has to be chosen such that $M \geq A - R_{\min}$. In our dataset $R_{\min}$ is -29.33. Hence, M must be at least $A + 29.33$.



(a) $\alpha = 10\%$

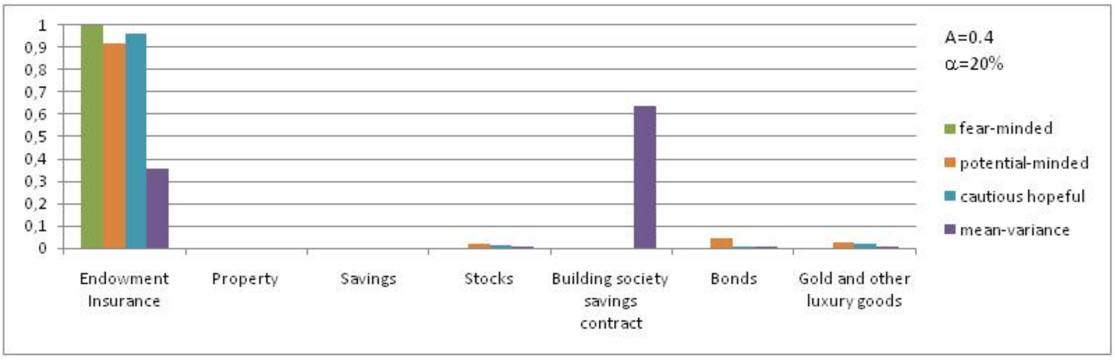


(b) $\alpha = 20\%$

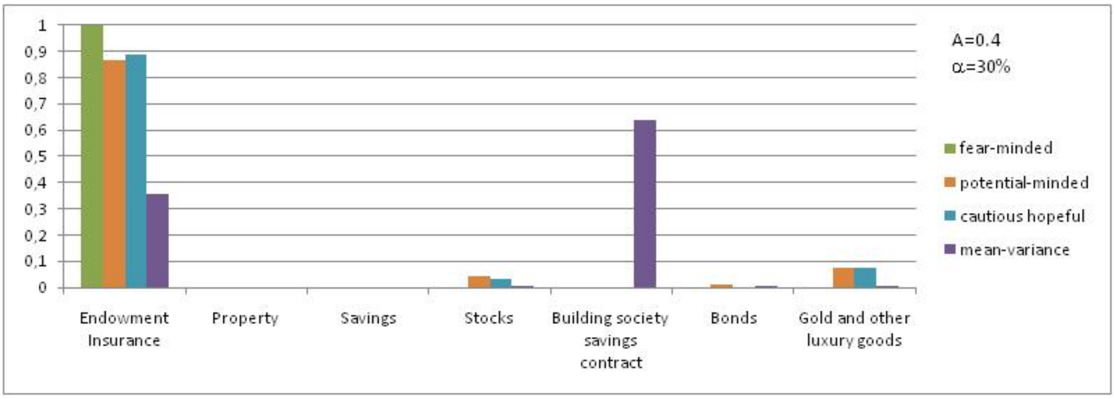


(c) $\alpha = 30\%$

Figure 2.3: Optimal portfolios for security-minded, potential-minded, cautiously hopeful and mean-variance behavior for $A = 0.2$



(a) $\alpha = 20\%$



(b) $\alpha = 30\%$

Figure 2.4: Optimal portfolios for security-minded, potential-minded, cautiously hopeful and mean-variance behavior for $A = 0.4$

3 Safety-First Portfolio Optimization: Fixed versus Random Target

This chapter is based on Singer (2010).

> *"There's no way to determine the probability of a loss (risk measure) without a model."*
>
> *Robert A. Jarrow, 2011*[1]

3.1 Introduction

Portfolio optimization under the safety-first criterion is concerned with maximizing the expected portfolio return, while, simultaneously, the probability of failing to achieve a specified (fixed) target must fall below a critical level and has its origins in the early papers of Roy (1952), Telser (1955) and Kataoka (1963). The safety-first risk measure is commonly expressed by a probability statement as $P(Z < T) \leq \alpha$ where Z is a random variable, e.g. portfolio return, T is a fixed target, e.g. a minimum desired portfolio return and α is a critical level on the probability of failing to achieve the target.

A comparative advantage of the safety-first criterion over deviation risk measures, such as the variance, is its consistency with the way investors perceive risk (see for example Atwood et al., 1988; Harlow, 1991; Brogan and Stidham, 2005). Empirically, this is shown for example in Lopes (1987), Kroll et al. (1988a), DeBondt (1998), Lopes and Oden (1999) and Neugebauer (2008). These behaviorally appealing feature have made the safety-first criterion attractive for behavioral portfolio theory, see for example Shefrin and Statman (2000), Singer (2009) and the recent paper by Das et al. (2010), in

[1] At his keynote speech at the annual meeting of the Southwestern Finance Association 2011 in Houston, Texas. Robert A. Jarrow is Professor of investment management, finance and economics at Cornell University.

which the authors transform the utility based interpretation of mean-variance portfolio theory to the more appealing target based interpretation of safety-first portfolio theory under the assumption of normal distributed asset returns. More generally, Kalin and Zagst (1999) show the equivalence of mean-variance and safety-first portfolio theory for a wide class of probability distributions.

What all the above cited papers about the safety-first model have in common is the assumption of a fixed target T, which, however, leads to significant conceptual disadvantages: Suppose an investment fund which seeks to achieve a fixed return T for the next period. According to this target the fund manager purchases and sells assets. But, what happen when the market return within the next period is greater than T? The fund would perform rather poorly. This situation could have been avoided if the manager had reallocated the assets according to the expected performance of the market, which is common practice in passive portfolio management. Or, suppose a fund which seeks to outperform the market (active portfolio management), i.e. the target for the next investment period is the expected market performance plus some extra return.[2] Or, from an individual perspective the target may not even be known. Many individuals have the target of "being successful", but only a very few know precisely which selection of money, leisure time, culture etc. must be attained to achieve this target (Bordley and LiCalzi, 2000). There are, thus, several situations in which an unknown or random target seems to be a more suitable choice. But, does in all these situations a random target lead to better results in terms of higher expected returns? Or, are there situations in which a fixed target should be the preferred choice? As this chapter pays particular attention to a portfolio optimization model, the question is, which target choice leads ceteris paribus to larger optimal expected portfolio returns? This chapter suggests a first answer to these questions.

In detail, assuming normal distributed asset returns, we know for example from Kalin and Zagst (1999) that the (probabilistic) safety-first risk measure can easily be transformed to a deterministic risk measure in terms of standard deviation. In section 3.2, I use this result to transform the safety-first portfolio model to an equivalent deterministic version, which is general enough to consider both, fixed and random targets. In section 3.3, which contains the main results, I compare optimal expected portfolio returns of the fixed and random target strategy and obtain following results: The random target strategy outperforms the fixed target strategy, if the portfolio return and the random target are positively correlated and riskless investing is prohibited, whereas the fixed target strategy outperforms the random target strategy, if the portfolio return and the random target are not positively correlated and riskless investing is allowed. By providing empirical evidence for the German stock market in section 3.4, I point out that the first

[2] For financial risk management with benchmarking see for example Basak et al. (2006), Browne (2000) and Gaivoronski et al. (2005).

case, in which the portfolio return and the random target are positively correlated, is practically most relevant. As the normal distribution is a good starting point analyzing the safety-first model (see Leibowitz and Henriksson, 1989; Leibowitz and Kogelman, 1991; Das et al., 2010), but typically violated in empirical finance, I relax this assumption in section 3.5 and show, using a well-known approximation, that all results from section 3.3 remain the same. Section 3.6 offers a discussion and concludes the chapter.

3.2 Safety-first portfolio optimization with normal distributed asset returns

Consider an investment universe of n different financial assets with $\mathbf{R} := (R_1, \ldots, R_n)^T$ presenting the vector of random asset returns. A portfolio where short sales are prohibited is defined as a vector $\mathbf{x} \in [0,1]^n$ with x_i being the proportion invested in asset i and the proportions sum to one, which is also known under the "fully invested constraint". Let the product $\mathbf{x}^T\mathbf{R}$ be the random portfolio return, T be a fixed or random target and α be a critical probability, then the safety-first portfolio model which maximizes the expected portfolio return subject to a safety-first constraint can be expressed as

Problem 4.

$$\begin{aligned} \max_{\mathbf{x}\in[0,1]^n} \mu(\mathbf{x}^T\mathbf{R}) &\quad s.t. \\ P(\mathbf{x}^T\mathbf{R} < T) &\leq \alpha\,, \qquad (3.1) \\ \mathbf{1}^T\mathbf{x} &= 1\,. \qquad (3.2) \end{aligned}$$

For a numerical treatment of problem 4, it is useful to provide a deterministic rather than a probabilistic expression of the safety-first constraint (3.1), which can be easily achieved under the normal assumption (see for example Kalin and Zagst, 1999). Many other papers studying the safety-first framework, among them Leibowitz and Henriksson (1989), Leibowitz and Kogelman (1991), Albrecht (1993) and Das et al. (2010), assume normal distributed asset returns. Define therefore $Z := \mathbf{x}^T\mathbf{R} - T$ with $Z \sim \mathcal{N}(\mu(Z), \sigma^2(Z))$ as a normal distributed random variable with expected value $\mu(Z)$ and variance $\sigma^2(Z)$. Then, employing the common textbook transformation for the normal distribution, safety-first constraint (3.1) can be equivalently expressed as

$$\begin{aligned} P(Z<0) \leq \alpha &\quad \Leftrightarrow \\ P\left(\frac{Z-\mu(Z)}{\sigma(Z)} < -\frac{\mu(Z)}{\sigma(Z)}\right) \leq \alpha &\quad \Leftrightarrow \\ \Phi\left(-\frac{\mu(Z)}{\sigma(Z)}\right) \leq \alpha &\quad \Leftrightarrow \\ \Phi^{-1}(\alpha)\sigma(Z) + \mu(Z) \geq 0\,, & \qquad (3.3) \end{aligned}$$

where $\Phi^{-1}(\alpha)$ is the α-quantile of the standard normal distribution and $\sigma(Z)$ is the standard deviation obtained from drawing the positive square root of $\sigma^2(Z)$. The following theorem, found in a slightly modified version in Kall and Mayer (2005, pp. 103-106) and Ruszczyński and Shapiro (2003, pp. 10), provides precise expressions for $\mu(Z)$ and $\sigma^2(Z)$ (and $\sigma(Z)$).

Theorem 1. *Let $\mathbf{R} = (R_1, \ldots, R_n)^T$ be n-variate normal distributed with expected value vector $\boldsymbol{\mu}$ and covariance matrix Σ, $\mathbf{R} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$, and T be univariate normal distributed with expected value $\mu(T)$ and variance $\sigma^2(T)$, $T \sim \mathcal{N}(\mu(T), \sigma^2(T))$, then the random variable $Z = \mathbf{x}^T\mathbf{R} - T$ is univariate normal distributed with expected value $\boldsymbol{\mu}^T\mathbf{x} - \mu(T)$ and variance $\|L^T\mathbf{x} - \mathbf{b}\|^2$, where $L \in \mathbb{R}^{n\times r}$ and $\mathbf{b} \in \mathbb{R}^r$ are obtained from the factorization of the covariance matrix $cov(\mathbf{R}^T, T)$ and $\|.\|$ denotes the Euclidean norm.*

Proof: A detailed proof, following Kall and Mayer (2005, pp. 103-106), and a remark on the numerical treatment of the factorization of the covariance matrix are provided in the separate appendix.

$\square$

Note, the variance, $\sigma^2(Z)$, can be decomposed as

$$\sigma^2(Z) = \|L^T\mathbf{x} - \mathbf{b}\|^2 = (L^T\mathbf{x} - \mathbf{b})^T(L^T\mathbf{x} - \mathbf{b}) = \mathbf{x}^T \underbrace{LL^T}_{=\Sigma} \mathbf{x} - 2(L\mathbf{b})^T\mathbf{x} + \mathbf{b}^T\mathbf{b}\,,$$

where the first addend in the last equation is the variance of $\mathbf{x}^T\mathbf{R}$. The middle addend is two times the covariance between $\mathbf{x}^T\mathbf{R}$ and T with $L\mathbf{b} \in \mathbb{R}^n$ being the cross-covariance vector between $\mathbf{R}$ and T. The third addend is the variance of T.

Applying then the deterministic safety-first constraint (3.3) with $\mu(Z) = \boldsymbol{\mu}^T\mathbf{x} - \mu(T)$ and $\sigma(Z) = \|L^T\mathbf{x} - \mathbf{b}\|$, obtained from Theorem 1, problem 4 can be reformulated as

Problem 5.

$$\begin{aligned} \max_{\mathbf{x}\in[0,1]^n} \mathbf{x}^T\boldsymbol{\mu} &\quad s.t. \\ \Phi^{-1}(\alpha)\|L^T\mathbf{x} - \mathbf{b}\| + \boldsymbol{\mu}^T\mathbf{x} &\quad \geq \quad \mu(T)\,, \\ \mathbf{1}^T\mathbf{x} &\quad = \quad 1\,. \end{aligned} \tag{3.4}$$

Problem 5 is the deterministic equivalent of problem 4 and can be solved numerically. It is linear in its objective but quadratic in its constraints.[3] It therefore relates to the class of quadratic optimization problems. If $\alpha \in (0, 1/2)$, the deterministic safety-first constraint (3.4) is concave, which was first shown by Kataoka (1963). Under this assumption, problem 5 can easily be solved by concave optimization methods.[4] As α

[3] More precisely, (3.4) is a second-order cone constraint, also called ice-cream cone or Lorentz cone. For a detailed discussion on that, consult Kall and Mayer (2005, pp. 273-276) and the references therein.

[4] If the feasible domain is concave and not empty, there exists a unique maximum. For an overview of convex optimization see Boyd and Vandenberghe (2007)

represents the maximum probability of failing to achieve the target and is moreover specified by the investor herself, it is usually chosen to be low, e.g. 5% or 10%. Thus, the restriction of α not exceeding 50% does not limit the practical value of the results. I therefore stick throughout this chapter to this assumption.

Note, modeling a riskfree asset j can be easily achieved by setting the j-th row of L to the zero-vector. Then, we have $R_j = \mu_j$ with μ_j being the riskfree rate. Analogously, a fixed target instead of a normal distributed target can be achieved by setting $\mathbf{b}$ to the zero-vector. Then, the correlation between the target and the asset returns is zero and T collapses to a fixed target with value $\mu(T)$.

3.3 Fixed versus random target

This section concerns the comparison of the fixed target strategy (S_1) and the random target strategy (S_2). The fixed and the random target is denoted as T_1 and T_2, respectively. Both strategies face the same investment universe and the same estimates for the expected returns, $\boldsymbol{\mu} = (\mu(R_1), \ldots, \mu(R_n))^T$, and covariances, Σ. Additionally, the crosswise covariations between the random target and the asset returns are denoted by the n-vector $(L\mathbf{b}) = cov(\mathbf{R}, T_2)$. Note, the $n \times r$ matrix L and the r-vector $\mathbf{b}$ are obtained from the factorization of the covariance matrix $cov(R_1, \ldots, R_n, T_2)$ (see Theorem 1).

As assumed in Theorem 1, T_2 is normal distributed with expected value $\mu(T_2)$ and variance $\sigma^2(T_2)$. However, it appears reasonable to assume, $\mu(T_2) > T_1$, so that the expected target return of S_2 is greater than T_1. This is due to the uncertainty about T_2, making S_2 riskier than S_1. This extra risk must thus be compensated by a greater expected target return. But, to keep the results as simple as possible, I assume that $\mu(T_2) = T_1$. Nevertheless, all results presented here can be straightforwardly modified such that $\mu(T_2) > T_1$, for example by defining $\mu(T_2) := T_1 + \varepsilon$, $\varepsilon > 0$ and adapting the calculations.

The following two subsections evaluate the differences in the performance of S_1 and S_2 by comparing optimal expected portfolio returns. Subsection 3.3.1 discusses the practical more interesting case, in which the random target and the asset returns are crosswise positively correlated, whereas subsection 3.3.2. investigates the opposite case.

3.3.1 The positive correlated case

Consider strategy S_2, in which a portfolio is managed subject to the performance of a benchmark, such as a stock index like the S&P500, without directly investing into the benchmark.[5] There are at least three situations where this appears reasonable:

[5] A direct investment in an index can be obtained by purchasing an exchange traded fund (ETF) on the index, which explicitly tracks the index and is, moreover, attractive because of low transactions costs and tax efficiency.

First, the portfolio manager seeks to outperform the benchmark, which is typical for actively traded funds. Second, the manager seeks to invest in a foreign market without suffering from foreign exchange risk. This can be realized by tracking or outperforming a foreign representative market index, for instance the S&P500 for the U.S., by a domestic portfolio. Third, the latter situation can also be derived from an individual perspective, where a skilled private investor is attracted by the performance of a foreign market, but does not want to invest directly into the market. The individual investor rather seeks to track the performance of the foreign market by only investing in domestic stocks. This situation is derived from a behavioral phenomenon called *home bias*, which was first documented by French and Poterba (1991).

Reducing all these examples to the stock market, they have in common to track or outperform one stock market by investing in similar but different stocks from another market. I therefore assume that asset returns are positively correlated with the benchmark return, i.e. the cross-covariance vector between $\mathbf{R} = (R_1, \ldots, R_n)^T$ and T_2 is

$$(L\mathbf{b}) = cov(\mathbf{R}, T_2) > \mathbf{0}\,. \tag{3.5}$$

This assumption is generally justified for most of the risky financial assets, in particular for stock markets. Section 3.4 provides empirical evidence that even a stronger version of (3.5) is justified for the stock market. Remark that assumption (3.5) excludes riskless investing because it does not allow for a zero covariance. As a whole, I compare S_1 with fixed target, T_1, and S_2 with normal distributed target, $T_2 \sim \mathcal{N}(T_1, \sigma^2(T_2))$. Applying Theorem 1 yields

$$\begin{aligned} Z_1 &= \mathbf{x}^T\mathbf{R} - T_1 \quad \text{with} \quad Z_1 \sim \mathcal{N}(\boldsymbol{\mu}^T\mathbf{x} - T_1, \|L^T\mathbf{x}\|^2) \\ Z_2 &= \mathbf{x}^T\mathbf{R} - T_2 \quad \text{with} \quad Z_2 \sim \mathcal{N}(\boldsymbol{\mu}^T\mathbf{x} - T_1, \|L^T\mathbf{x} - \mathbf{b}\|^2)\,. \end{aligned}$$

Note, the expected values of Z_1 and Z_2 coincide, but the variances differ. Thus, in the normal distributed case, the question, whether S_1 outperforms S_2 or vice versa, is simply the question of comparing variances. The following theorem shows that, under a weak additional assumption, the variance of Z_2 is smaller than the variance of Z_1 and therefore, S_2 outperforms S_1. The proof of Theorem 2 makes use of

Lemma 1. *If*

$$cov(R_i, T_2) > \frac{1}{2}\sigma^2(T_2)\,, \quad i = 1, \ldots, n\,, \tag{3.6}$$

holds, then for any critical probability $\alpha \in (0, 1/2)$ the following inequality is true:

$$\Phi^{-1}(\alpha)\sigma(Z_2) + \mu(Z_2) > \Phi^{-1}(\alpha)\sigma(Z_1) + \mu(Z_1)$$

Proof: From the fully invested constraint (3.2) together with (3.6) follows

$$cov(\mathbf{R}, T_2)^T\mathbf{x} > \frac{1}{2}\sigma^2(T_2)\,,$$

which is equivalent to

$$\begin{aligned}
0 &> -2cov(\mathbf{R},T_2)^T\mathbf{x}+\sigma^2(T_2) \quad \Leftrightarrow \\
\|L^T\mathbf{x}\|^2 &> \|L^T\mathbf{x}\|^2-2(L\mathbf{b})^T\mathbf{x}+\mathbf{b}^T\mathbf{b} \quad \Leftrightarrow \\
\|L^T\mathbf{x}\| &> \|L^T\mathbf{x}-\mathbf{b}\| \quad \Leftrightarrow \\
\Phi^{-1}(\alpha)\|L^T\mathbf{x}\|+\boldsymbol{\mu}^T\mathbf{x}-T_1 &< \Phi^{-1}(\alpha)\|L^T\mathbf{x}-\mathbf{b}\|+\boldsymbol{\mu}^T\mathbf{x}-T_1 \quad \Leftrightarrow \\
\Phi^{-1}(\alpha)\sigma(Z_1)+\mu(Z_1) &< \Phi^{-1}(\alpha)\sigma(Z_2)+\mu(Z_2)\,.
\end{aligned}$$

Notice that the second last inequality reverses because $\Phi^{-1}(\alpha)<0\ \forall\alpha\in(0,1/2)$.

□

Theorem 2. *Provided (3.6) holds, for any critical probability $\alpha\in(0,1/2)$ the optimal expected portfolio return of S_2 is larger or equal than the optimal expected portfolio return of S_1.*

Proof: It is sufficient to show that the set of feasible portfolios of S_1, denoted as $\mathcal{F}_1$, is a subset of the feasible domain of S_2, denoted as $\mathcal{F}_2$. If $\mathcal{F}_1=\emptyset$, then clearly $\mathcal{F}_1\subseteq\mathcal{F}_2$. All portfolios $\mathbf{x}\in\mathcal{F}_1\neq\emptyset$ satisfy

$$P(Z_1<0)\le\alpha \quad\Leftrightarrow\quad \Phi^{-1}(\alpha)\sigma(Z_1)+\mu(Z_1)\ge 0 \quad \overset{\text{Lemma 1}}{\Leftrightarrow}$$

$$\Phi^{-1}(\alpha)\sigma(Z_2)+\mu(Z_2)>0 \quad\Leftrightarrow\quad P(Z_2<0)<\alpha\,.$$

Thus, $\mathbf{x}\in\mathcal{F}_2$.

□

Remark, in the special case where the safety-first constraint for S_1 is satisfied for all portfolios $\mathbf{x}$, i.e. $P(\mathbf{x}^T\mathbf{R}<T_1)<\alpha\ \forall\mathbf{x}\in[0,1]^n$, the entire wealth is invested in the single asset with the highest expected return and, thus, the same is true for S_2. In this case, both investors obtain the same optimal expected portfolio return and only one asset is hold. Remark, if inequality (3.6) reverses, the result clearly reverses, i.e. S_1 outperforms S_2. But this is practically not the case as supported by the empirical evidence in section 3.4.

Figure 3.1 in which the safety-first efficient frontiers[6] for S_1 and S_2 are sketched, illustrates the result: Suppose, both investors choose a critical probability of α_1, then the portfolio problem is neither feasible for S_1 nor for S_2. Suppose, both choose α_2, then S_2 outperforms S_1 as $\boldsymbol{\mu}^T\mathbf{x}_2^*>\boldsymbol{\mu}^T\mathbf{x}_1^*$. Finally, suppose that they choose α_3, then their optimal expected portfolio returns coincide and their entire wealth is invested in the single asset with the highest expected return.

[6]Shefrin (2005) uses the term *SP/A efficient frontier*, which is the same as the safety-first efficient frontier, plotted in $(\boldsymbol{\mu}^T\mathbf{x}^*,\alpha)$-space. The safety-first efficient frontier is monotone non-decreasing as investors prefer higher portfolio returns $(\boldsymbol{\mu}^T\mathbf{x}^*)$ but lower risk (α).

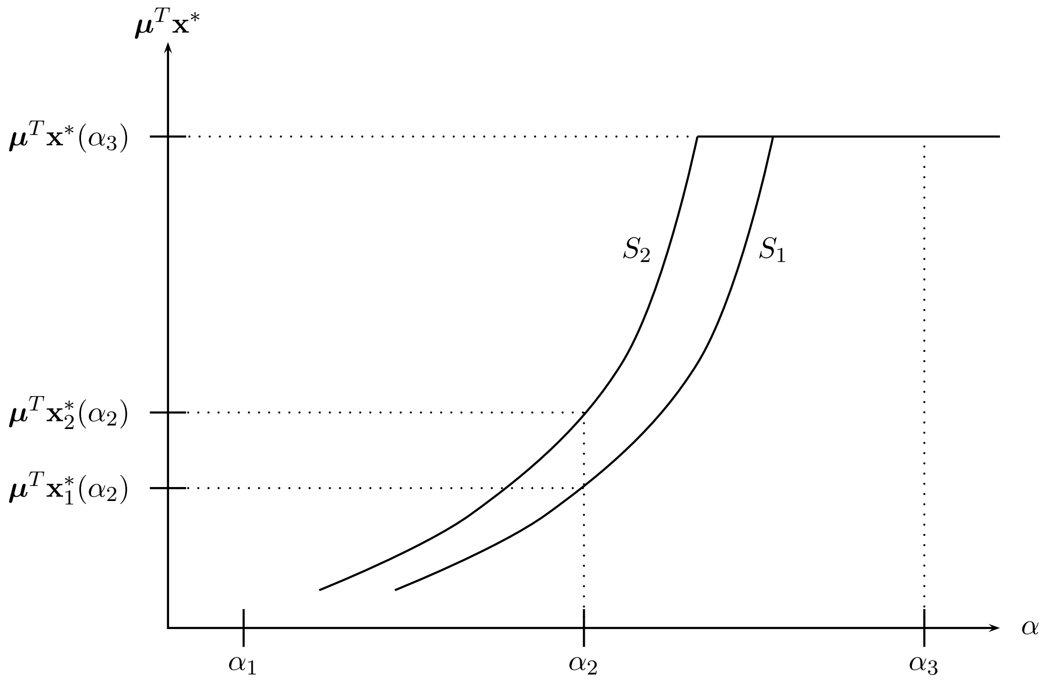


Figure 3.1: Safety-first efficient frontiers for S_1 and S_2.

3.3.2 The non-positive correlated case

This case assumes a non-positive correlation between the random target and the asset returns, i.e. the cross-covariance vector between T_2 and $\mathbf{R}$ is non-positive,

$$(L\mathbf{b}) = cov(\mathbf{R}, T_2) \leq \mathbf{0}\,. \tag{3.7}$$

This situation can be motivated by at least three examples: First, from an individual perspective the target may not even be known. Many individuals have the target of "being successful", but only a very few know precisely which selection of money, leisure time, culture etc. must be attained to achieve this target (Bordley and LiCalzi, 2000). Second, individuals may follow a group target, because they are uncertain about their individual target. This can be interpreted as *herd behavior* (see for example Shiller, 2005, pp. 157-172). In these two examples the target is not clearly specified and can thus be interpreted as unknown or random and stochastically independent from the portfolio. Third, there exists a negative correlation between T_2 and $\mathbf{R}$, which for example occurs when T_2 presents the return of a bond market whereas $\mathbf{R}$ are stock returns.

As in the previous subsection, the corresponding random variables for the strategies

S_1 and S_2 are

$$Z_1 = \mathbf{x}^T\mathbf{R} - T_1 \quad \text{with} \quad Z_1 \sim \mathcal{N}(\boldsymbol{\mu}^T\mathbf{x} - T_1, \|L^T\mathbf{x}\|^2) \text{ and}$$
$$Z_2 = \mathbf{x}^T\mathbf{R} - T_2 \quad \text{with} \quad Z_2 \sim \mathcal{N}(\boldsymbol{\mu}^T\mathbf{x} - T_1, \|L^T\mathbf{x} - \mathbf{b}\|^2)\,,$$

respectively, obtained from Theorem 1. Notice, in the case where T_2 and $\mathbf{R}$ are stochastically independent, the variance of the sum is the sum of variances, that is

$$\sigma^2(Z_2) = \sigma^2(\mathbf{x}^T\mathbf{R} - T_2) = \sigma^2(\mathbf{x}^T\mathbf{R}) + \sigma^2(T_2) = \|L^T\mathbf{x}\|^2 + \sigma^2(T_2)\,.$$

From this, it follows immediately that S_2 is riskier than S_1 and therefore S_1 outperforms S_2. In the following I show that this is also true, when asset returns and the random target are non-positively correlated. The proof of Theorem 3 makes us of

Lemma 2. *Provided (3.7) holds, for any critical probability $\alpha \in (0, 1/2)$ the following inequality is true:*

$$\Phi^{-1}(\alpha)\sigma(Z_2) + \mu(Z_2) < \Phi^{-1}(\alpha)\sigma(Z_1) + \mu(Z_1)$$

Proof:

$$\begin{aligned}
0 \quad &< \quad -2\,\underbrace{cov(\mathbf{R}, T_2)^T\mathbf{x}}_{\leq 0} + \underbrace{\sigma^2(T_2)}_{>0} \quad \Leftrightarrow \\
\|L^T\mathbf{x}\|^2 \quad &< \quad \|L^T\mathbf{x}\|^2 - 2(L\mathbf{b})^T\mathbf{x} + \mathbf{b}^T\mathbf{b} \quad \Leftrightarrow \\
\|L^T\mathbf{x}\| \quad &< \quad \|L^T\mathbf{x} - \mathbf{b}\| \quad \Leftrightarrow \\
\Phi^{-1}(\alpha)\|L^T\mathbf{x}\| + \boldsymbol{\mu}^T\mathbf{x} - T_1 \quad &> \quad \Phi^{-1}(\alpha)\|L^T\mathbf{x} - \mathbf{b}\| + \boldsymbol{\mu}^T\mathbf{x} - T_1 \quad \Leftrightarrow \\
\Phi^{-1}(\alpha)\sigma(Z_1) + \mu(Z_1) \quad &> \quad \Phi^{-1}(\alpha)\sigma(Z_2) + \mu(Z_2)
\end{aligned}$$

Notice that the second last inequality reverses because $\Phi^{-1}(\alpha) < 0 \;\forall\alpha \in (0, 1/2)$.

□

Theorem 3. *Provided (3.7) holds, for any critical probability $\alpha \in (0, 1/2)$ the optimal expected portfolio return of S_1 is larger or equal than the optimal expected portfolio return of S_2.*

Proof: Employing Lemma 2 instead of 1 and redoing the proof of Theorem 2 yields the proposition.

□

This result requires, compared to the positive correlated case discussed in subsection 3.3.1, no additional assumption. Furthermore, it allows riskless investing, which is prohibited in the previous case. As the fixed target strategy, S_1, outperforms the random target strategy, S_2, it reveals two issues: First, a random target which is stochastically independent from the portfolio return, induced by a general uncertainty about the target, leads to a poor performance and should be avoided. Second, an inappropriate target choice characterized by a negative covariation with the portfolio return leads also to a poor performance in terms of an expected return loss.

3.4 Empirical evidence for condition (3.6)

This section provides empirical evidence for condition (3.6) and supports the practical relevance of the result from section 3.3.1. To do this, I estimate

$$cov(R_i, T_2) - 1/2\sigma^2(T_2)\,,\ i = 1, \dots, n\,, \tag{3.8}$$

in which R_i is the return of stock i and T_2 is the return of a stock index. I use German stock data from the DAX (blue chip stocks) and the MDAX (mid cap stocks). For T_2 I use data from important national and international indices. Condition (3.6) is satisfied if (3.8) is positive for all i.

Table 3.1 reports empirical estimators for (3.8), where $R_i,\ i = 1, \dots, 30$, are the returns of all 30 DAX stocks and for T_2 I use the following stock indices: MSCI WORLD, DJ STOXX 50, DJ EURO STOXX 50, MSCI EUROPE, FTSE 100, S&P 500 and NYSE. Estimates are based on a sample period for which monthly return data for all DAX stocks contained in the index in February 2010 is available, that is a period from February 2001 to February 2010. Table 3.1 shows that (3.8) is positive and thus (3.6) is true for all stocks in the DAX, except for three (BEIX: Beiersdorf, FMEX: Fresenius Medical Care and DTEX: Deutsche Telekom, for which only the condition using the MSCI EUROPE is not satisfied). Undocumented results for a larger sample period from September 1991 to February 2010, for which complete return data of 14 DAX stocks exist, reveal that (3.8) is entirely positive.

Table 3.2 reports the empirical estimator for (3.8) using the same index data, but stock data from the MDAX. Again, estimates are based on a sample period ranging from February 2001 to February 2010, for which complete return data of 33 out of 40 MDAX stocks is available. Table 3.2 shows that (3.8) is positive and thus (3.6) is true for all stocks in the MDAX, except for four (CLS1: Celesio, DEQ: Deutsche Euroshop, FIE: Fielmann and VOS: Vossloh).

The empirical results presented here indicate that condition (3.6) is mainly true, at least for the German stock market. This provides additional support that, in the context of section 3.3.1, the random target strategy S_2 should be preferred over the fixed target strategy S_1.

3.5 Return distribution is unknown

It is shown empirically that return distributions are fat-tailed (see for example Adler, 1998, and the references therein) and skewed to the left, i.e. losses weigh heavier than gains, discussed for example in Harlow (1991). These findings indicate that the normal assumption does not necessarily hold in general. This section therefore relaxes the normal assumption and assumes only that good estimates for the first two moments of the return distributions exist. Then, a well-known textbook inequality - the Chebyshev inequality - can be used. In the following, this inequality is applied to the safety-first constraint (3.1) yielding a stronger but deterministic version of (3.1).[7] Providing this, it is easy to verify that under unknown return distributions the results from section 3.3 follow analogously.

Again, let $Z = \mathbf{x}^T\mathbf{R} - T$ be a random variable with $\mathbf{x}^T\mathbf{R}$ being the random portfolio return and T being a target, either fixed or random. For the expected value and the variance of Z we get

$$\begin{aligned} \mu(Z) &= \boldsymbol{\mu}^T\mathbf{x} - \mu(T)\,, \\ \sigma^2(Z) &= \|L^T\mathbf{x} - \mathbf{b}\|^2\,. \end{aligned}$$

The following inequalities provide an upper bound for the safety-first constraint (3.1):

$$\begin{aligned} & P(\mathbf{x}^T\mathbf{R} < T) = P(Z < 0) \\ \leq\; & P(Z \leq 0) = P(Z - \mu(Z) \leq -\mu(Z)) = P(\mu(Z) - Z \geq \mu(Z)) \\ \leq\; & P(|\mu(Z) - Z| \geq \mu(Z)) \\ \leq\; & \frac{\sigma^2(Z)}{\mu(Z)^2} = \frac{\|L^T\mathbf{x} - \mathbf{b}\|^2}{(\boldsymbol{\mu}^T\mathbf{x} - \mu(T))^2}\,, \end{aligned}$$

where the last inequality is obtained from Chebyshev's rule.[8] Instead of (3.1), the stronger inequality

$$\frac{\|L^T\mathbf{x} - \mathbf{b}\|^2}{(\boldsymbol{\mu}^T\mathbf{x} - \mu(T))^2} \leq \alpha$$

can be applied. Drawing the square root and rearranging yields

$$-\alpha^{-\frac{1}{2}}\|L^T\mathbf{x} - \mathbf{b}\| + \boldsymbol{\mu}^T\mathbf{x} \geq \mu(T)\,. \tag{3.9}$$

Comparing (3.9) to the deterministic safety-first constraint (3.4) obtained for the multivariate normal case (see section 3.2), the sole difference is the multiplier for the term

[7] This idea has been first suggested by Roy (1952). For a detailed discussion on the application of Chebyshev's inequality to the safety-first criterion consult Kall and Mayer (2005) or Birge and Louveaux (1997).

[8] A detailed illustration for the univariate case is provided in Breuer et al. (2006, pp. 119-121). For the Chebyshev inequality in general consult a textbook on probability theory, such as Behnen and Neuhaus (1995).

$\|L^T\mathbf{x} - \mathbf{b}\|$. In (3.4) the multiplier is $\Phi^{-1}(\alpha)$, whereas it is $-\alpha^{-\frac{1}{2}}$ for (3.9), both are negative for all $\alpha \in (0, 1/2)$. Thus, redoing the proofs from section 3.3 with (3.9) instead of (3.4) yields the same results. Moreover, as (3.9) is concave for all α the same efficient solving methods as for problem 5 can be used.

Note, applying the stronger inequality (3.9) instead of (3.1) reduces the number of feasible portfolios, i.e. the feasible domain of problem 5 with (3.9) instead of (3.1) is a subset of the feasible domain of the original problem 4. Thus, choosing α very small may lead to infeasibility of the safety-first problem under (3.9), but not necessarily under the true safety-first constraint (3.1). Nevertheless, this approach provides a useful and tractable alternative to the multivariate normal case.

3.6 Conclusion

This chapter investigates the safety-first portfolio model under two different target assumptions, the fixed target, which is commonly assumed in the literature, and the random target, which has played no role in the existing literature on the safety-first model so far. As a random target can be easily motivated for this framework, the open question is, which target choice leads to a better performance? I answer the question by comparing optimal expected portfolio returns of the fixed and the random target strategy. Assuming multivariate normal returns the answer is: (1) The random target strategy outperforms the fixed target strategy if the portfolio return and the random target are positively correlated and riskless investing is prohibited; (2) the fixed target strategy outperforms the random target strategy if the portfolio return and the random target are not positively correlated and riskless investing is allowed. The first result is practically most relevant, in particular for institutional portfolio management and skilled private investors, which is supported by empirical evidence in section 3.4. The second result suggests general uncertainty about the target and an inappropriate target choice, characterized by a negative correlation between the portfolio return and the target, should be avoided. As the normal assumption is in general violated in empirical finance, section 3.5 relaxes this assumption and illustrates that both results hold when approximating the safety-first statement by the well-known Chebyshev inequality.

The normal distribution and Chebyshev's inequality are on the one hand very tractable and easy to implement, but on the other hand not very accurate. To overcome this limitation several extensions are possible: One, the normal assumption can be relaxed to the general distribution family depending on a shift and a scale parameter (see Kalin and Zagst, 1999). Two, the normal distribution can be generalized to the elliptical distribution. Third, a copula function, which provides a general technique for formulating a multivariate distribution, can be used. Four, considering higher order moments a more accurate probability inequality can be used. I recommend this issues for further research.

3.A Appendix

3.A.1 Proof of Theorem 1

As $\mathbf{R}$ is n-variate and T univariate normal distributed, the vector $(\mathbf{R}^T, T)^T$ is $n+1$-variate normal distributed and can be written as

$$\begin{pmatrix}\mathbf{R}\\ T\end{pmatrix} = \begin{pmatrix}L\\ \mathbf{b}^T\end{pmatrix}\mathbf{Y} + \begin{pmatrix}\boldsymbol{\mu}\\ \mu(T)\end{pmatrix},$$

in which $L \in \mathbb{R}^{n\times r}$, $\mathbf{b} \in \mathbb{R}^r$ and $\mathbf{Y} = (Y_1, \ldots, Y_r)^T$ has a r-variate normal distribution with mean $\mathbf{0}$ and identity matrix as covariance matrix (see for example Giri, 2004, pp. 81-82). Simple algebra yields

$$\begin{aligned} Z &= \mathbf{x}^T\mathbf{R} - T = (\mathbf{R}^T, T)\left(\begin{smallmatrix}\mathbf{x}\\ -1\end{smallmatrix}\right)\\ &= \mathbf{Y}^T L^T\mathbf{x} + \boldsymbol{\mu}^T\mathbf{x} - \mathbf{b}^T\mathbf{Y} - \mu(T)\\ &= \mathbf{Y}^T(L^T\mathbf{x} - \mathbf{b}) + \boldsymbol{\mu}^T\mathbf{x} - \mu(T)\,. \end{aligned}$$

Hence, we get the expected value and the variance of Z, respectively, as

$$\begin{aligned} \mu(Z) &= \boldsymbol{\mu}^T\mathbf{x} - \mu(T)\,,\\ \sigma^2(Z) &= (L^T\mathbf{x} - \mathbf{b})^T(L^T\mathbf{x} - \mathbf{b}) = \|L^T\mathbf{x} - \mathbf{b}\|^2\,. \end{aligned}$$

□

3.A.2 A note on the numerical treatment of the covariance matrix factorization

Kall and Mayer (2005) suggest the Cholesky-factorization

$$cov(\mathbf{R}^T, T) = cov(R_1, \ldots, R_n, T) = \begin{pmatrix}L\\ \mathbf{b}^T\end{pmatrix}\begin{pmatrix}L\\ \mathbf{b}^T\end{pmatrix}^T,$$

with L being a lower triangular matrix, which, however, may lead to numerical problems. To compute the Cholesky-factorization the matrix must be positive definite, which may, due to stochastic independences between two or more assets, not be the case. In this case, the covariance matrix is positive semidefinite and the Cholesky-factorization can not be computed. To correct this drawback, I suggest a more general factorization for symmetric matrices, in which the covariance matrix is factorized as

$$cov(\mathbf{R}^T, T) = QDQ^T\,.$$

The matrix D is a diagonal matrix and contains the eigenvalues of $cov(\mathbf{R}^T, T)$ and Q is an orthogonal matrix. Numerically, if the covariance matrix is positive semidefinite, small rounding errors may induce negative eigenvalues close to zero, which is theoretically impossible. This is corrected by hand by setting eigenvalues smaller than $\varepsilon > 0$ to zero. Doing this, $\tilde{D}$ arise from D and the covariance matrix can finally be factorized as

$$cov(\mathbf{R}^T, T) = Q\sqrt{\tilde{D}}\sqrt{\tilde{D}}Q^T = \begin{pmatrix} \tilde{L} \\ \tilde{\mathbf{b}}^T \end{pmatrix} \begin{pmatrix} \tilde{L} \\ \tilde{\mathbf{b}}^T \end{pmatrix}^T,$$

in which $\tilde{L}$ is necessarily no triangular matrix.

Datastream code	MSCI	DJST	DJES	MSCE	FTSE	S&P	NYSE
ADSX	14.44	15.08	15.16	14.25	13.76	12.53	13.45
ALVX	39.31	41.08	46.68	40.00	35.37	39.63	39.00
BASX	22.65	22.63	24.30	22.93	21.00	20.57	22.53
BAYX	22.05	27.04	29.86	23.39	21.51	20.90	21.76
BEIX	-0.20	0.39	-3.34	-3.13	3.31	-1.18	-0.90
BMWX	16.45	20.41	20.58	14.25	18.49	16.07	16.53
CBKX	65.60	66.66	76.66	70.82	57.34	62.97	65.72
DAIX	31.76	32.08	34.93	30.78	29.45	30.56	31.87
DBKX	45.33	43.78	49.99	46.89	37.13	44.17	44.48
DB1X	27.61	26.80	28.50	27.31	24.02	26.10	27.39
LHAX	32.18	33.99	38.34	31.25	29.23	32.15	31.32
DPWX	28.87	27.00	29.15	29.76	26.06	28.46	29.59
DTEX	2.24	9.28	7.61	-1.60	4.37	3.88	0.92
EONX	10.02	11.04	9.45	9.54	10.96	7.69	10.16
FMEX	-1.99	-0.83	-3.92	-5.75	0.76	-1.76	-3.68
FR3X	12.78	12.86	10.77	10.79	14.21	11.55	10.76
HE3X	6.19	7.03	5.15	4.11	7.95	5.12	5.54
IFXX	92.54	94.40	106.32	97.91	75.35	87.42	90.11
SDFX	28.49	21.49	22.50	28.44	25.27	24.87	28.63
LINX	15.52	19.06	18.97	15.67	17.30	12.90	14.21
MANX	40.51	39.68	44.31	44.05	35.71	35.44	38.34
MRKX	4.16	4.78	2.38	2.50	5.41	3.45	3.52
MEOX	27.65	27.79	29.33	26.96	24.74	27.41	27.52
MU2X	19.46	23.84	27.02	17.88	19.18	20.06	18.25
RWEX	5.61	8.97	8.14	4.27	7.60	4.54	5.93
SZGX	30.44	26.91	27.81	28.36	29.78	29.04	32.05

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	MSCI	DJST	DJES	MSCE	FTSE	S&P	NYSE
SAPX	30.61	29.57	31.68	26.44	26.26	32.52	28.66
SIEX	38.23	40.86	45.52	38.91	35.73	37.17	36.20
TKAX	43.44	39.91	43.89	45.01	39.07	39.91	42.94
VO3X	26.67	22.06	25.04	26.77	19.64	24.52	26.72

This table reports empirical estimators for $cov(R_i, T_2) - 1/2\sigma^2_{T_2}$ where $R_i,\ i = 1, \ldots, 30$, are the returns of all 30 DAX (blue chip) stocks. For T_2 I use the following international stock indices: MSCI: MSCI WORLD, DJST: DJ STOXX 50, DJES: DJ EURO STOXX 50, MSCE: MSCI EUROPE, FTSE: FTSE 100, S&P: S&P 500 and NYSE. I use a sample period for which monthly return data for all DAX stocks contained in the index in February 2010 is available, that is a period from February 2001 to February 2010.

Table 3.1: Cross-covariances of all DAX stocks and international stock indices

Datastream code	MSCI	DJST	DJES	MSCE	FTSE	S&P	NYSE
NDA	8.93	7.03	4.79	4.40	9.11	8.31	9.73
BYW6	25.17	18.38	18.15	27.75	21.58	20.71	24.68
GBF	30.56	26.51	29.02	30.74	29.80	27.83	30.16
BOS3	38.78	38.16	40.06	40.33	37.32	35.83	39.05
CLS1	-8.01	-4.11	-9.96	-12.27	-2.44	-7.60	-7.01
CON	41.96	36.63	39.87	44.77	37.72	38.90	42.12
DEQ	-2.68	-5.78	-9.52	-5.79	-0.51	-3.70	-2.36
DOU	3.91	4.84	3.09	1.24	7.19	2.69	3.51
EAD	27.13	32.06	34.40	26.37	28.79	25.60	26.65
ZIL2	30.67	24.83	25.85	28.54	25.50	29.57	31.17
FIE	-1.88	1.57	-0.96	-5.10	2.43	-2.23	-2.03
FPE3	21.53	15.38	14.73	23.08	17.90	17.82	21.49
G1A	40.42	35.13	37.35	41.38	37.10	37.63	40.13
GIL	41.06	38.18	40.60	47.72	36.16	34.86	40.80
HNR1	19.12	17.36	19.93	17.95	16.89	19.65	19.36
HDD	48.36	48.48	52.77	52.95	45.57	43.61	50.12
HEI	28.42	32.58	34.58	30.92	27.73	25.43	29.12
HOT	47.38	41.22	45.74	49.83	44.43	42.49	46.52
IVG	46.05	38.32	42.45	53.07	41.19	38.64	45.44
KRN	12.27	13.42	12.01	9.57	15.04	10.96	11.85
LEO	37.71	31.30	31.46	38.85	32.64	35.88	38.90
MLP	60.65	62.87	75.20	60.85	48.68	62.38	59.61
PFD4	47.96	43.30	48.20	51.23	39.89	44.97	48.77
PSM	83.52	81.32	91.31	88.84	69.61	78.76	81.95
PUM	25.11	22.51	22.50	25.40	24.67	23.00	24.71
RAA	17.86	16.47	16.84	18.11	18.14	14.99	16.42

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	MSCI	DJST	DJES	MSCE	FTSE	S&P	NYSE
RHM	12.25	11.20	8.60	11.95	14.90	9.81	12.63
RHK	4.70	7.55	4.61	2.74	9.69	3.34	4.26
SGL	52.18	39.24	46.29	52.16	40.80	50.59	50.24
SAZ	23.76	18.15	17.88	20.45	18.62	24.77	25.94
SZU	4.48	7.66	6.23	2.81	8.12	2.69	2.92
TUI1	51.02	52.12	59.03	55.17	46.99	48.36	52.19
VOS	-1.06	-3.47	-6.97	-4.25	2.55	-1.65	-0.62

This table reports the empirical estimator for $COV(R_i, T_2) - 1/2\sigma_{T_2}^2$ where $R_i,\ i = 1, \ldots, 33$, are the returns of 33 MDAX (mid cap) stocks, for which complete return data for the sample period from February 2001 to February 2010 is available. For T_2 I use the following international stock indices: MSCI: MSCI WORLD, DJST: DJ STOXX 50, DJES: DJ EURO STOXX 50, MSCE: MSCI EUROPE, FTSE: FTSE 100, S&P: S&P 500 and NYSE.

Table 3.2: Cross-covariances of 33 MDAX stocks and international stock indices

4 Goal-Specific Asset Selection and Utility in Behavioral Portfolio Theory with Mental Accounts

This chapter is based on Singer (2011).

"My intention was to minimize my future regret. So I split my contributions fifty-fifty between bonds and equities."

Harry M. Markowitz, 1998[1]

4.1 Introduction

More than 60 years ago, Friedman and Savage (1948) noted that risk aversion and risk seeking share roles in our behavior: People who buy insurance policies often buy lottery tickets as well. Four years later, Markowitz wrote two papers that reflect two very different views of behavior. In 1952b, he created the mean-variance (MV) framework, in 1952a, he extended Friedman and Savage's insurance lottery framework. People in the MV framework, unlike those in the insurance lottery framework, never buy lottery tickets; they are always risk averse, never risk seeking. Nevertheless, because of its tractability the MV model has become the leading textbook theory for portfolio selection despite the fact that it fails to explain the insurance lottery puzzle.

The key element of the portfolio model I present here, contributing to explain phenomena such as the insurance lottery puzzle, is mental accounting. This model, which is closely related to that by Das, Markowitz, Scheid, and Statman (2010), combines features of MV theory and Shefrin and Statman's (2000) behavioral portfolio theory (BPT). In this model, as well as in that by Das et al. (2010) investors consider their

[1](Shefrin, 2002, p. 31)

portfolios as collections of mental accounting subportfolios where each subportfolio is associated with a goal. In each mental account (MA), investors care about the expected return and its risk, measured by the probability of failing to reach the goal of that MA.

In contrast to Das et al. (2010), I follow two different approaches: First, I analyze the case in which goal-specific asset selection is allowed, e.g. for a secure retirement the investor is allowed to select assets which appeal to that goal, such as bonds or dividend stocks. This case has not been analyzed yet, but is of high importance since it appeals to individual investors' intuition. This relaxation implies that solutions to subportfolios are points on different MV efficient frontiers. Based on these efficient subportfolios, I suggest three distinct ways of arriving at the solution to the aggregate portfolio. Second, I analyze the case in which goal-specific asset selection is disallowed, i.e. putting the same assumption as Das et al. (2010), but relying on a different interpretation. They follow a two step approach where in the first step each subportfolio is solved separately without considering relations between mental accounts and, second, calculate the aggregate portfolio on the basis of a predefined allocation rule. In this chapter, however, I present a one step approach where the mental accounting model with respect to relations between mental accounts is solved as a whole. Assuming this case, I also present a utility analysis and show that mental accounting investors are consistent with Friedman and Savage (1948) and Markowitz (1952a) investors. Thus, this chapter provides new theoretical evidence that mental accounting is a key driver of the insurance lottery-puzzle.

The remainder is organized as follows: Section 4.2 reviews related work. Based on the setting by Das et al. (2010) section 4.3 presents a new behavioral portfolio model with mental accounts. Section 4.4 offers issues on the relation between MA and utility while section 4.5 concludes.

4.2 Related work

According to Thaler (1999, p. 183) "[m]ental accounting is [defined as] the set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities." Translated into a portfolio context, assets are grouped into categories and each category maintains a specific investment goal. Individual investors want to satisfy goals such as a secure retirement, college education for the children, or a chance for great riches. The first paper which formally put the MA approach into a portfolio setting is that by Shefrin and Statman (2000). In their BPT they present a single MA version, which is based on SP/A theory (Lopes, 1987; Lopes and Oden, 1999), and a multiple MA version, which was later advanced by Das et al. (2010). In their MA framework they integrate appealing features of MV theory and BPT. The main contribution of Das et al. (2010) is that in their setting, when short selling is allowed, portfolios that follow from MV analysis with mental accounting belong to the MV efficient frontier, i.e. mental accounting does not introduce inefficiency in MV sense.

In the same year, DeGiorgi (2010) developed a dynamic portfolio model with mental accounts, which is, in contrast to Das et al. (2010), based on cumulative prospect theory (Tversky and Kahneman, 1992), and obtains similar results. Hoffmann, Shefrin, and Pennings (2010) developed a different dynamic portfolio model and provide empirical evidence for BPT. Empirically, Choi, Laibson, and Madrian (2009) study 401(k) savings accounts of a large U.S. firm and document "a lack of coordination between the asset allocations of different financial accounts in a household portfolio." They conclude that it is not possible to explain the effect without mental accounting.

4.3 The model

As the model presented here is based on the setting by Das, Markowitz, Scheid, and Statman (2010), subsection 3.3.1 reviews basic features of that model while subsections 3.3.2 and 3.3.3 suggest new issues.

4.3.1 Review of the model by Das et al. (2010)

In the mental accounting model by Das et al. (2010) investors consider their portfolios as collections of mental accounting subportfolios where each subportfolio is associated with a goal and each goal has a threshold level. In each MA subportfolio, investors care about the expected return of the subportfolio and its risk, measured by the probability of failing to reach the threshold level of that MA. This risk measure is often called shortfall probability and is central in BPT. Formally, for each subportfolio the following optimization model is solved

$$\max_x x^T \mu \,, \tag{4.1}$$

subject to the shortfall constraint

$$P(r < T) \leq \alpha \,, \tag{4.2}$$

and the fully invested constraint

$$x^T \mathbf{1} = 1 \,, \tag{4.3}$$

where $\mathbf{1} = (1, \ldots, 1)^T \in R^n$, $x = (x_1, \ldots, x_n)^T$ being the vector of portfolio weights of n assets with $\mu \in R^n$ being the mean return vector, P denotes probability, T is the threshold level of portfolio return r and the maximum probability of the portfolio failing to reach the return T is α.

Das et al. (2010) use a numerical example to illustrate their results. They assume three assets with mean vector and covariance matrix of returns

$$\mu = \begin{bmatrix} 0.05 \\ 0.10 \\ 0.25 \end{bmatrix} , \quad \Sigma = \begin{bmatrix} 0.0025 & 0.0000 & 0.0000 \\ 0.0000 & 0.0400 & 0.0200 \\ 0.0000 & 0.0200 & 0.2500 \end{bmatrix} . \tag{4.4}$$

The first asset is a low risk asset, analogous to a bond. It has low return and low variance compared to the other two more "risky" assets, analogous to a low and a high risk stock. They suppose further an investor who divides an aggregate portfolio into three subportfolios: a secure retirement, college education for the children, and a gambling subportfolio, which provides a chance for great riches.[2] The retirement subportfolio is associated with the parameter pair $(T_1, \alpha_1) = (-0.10, 0.05)$, that is, the investor stipulates that she does not want the probability of failing to reach $T_1 = -10\%$ to exceed $\alpha_1 = 5\%$. For the eduction and gambling subportfolio they assume parameter pairs $(T_2, \alpha_2) = (-0.05, 0.15)$ and $(T_3, \alpha_3) = (-0.15, 0.20)$, respectively.

For each MA, they solve optimization model (4.1)-(4.3) using the same three assets and calculate an aggregate portfolio which invests 60% in the retirement, 20% in the education and 20% in the gambling subportfolio. Their main contribution is that in their setting, when short selling is allowed, portfolios that follow from MV analysis with mental accounting belong to the MV efficient frontier, i.e. mental accounting does not introduce inefficiency in MV sense. However, this does not hold when goal-specific asset selection is allowed, e.g. the investor is allowed to select retirement-specific assets which may significantly differ from those selected for gambling purposes. Subsection 3.3.2 presents a solution to that problem. Das et al. (2010) find the solution for the aggregate portfolio assuming that the investor allocates her wealth according to the 60:20:20 rule. Subsection 3.3.3 follows a different interpretation and presents a closed form solution to the mental accounting portfolio problem which does not require any exogenous assumption about the final wealth allocation.

4.3.2 Goal-specific asset selection allowed

In this subsection the investor is allowed to select goal-specific assets for each mental account, which has not been analyzed yet. To appeal to real investors' behavior, only "safe" assets like bonds and low risk stocks are considered for a secure retirement, whereas a bond seems to be inappropriate for gambling purposes. I therefore consider only the bond and the low risk stock from the previous example for the retirement account and the low and high risk stock for the gambling account. The education account remains unchanged. Employing these modifications yields three subproblems with different investment universes. Each subproblem is solved using the methodology in Das et al. (2010), who assume normal distributed asset returns with parameters given in (4.4). As asset returns are assumed to be jointly normal distributed, the portfolio return is univariate normal distributed with mean μ and standard deviation σ. Hence, the shortfall probability can be transformed to

$$\mu \geq T - \Phi^{-1}(\alpha)\sigma \,, \tag{4.5}$$

[2] Originally, Das et al. (2010) call the third goal "bequest".

which is a straight line in (μ, σ)-space with intercept T and slope $\Phi^{-1}(\alpha)$ being the α-quantile of the standard normal distribution. The solution to each subproblem, if it exists, is the intersection point between (4.5) and the upper branch of the MV efficient frontier (Pyle and Turnovsky, 1970). For example, the optimal solution to the retirement portfolio is the intersection point between (4.5) with $T = -0.10$ and the MV efficient frontier induced by the bond and the low risk stock, that is the point $(\mu, \sigma) = (0.08, 0.11)$ shown in figure 4.1. Figure 4.1 also contains the optimal solution to the education and gambling subportfolio, respectively, indicated by the marked points. Note that the solution to each MA lies on a different efficient frontier. For that reason, the result obtained by Das et al. (2010) does not hold when goal-specific asset selection is allowed.

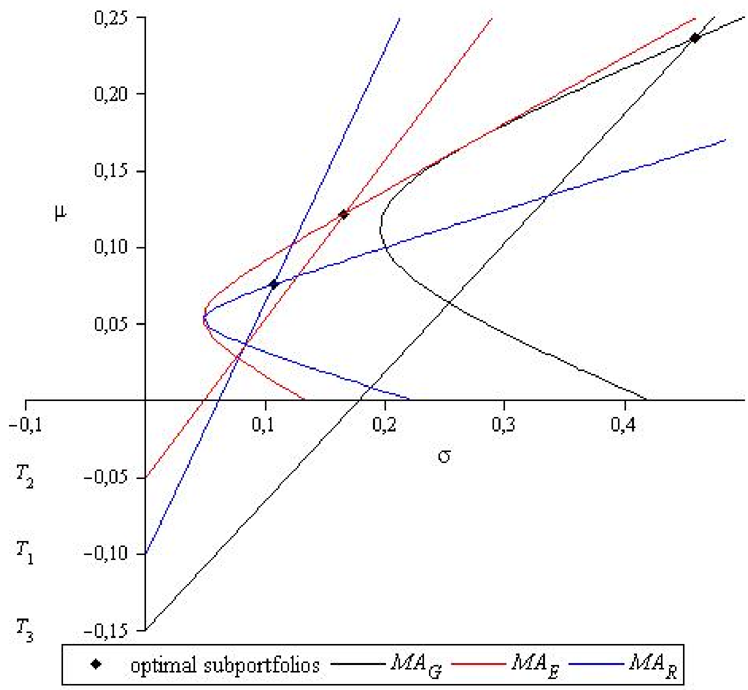


Figure 4.1: Graphical solution to each mental account. MA_R, MA_E and MA_G denote retirement, education and gambling mental account, respectively.

I suggest three ways to obtain a solution to that problem:

a) Exogenous solution:
As in Das et al. (2010) the solution to the aggregate portfolio can be found by applying the 60:20:20 rule or any other by the investor freely chosen allocation rule, which is the main advantage of that approach. However, such allocation rules may lead to suboptimal solutions in MV sense.

b) Behavioral solution:

Assets	MA_R	MA_E	MA_G	60:20:20	Behavioral	Rational
x_1	0.48	0.38	-	0.36	0.68	-6.58
x_2	0.52	0.35	0.09	0.40	0.28	10.78
x_3	-	0.27	0.91	0.24	0.04	-3.20
μ	0.08	0.12	0.24	0.12	0.09	0.05
σ	0.11	0.16	0.46	0.16	0.09	0.05

Note: MA_R, MA_E and MA_G denote retirement, education and gambling MA, respectively. The solution to the behavioral and rational approach is the global minimum variance portfolio.

Table 4.1: Results for all mental accounting subportfolios and aggregate portfolios

Empirically, it has been observed that individual (Kroll et al., 1988a) and institutional (Jorion, 1994) investors ignore correlations between mental accounts, i.e. mental accounts are assumed to be uncorrelated.[3] Based on this assumption, a "behavioral" MV efficient frontier, induced by the parameters of the subportfolios, can be calculated, on which a "behavioral" solution to the aggregate portfolio can be found, see figure 4.2. For a fixed return μ the solution to the aggregate "behavioral" portfolio can be taken from the efficient frontier. But, to define a threshold return for the aggregate portfolio might not appeal to the intuition of mental accounting investors. This can be avoided by delegating the task to a portfolio manager or by choosing the *global minimum variance portfolio*.

c) Rational solution:
The rational solution in MV sense is obtained by taking correlations between mental accounts into account. Calculations of correlations between mental accounts are provided in the separate appendix. Based on this assumption, the MV efficient frontier, induced by the parameters of the subportfolios given in the appendix, can be calculated, on which a rational solution to the aggregate portfolio can be found, see figure 4.2.

Figure 4.2 illustrates these different approaches graphically while table 4.1 contains the corresponding numerical values. Note that the *global minimum variance portfolio* of the rational approach contains large short positions, which may not be desired by individual investors. This can be circumvent by moving upwards to another point on the efficient frontier.

[3] The same assumption is made by Siebenmorgen and Weber (2003) in their static and by DeGiorgi (2010) in his dynamic portfolio model.

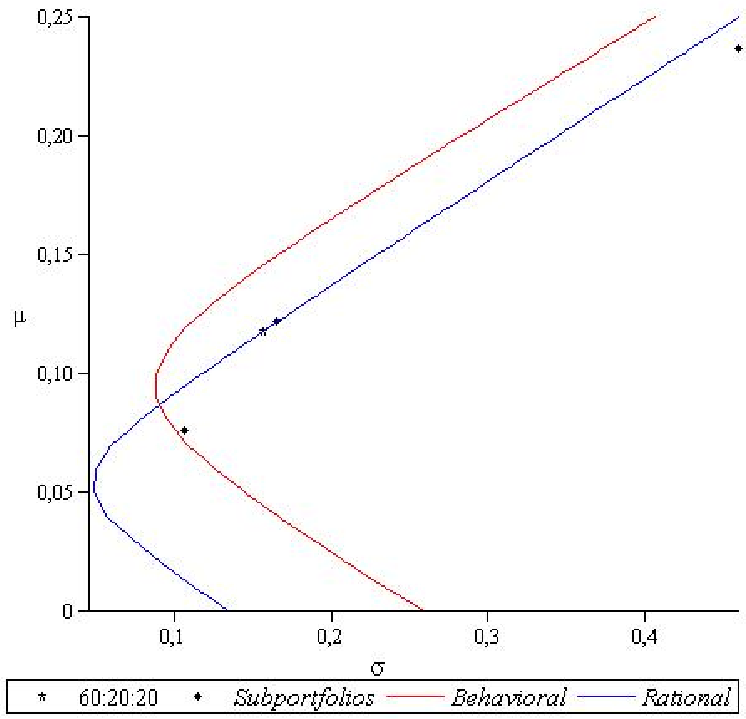


Figure 4.2: Graphical solutions to the aggregate portfolio

4.3.3 Goal-specific asset selection disallowed

In this subsection, I assume, as well as Das et al. (2010), identical assets in each mental account. Their solution to the mental accounting model is a two step approach: First, each subproblem is solved separately without considering relations between mental accounts. Second, the solution to the aggregate portfolio is calculated on the basis of a predefined allocation rule, such as the 60:20:20 rule. In this section, however, I follow a different interpretation and suggest a one step approach in which the mental accounting model with respect to relations between mental accounts is solved as a whole. Comparing the retirement goal with $(T_1, \alpha_1) = (-0.10, 0.05)$ and the gambling goal with $(T_3, \alpha_3) = (-0.15, 0.20)$, reveals an interesting observation: As the gambling threshold is lower than the retirement threshold, but the probability of failing to achieve the gambling threshold is larger, any portfolio that meets the retirement goal automatically meets the gambling goal. As the MA problem is solved as a whole, with no loss of generality, the gambling goal or technically spoken, the constraint representing the gambling goal, can be excluded from the analysis before solving the problem. To avoid such dominance relations, an ordering rule for (T, α) pairs must be imposed. Define therefore the shortfall probability vector (SPV) as[4]

$$P(r < T_k) \leq \alpha_k \quad \text{with} \quad T_k < T_l \quad \Leftrightarrow \quad \alpha_k < \alpha_l\,,\ k,l = 1,\ldots,m,\ k \neq l\,. \tag{4.6}$$

The SPV model for portfolio optimization with mental accounts maximizes expected portfolio return subject to a SPV and the fully invested constraint. Applying the three MA example introduced earlier in this chapter, formally, the following optimization model is solved

$$\max_x x^T \mu\,, \tag{4.7}$$

subject to the SPV constraints

$$\begin{aligned} P(r < -0.10) &\leq 0.05\,, \\ P(r < -0.05) &\leq 0.15\,, \\ P(r < -0.15) &\leq 0.20 \end{aligned} \tag{4.8}$$

and the fully invested constraint

$$x^T \mathbf{1} = 1\,. \tag{4.9}$$

First, note that the ordering rule defined in (4.6) is not satisfied. However, to demonstrate the linkage to Das et al. (2010), I stick for this moment to this example. Second, the solution to (4.7)-(4.9) is the optimal solution to the aggregate portfolio, which does not require any predefined allocation rule. Third, finding the solution to (4.7)-(4.9) is as simple as finding the solution to the model by Das et al. (2010), as illustrated in figure 4.3: The three straight lines correspond to the SPV constraint (4.8). The feasible domain is the shaded area below the efficient frontier and above the straight lines. Thus, the feasible portfolio with maximal expected return is the point labeled A.

The optimal portfolio invests 54% in the bond, 27% in the low risk stock and 19% in the high risk stock and coincides with the optimal solution of the retirement subportfolio. It can be observed from figure 4.3 that this solution meets all three goals simultaneously, whereas this does not hold for the 60:20:20 portfolio, also labeled in figure 4.3.

4.4 Mental accounting and utility

Let r be the random portfolio return. If a mental accounting investor orders assets solely on the basis of expected return $E(r)$ and SPV and accepts the axioms supporting the expected utility theorem, her preference ordering over $E(r)$, SPV combinations is uniquely represented by the positive linear transformations of the function $E(r) - g^T SPV$, see

[4]To the best of my knowledge the SPV model goes back to a working paper by Schubert (2002). His motivation was the drawback of the shortfall probability that it fails to provide any indication of how severe the shortfall will be, should it occur.

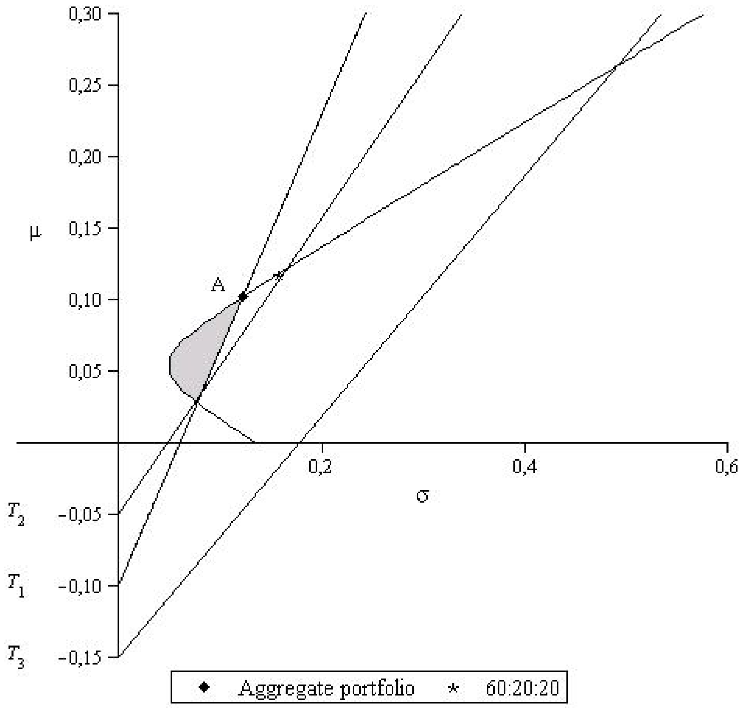


Figure 4.3: Graphical solutions to the SPV model

Arzac (1974).[5] Sticking to the three MA example this function uniquely implies the following utility function

$$u(r) = \begin{cases} r\,, & \infty > r \geq T_1 \\ r - g_1\,, & T_1 > r \geq T_2 \\ r - g_1 - g_2\,, & T_2 > r \geq T_3 \\ r - g_1 - g_2 - g_3\,, & T_3 > r > -\infty \end{cases}\,, \tag{4.10}$$

in which $g_i > 0$ presents the individual utility reduction of not achieving threshold $T_i\,,\ i = 1, 2, 3$. Figure 4.4 shows utility function (4.10) for the case $g_1 = g_2 = g_3 = 1$. This piecewise linear function with jump discontinuities at the threshold levels can be approximated by a compound arctangent function, that has appealing behavioral properties, see figure 4.4: It is monotonically increasing, continuously differentiable and exhibits both concave and convex regions, which is equivalent to investors who are risk averse for some returns and risk loving for others. This utility function is consistent with the solution to the insurance lottery puzzle by Friedman and Savage (1948), see figure 4.5(a), and the customary wealth theory by Markowitz (1952a), see figure 4.5(b), stating that

[5] Arzac (1974) analysis the case with a single shortfall probability, which can be straightforwardly extended to the case with a shortfall probability vector.

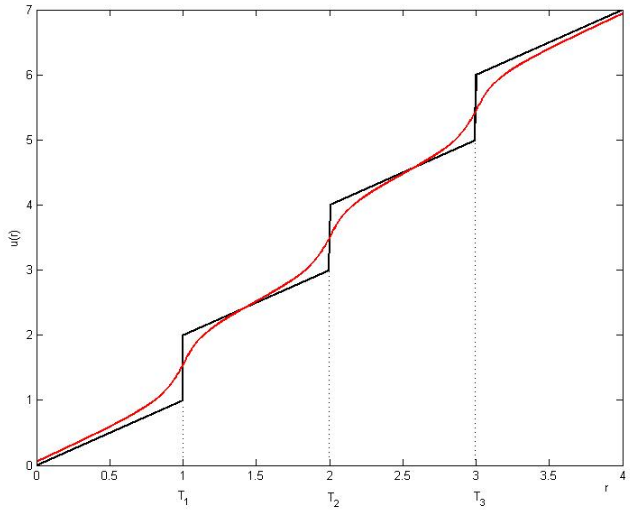
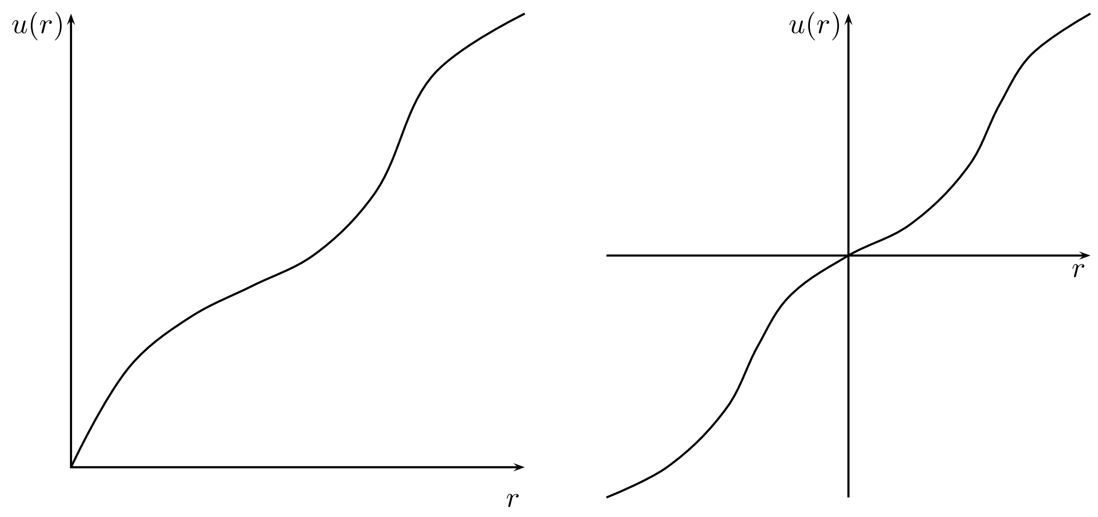


Figure 4.4: Piecewise utility function for MA investors with three thresholds and approximated function: $f(x) = x + \frac{\arctan(10x-10)}{\pi} + \frac{\arctan(10x-20)}{\pi} + \frac{\arctan(10x-30)}{\pi} + 1.5$

investors purchase both insurance policies and lottery tickets at the same time. Their solutions are based on utility functions that feature both concave and convex portions. The concave portion is consistent with the purchase of insurance policies and the convex portion is consistent with the purchase of lottery tickets. Notably, the insurance lottery puzzle is a thorn in the side of conventional expected utility theory, which is based upon concave utility functions, but it is not in the side of mental accounting utility theory.

4.5 Conclusion

This chapter investigates goal-specific asset selection and utility in a behavioral portfolio framework with mental accounts. When goal-specific asset selection is allowed optimal subportfolios do not necessarily lie on the same MV efficient frontier. Nevertheless, they induce two new MV efficient frontiers; one that ignores correlations and one that incorporates correlations between subportfolios. On both frontiers a different solution to the aggregate portfolio can be found. A more ad hoc approach to obtain a solution to the aggregate portfolio is to employ a predefined allocation rule, i.e. putting weight on each subportfolio and aggregating these. When goal-specific asset selection is not allowed, I suggest a closed form solution which is a point on the MV efficient frontier and coincides with one optimal subportfolio. This case implies a utility function for MA investors consistent with Friedman and Savage's (1948) and Markowitz's (1952a) solution to the insurance lottery puzzle. Taken as a whole, this chapter complements



(a) Friedman-Savage's utility function (b) Markowitz's customary wealth utility function

Figure 4.5: Utility functions explaining the insurance lottery puzzle

and extends existing literature on static BPT by Shefrin and Statman (2000) and Das et al. (2010).

A potential limitation is the assumption of normal distributed asset returns. I suggest two relevant extensions: First, this analysis can be straightforwardly rerun assuming two parameter distributions in general (Pyle and Turnovsky, 1970; Kalin and Zagst, 1999). Second, a more comprehensive approach that allows for fat tailed distributions assumes elliptical distributed asset returns. A second limitation concerns the utility analysis that is rather crude. This can be run with more detail for example by analyzing local risk aversion (seeking) around threshold levels.

4.A Appendix: Correlation between mental accounts

The random returns of each mental account are given by

$$\begin{aligned}
MA_R &= u_1 r_1 + u_2 r_2\,, & u_1 + u_2 &= 1\,,\\
MA_E &= v_1 r_1 + v_2 r_2 + v_3 r_3\,, & v_1 + v_2 + v_3 &= 1 \text{ and}\\
MA_G &= w_1 r_2 + w_2 r_3\,, & w_1 + w_2 &= 1\,,
\end{aligned}$$

in which $(r_1, r_2, r_3)^T$ is multivariate normal distributed with parameters given in (4.4). The covariance between the retirement and the education mental account is calculated as follows:

$$\begin{aligned}
Cov(MA_R, MA_E) &= Cov(u_1r_1 + u_2r_2, v_1r_1 + v_2r_2 + v_3r_3) \\
&= Cov(u_1r_1, v_1r_1) + Cov(u_1r_1, v_2r_2) + Cov(u_1r_1, v_3r_3) \\
&\quad + Cov(u_2r_2, v_1r_1) + Cov(u_2r_2, v_2r_2) + Cov(u_2r_2, v_3r_3) \\
&= u_1v_1\sigma_1^2 + u_1v_2\sigma_{12} + u_1v_3\sigma_{13} + u_2v_1\sigma_{12} + u_2v_2\sigma_2^2 + u_2v_3\sigma_{23}\,.
\end{aligned}$$

The covariances between the retirement and gambling and between the education and gambling mental account are

$$\begin{aligned}
Cov(MA_R, MA_G) &= u_1w_1\sigma_{12} + u_1w_2\sigma_{13} + u_2w_1\sigma_2^2 + u_2w_2\sigma_{23} \text{ and} \\
Cov(MA_E, MA_G) &= v_1w_1\sigma_{12} + v_1w_2\sigma_{13} + v_2w_1\sigma_2^2 + v_2w_2\sigma_{23} + v_3w_1\sigma_{23} + v_3w_2\sigma_3^2\,,
\end{aligned}$$

respectively, and are calculated analogously. Substituting the asset weights belonging to the optimal solutions to each subproblem, that are given in table 4.1, yields mean vector and covariance matrix of returns

$$\mu = \begin{bmatrix} 0.08 \\ 0.12 \\ 0.24 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 0.0115 & 0.0106 & 0.0114 \\ 0.0106 & 0.0275 & 0.0699 \\ 0.0114 & 0.0699 & 0.2111 \end{bmatrix}. \tag{4.11}$$

5 Lottery Tickets and Common Stocks: Complements or Substitutes?

This chapter is based on Johansen and Singer (2011).[1]

> *"It is principle at games of chance that a multitude of illusions support hope and sustain it against unfavorable chances."*
>
> *Simon Laplace, 1796*[2]

5.1 Introduction

More than 60 years ago, Friedman and Savage (1948) noted that gambling and investment decisions are closely related. The most common form of gambling are state lotteries while the most popular investment security with gambling features are common stocks. On the one hand they serve as financial investment products providing a chance for great riches; on the other hand they significantly differ in their risk-return profiles and in the way they are distributed. Thus, the relation between both products is not clear. In this chapter we shed light on this problem and provide a clear answer to the question of whether they act as complements or as substitutes.

In a 2009 paper, Kumar analyzes the extent to which people's overall attitudes toward gambling influence their stock investment decisions using panel data from 1991 to 1996 of portfolio holdings and trades of 77,995 individual investors at a large U.S. discount broker house. As attitude toward gambling cannot be observed in the data Kumar uses socioeconomic characteristics, derived from lottery studies, to infer individual investors gambling preferences and attempts to detect traces of gambling in their

[1] This unpublished research paper is entitled "Lottery Tickets and Common Stocks: Complements or Substitutes?" and is available upon request.

[2] (Peterson, 2007, p. 176)

stock market decisions. Kumar defines lottery-type stocks as low-priced stocks with high idiosyncratic volatility and high idiosyncratic skewness and provides empirical evidence for his hypothesis *"that state lotteries and lottery-type stocks act as complements and attract very similar socioeconomic clienteles."* (Kumar, 2009, p. 1892)

However, Kumar's result is a thorn in the side of behavioral portfolio theory (BPT) (Das et al., 2010; Statman, 2004; Shefrin, 2002; Shefrin and Statman, 2000). In BPT, investors form portfolios using layered pyramids where each layer acts as a mental account or subportfolio with a different investment goal. In the bottom layer of the pyramid are securities designed to provide investors with security, such as savings accounts, insurance policies and money market funds. Further up the pyramid come riskier securities such as investment funds and real estate. At the pinnacle of the pyramid lie the most speculative investments, such as out-of-the-money call options, stocks and lottery tickets (Shefrin, 2002, p. 122). We refer this pinnacle layer to a gambling mental account intended for a shot at getting rich. We argue that less diversified stock portfolios, frequently observed among individual investors (e.g. Kelly, 1995; Goetzmann and Kumar, 2008), act as gambling accounts. Is such a gambling account complemented by lottery tickets or other stocks, diversification increases, risk and expected return decrease, which, however, contradicts the nature of gambling. Thus, we hypothesize that the opposite is true, namely, that lottery tickets and common stocks act as substitutes.

We test this hypothesis using cross-sectional data of more than 40,000 German households from 1993, which ranges in the observation period of Kumar (2009) and in a second step for 2008 in order to capture effects of changes in attitudes toward financial assets over time. To the best of our knowledge we are the first who investigate this relationship using a dataset that contains information about both lottery demand and stock ownership. The remainder is organized as follows: Section 5.2 presents data and research methodology while section 5.3 contains the main results. Section 5.4 provides a brief summary and implications for other economic phenomena.

5.2 Data and methodology

Earlier studies have dealt with lottery demand empirically (e.g. Beckert and Lutter (2008); Clotfelter and Cook (1991), but whether Kumar´s hypothesis of a complementary relationship between lotteries and lottery-type stocks holds, is still an open question. Looking at German households we are interested in testing the hypothesis *H: Lottery tickets and common stocks act as substitutes.*

We use data from the Einkommens- und Verbrauchsstichprobe (EVS) (German survey of household income and expenditure) which covers 40,230 households age 18 and older representative for the German population structure. German households are requested every fifth year to supply data on household income and expenditure, savings, durable consumer goods and the housing situation. In contrast to Kumar's portfolio data, in which explicit information about lottery demand is not available, the EVS contains such information. First of all, we use the 1993 survey to better understand and interpret our results in the light of Kumar's results, which are based on panel data from 1991 to 1996. In a future part of our analysis, we also use the latest available wave from 2008 to control for possible changes over time, since financial market structure and individual attitudes towards financial activities have changed. All descriptive results are weighted with an expansion factor in order to control for representativity.

We evaluate whether ownership of stocks (measured by receipt of returns on stocks) has an impact on lottery demand.[3] In order to avoid a sample selection bias due to a large number of households neither gambling nor owning stocks at all, we drop those households from our analysis answering both questions "no". Thus, the sample used in the multiple regression analysis is reduced to 18,866. As we are not able to make use of Kumar´s measure of lottery-type stocks, we define a new proxy for lottery-type stocks: As Kumar (2009) shows lottery-type stocks are mostly nondividend-paying stocks and information about income from paid dividends is available in our data, we define those stocks as of lottery-type that pay a small or zero dividend.[4] In this analysis, a small dividend yield is defined as to be smaller than the average dividend yield of the DAX.

We apply different multiple regressions in our cross-sectional sample. Due to the fact that we observe an action with two possible outcomes as denoted in (5.1), we perform Logit and Probit regressions on the probability of owning lottery tickets taking ownership of common stocks as exogenous.[5]

$$Y_i = \begin{cases} 1, & \text{with probability } p \text{ that an individual demands lottery tickets} \\ 0, & \text{with probability } 1-p \text{ that it does not.} \end{cases} \tag{5.1}$$

This can also be formulated as

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2^T Z_i + \epsilon_i \tag{5.2}$$

with Y_i being the observable case that person i spends money on lottery tickets or not, β_0 is the constant of the empirical model, β_1 is the marginal effect for our variable of interest, namely, that somebody holds (lottery-type) stocks or not, so that X_i takes on value 1, and Z_i is a vector of exogenous control variables also estimated with their corresponding coefficients and marginal effects indicated by vector β_2. The vector of exogenous control variables contains information on socioeconomic characteristics of the households like age, gender, income, education and others, which have also been of

[3] Originally, the variable records expenses for state lotteries and other games of chance. As our theoretical arguments and conclusions can be drawn for games of chance in general we define this variable for convenience as demand for lottery tickets.

[4] The EVS contains only one person owning stocks but receives zero dividend.

[5] A more detailed description of parameterizing probabilty p and formal expressions of marginal effects interpretation can be gathered e.g. from Cameron and Trivedi (2005).

interest in other lottery studies, so that we are able to compare our results to that of Kumar (2009); Beckert and Lutter (2008); Clotfelter and Cook (1991) and Brunk (1981). A more detailed description of these variables can be found in table 5.1.

5.3 Results

To begin with, we are interested in how many people demand lottery tickets or stocks. Descriptive results show that 47% of the full sample own either lottery tickets, stocks or both, while the rest neither hold stocks nor lottery tickets, so they do not gamble at all. Therefore, we leave this number of people out of our focus in order to avoid a selection bias. This leaves a number of 18,866 people for deeper analysis. These clearly separate into a group of 77% who demand only lottery tickets, 13% who only demand stocks, but still 10% who own both types of assets. If we define lottery-type stock owners as people who receive a return on dividends less than the two-year average return on dividends of the DAX in 1992-1993, which is 2.47%, in the reduced sample of 15,753 lottery players and/or lottery-type stock owners, we find that 96% solely play the lottery, 2% solely hold lottery-type stocks, and 2% have both. This provides first evidence for a substitional relationship between stocks and lottery tickets but needs to be investigated more detailed on the multivariate level.

Both Logit and Probit estimations are possible to evaluate the probability of playing the lottery, so that the decision which model is the adequate one has been made by comparing log likelihoods,[6] chosing the model with the highest value according to Cameron and Trivedi (2005). Wald statistics are tabulated to document the overall significance of the model with pseudo R^2 as a measure for the degree of variation of the endogenous variable explained by the regression.[7] Both coefficient estimates and marginal effects are tabulated using heteroscedasticity robust standard errors so that Logit models can be interpreted at odds and Probit models can be interpreted as marginal effects at means. We drop those people from our analysis who do not have stocks and do not play the lottery, so that a sample of 18,866 people is left for multiple analyzes. Table 5.2 shows the results of the Logit regressions.

It can be seen that across the estimation models 1 to 3, ownership of stocks has a significant negative influence on the probability of playing lottery. We are therefore not able to reject our main hypothesis that lottery tickets and common stocks act as substitutes, so that we oppose the results found by Kumar (2009). If we restrict the stock variable to the new lottery-type stock measure, the results ramain significant as can be gathered from model 4 to model 6 in table 5.3. In this case, lottery tickets and

[6] AIC and BIC criteria have also been used for model comparison but with the model showing the lowest value to be adequate. These criteria led to the same results as comparison of log likelihoods.

[7] Before we performed multiple regressions we tested for correlation among the exogenous variables but no strong correlation has been found so that problems due to collinearity are not suspected.

variable	description	expected sign
lottoD	dummy variable = 1 if the person spends money on lottery tickets	dep. var.
stockD	dummy variable = 1 if the person received returns on stocks	-
LTS	dummy variable = 1 if the person owns lottery-type stocks with a dividend per return on stocks in % lower than the 2-year average dividend of the DAX from 1992 to 1993 (which is 2.47%)	-
age	age of the first person the household in years	+
age^2	age squared	-
income	yearly household net income measured in Deutsche Mark (DM)	-
gender	dummy variable = 1 if the person is male	+/-
married	dummy variable = 1 if the person is married	+/-
job	dummy variable = 1 if the person is employed and works as a self-employed person, blue-collar worker, white-collar worker or public official	-
education	dummy variable = 1 if the persons highest school degree is university-entrance diploma (Abitur), polytechnical high-school or technical college entrance qualification	-
west	dummy variable = 1 if the person lives in West Germany	+
hhsize	number of people living in the household	+/-
german	dummy = 1 if person is German citizen	+

Table 5.1: Description of variables

lottery-type stocks show a clear substitutional relationship.

Furthermore, we do not find homogenous investors. Most of the socioeconomic control variables are highly statistically significant despite gender and job, which is significant only at 10% level so that it cannot be said that women behave differently from men or employed people behave different from unemployed people. Gender differences are in most empirical studies found despite that summarized by Rychlak (1992); all other studies show that men are more likely to play than women. Like Kumar we find that income is negatively related with demand for lottery tickets. This occurance of low income earners as the main group of people demanding lottery tickets may be explained by the same reasons given by Kumar (2009) and McCaffery (1994, p. 107) namely, that low income earners who are mostly less educated and have limited knowledge about other types of saving or other investment products, hope to win a large amount of money in a short period of time with all people being equal in distribution of chances.[8] Others like Beckert and Lutter (2008); Clotfelter and Cook (1991) and Brunk (1981) do not find a significant impact of income. With high income being positively associated with education we explain our findings of negative influence on lottery demand by the fact that high educated people have little interest in gambling but more in structured products. This also shows that financial literacy measured by education strongly influences gambling - the better educated the less likely they are to gamble. A negative relation is also being found by Kumar (2009) and Clotfelter and Cook (1991). Interestingly, age is of great importance as the probability of buying lottery tickets increases with age but the slope function follows an inverse u-shape which has also been provided by the results of Clotfelter and Cook (1991). On the other hand Kumar (2009) and Rychlak (1992) mention a negative impact of age on lottery demand whereas Beckert and Lutter (2008) and Brunk (1981) find strong positive age effects. Our findings as well as those of Clotfelter and Cook (1991) provide conflictive evidence to Kumar that married people are less likely to demand lottery tickets. If regional affiliation is concerned, we find that West German respondents are less likely to demand lottery tickets than people from East Germany but this should be investigated on lower aggregation level if the results should be compared to those of Kumar (2009) or Beckert and Lutter (2008).[9] A further difference to Kumar's results is that foreigners cannot be distinguished from German citizens because nationality does not have a significant impact on gambling activities. Finally, other socioeconomic aspects like religious interests and membership of a minority have been analyzed in the literature but cannot be covered by our dataset.[10]

[8]We use the education dummy as a proxy for financial education.

[9]Kumar's results show that people from economically fragile regions are more likely to play while Beckert and Lutter find people from urban areas to be more active.

[10]Clotfelter and Cook (1991) and Beckert and Lutter (2008) find positive impact of ethnic minorities and Clotfelter and Cook (1991) also show that Catholics are more likely to play.

5.4 Conclusion

This chapter provides strong empirical evidence that lottery tickets and common stocks act as substitutes. We conclude that it is not possible to explain this result without BPT, in which investors consider their portfolios as collections of mental accounting subportfolios virtually structured as layered pyramids.

Another explanation is financial education: We find a significant increase in lottery demand when education decreases. Thus, financially less educated investors prefer lottery tickets over common stocks, whereas the opposite can be deduced for financially educated people. However, financial education fails to sufficiently explain why so many people neither purchase lottery tickets nor common stocks. In contrast, BPT suggests that those investors do not maintain a gambling account at all.

At least three conclusions for the existing literature can be drawn: First, the results provide further evidence for BPT and are in line with recent empirical results by Choi, Laibson, and Madrian (2009), who find that mental accounting explains *"a lack of coordination between the asset allocations of different financial accounts in a household portfolio."* The results in this chapter further support theoretical implications of BPT by Shefrin and Statman (2000) and Das et al. (2010). Second, these results provide an explanation for the diversification puzzle (Statman, 2002, 2004), that is the observation that individual investors do not diversify among their stock portfolios: They simply view their undiversified stock portfolios as gambling accounts and do not diversify among stocks for the same reason that they do not diversify among lottery tickets. Third, it explains the great popularity of lottery bonds such as British Premium Bonds, which resemble index-linked certificates of deposit, but do not pay interest (Shefrin, 2002, p. 127). Instead, holders receive tickets to monthly lotteries that carry prices between £50 and £250,000. A Premium Bond packages a very safe security with a lottery ticket, and thus, serves two mental accounts.

As an implication for the financial sector, knowledge about customers could be used to advertise common stocks to those who participate in state lotteries. Furthermore, new segments for distributing financial products could be unlocked as lottery players are potential stock owners. This study leaves potential for future research: In the next step we analyze whether attitudes changed over time since the *dotcom* bubble, early influences of the subprime mortgage crisis and better access to financial markets have had a strong impact on financial decisions. Furthermore, we observe a fraction of 4,729 subjects who do not hold stocks but receive dividends. Those people sold their stock portfolios in the near past. It is interesting, whether these subjects have moved to lottery tickets or stopped gambling at all.

Logit estimations - dep. var.: lottoD						
	model 1		model 2		model 3	
variable	coeff.	marg. eff.	coeff.	marg. eff.	coeff.	marg. eff.
const.	***2.050		***0.500		**0.6637	
stockD	***-0.0000	***-0.0000	***-0.0000	***-0.0000	***-0.0000	***0.0000
	(-6.06)	(-5.95)	(-4.82)	(-4.70)	(-4.81)	(-4.69)
age			***0.0937	***0.0103	***0.0939	***0.0103
			(8.55)	(8.60)	(8.56)	(8.60)
age^2			***-0.0008	***-0.0001	***-0.0008	***-0.0001
			(-7.09)	(-7.14)	(-7.10)	(-7.14)
income			***-0.0000	***-0.0000	***-0.0000	***-0.0000
			(-11.03)	(-11.38)	(-10.58)	(-10.89)
gender			0.0092	0.0010	0.0087	0.0009
			(0.14)	(0.15)	(0.14)	(0.14)
married			***0.2501	***0.0288	***0.2670	***0.0309
			(3.96)	(3.81)	(3.86)	(3.69)
job			*0.1317	*0.0147	*0.1331	*0.0149
			(1.70)	(1.67)	(1.73)	(1.69)
education			***-0.5393	***-0.0618	***-0.5412	***-0.0620
			(-10.98)	(-10.84)	(-11.05)	(-10.90)
west			***-0.4152	***-0.0410	***-0.4198	***-0.0414
			(-5.16)	(-5.87)	(-5.23)	(-5.94)
hhsize					-0.0116	-0.0012
					(-0.49)	(-0.49)
german					-0.1465	-0.0153
					(-0.86)	(-0.91)
Wald Chi^2	***36.77		***852.37		***851.89	
pseudo R^2	0.0979		0.1332		0.1332	
n	18,866		18,866		18,866	

Note: heteroscedasticity robust standard errors, t and z-values in parantheses, significance levels: *** 1% -level, ** 5% -level, * 10%-level, stockD and income show a quantitative effect at the 6th decimal place, Wald Chi^2 $H_0 : \beta_0 = \beta_1 = \cdots = \beta_j = 0$. Numbers are truncated after the fourth decimal place.

Table 5.2: Logit model - estimation results

Probit estimations - dep. var.: lottoD						
	model 4		model 5		model 6	
variable	coeff.	marg. eff.	coeff.	marg. eff.	coeff.	marg. eff.
const.	***1.1437		***0.4035		***0.4548	
LTS	***-1.2468	***-0.4147	***-1.0558	***-0.3278	***-1.0551	***-0.3276
	(-26.60)	(-26.60)	(-21.32)	(-21.32)	(-21.30)	(-21.30)
age			***0.0531	***0.0108	***0.0530	***0.0108
			(9.04)	(9.04)	(9.01)	(9.01)
age^2			***-0.0004	***-0.0001	***-0.0004	***-0.0001
			(-8.06)	(-8.06)	(-7.97)	(-7.97)
income			***-0.0000	***-0.0000	***-0.0000	***-0.0000
			(-18.33)	(-18.33)	(-18.02)	(-18.02)
gender			0.0035	0.0007	0.0027	0.0005
			(0.11)	(0.11)	(0.08)	(0.08)
married			***0.1844	***0.0397	***0.1744	***0.0374
			(5.54)	(5.54)	(4.66)	(4.66)
job			***0.1062	***0.0222	***0.1050	***0.0220
			(2.61)	(2.61)	(2.58)	(2.58)
education			***-0.3221	***-0.0681	***-0.3215	***-0.0680
			(-12.36)	(-12.36)	(-12.33)	(-12.33)
west			***-0.2144	***-0.0401	***-0.2144	***-0.0401
			(-5.22)	(-5.22)	(-5.20)	(-5.20)
hhsize					0.0068	0.0014
					(0.82)	(0.82)
german					-0.0608	-0.0120
					(-0.65)	(-0.65)
Wald Chi^2	***707.41		***1446.98		***1449.95	
pseudo R^2	0.0456		0.1054		0.1054	
n	15,753		15,753		15,753	

Note: heteroscedasticity robust standard errors, t and z-values in parantheses, significance levels: *** 1% -level, ** 5% -level, * 10%-level, income shows a quantitative effect at the 6th decimal place, Wald Chi^2 $H_0 : \beta_0 = \beta_1 = \cdots = \beta_j = 0$. Numbers are truncated after the fourth decimal place.

Table 5.3: Probit model - estimation results

6 Summary and Conclusion

> *"I'd be a bum on the street with a tin cup if the markets were always efficient."*
>
> *Warren Buffet*[1]

The main body of this dissertation consists of four chapters dealing with selected topics in the field of behavioral portfolio management. A detailed summary of the results and concluding remarks are presented at the end of each chapter. This chapter addresses only the most significant contributions.

Chapter two investigates the role of behavioral portfolio theory in saving for retirement in Germany and documents (i) an impact of emotions on behavioral portfolios, since the security-minded (potential-minded) is the most conservative (aggressive) portfolio; (ii) concentrated behavioral portfolios with a large proportion in only one secure asset (endowment insurance) and a small proportion in risky assets; and (iii) mean-variance portfolios, which are more diversified than behavioral portfolios. Chapter three deals with a theoretical issue of the single mental account version of behavioral portfolio theory and addresses the question of whether a random threshold level should be preferred over a fixed threshold or vice versa. In a world with normal distributed returns and in a simple distribution-free setting, I obtain the following results: The random target strategy outperforms the fixed target strategy if the portfolio return and the random target are positively correlated and riskless investing is prohibited; meanwhile the fixed target strategy outperforms the random target strategy if the portfolio return and the random target are not positively correlated and riskless investing is allowed. Chapter four studies behavioral portfolio theory with multiple mental accounts and investigates goal-specific asset selection and utility in a setting similar to Das et al. (2010). When goal-specific asset selection is allowed, the solution to the aggregate portfolio can be

[1](Peterson, 2007, p. 9)

found on a new mean-variance efficient frontier, which is induced by the solutions to mental account subportfolios. When goal-specific asset selection is not allowed, the solution to the aggregate portfolio lies on the mean-variance efficient frontier induced by all assets under consideration and coincides with the optimal solution to one mental account subportfolio. In this case, I derive a utility function consistent with Friedman and Savage's (1948) solution to the insurance lottery puzzle. Chapter five uses behavioral portfolio theory to derive the hypothesis that lottery tickets and common stocks act as substitutes. We find strong empirical evidence for this hypothesis and conclude that it is not possible to explain this finding without behavioral portfolio theory.

Based on these findings, I draw one main conclusion: Since "real" investors do not behave like the *homo oeconomicus*, their financial decisions must not based on traditional economic theory. Portfolio selection decisions must not be made solely based on risk and return. Behavioral aspects should also be taken into account. Hence, behavioral portfolio management should play a greater role in financial industries. For instance, relevant behavioral theoretical elements should be collected and implemented in one tool that serves as what Shlomo Benartzi[2] calls "flight simulator" to simulate several behavioral scenarios and obtain a description rather than a prescription of how to react to these scenarios. Future research may involve all steps towards such a tool.

[2]In an interview in the Frankfurter Allgemeine Zeitung (November 2010), Shlomo Benartzi suggested a "flight simulator" as a tool investment advisers could use to test their clients' willingness to take risk.

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