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Traditio et Innovatio

Tempo effects in mortality:

Theoretical considerations and empirical solutions

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1. Contents

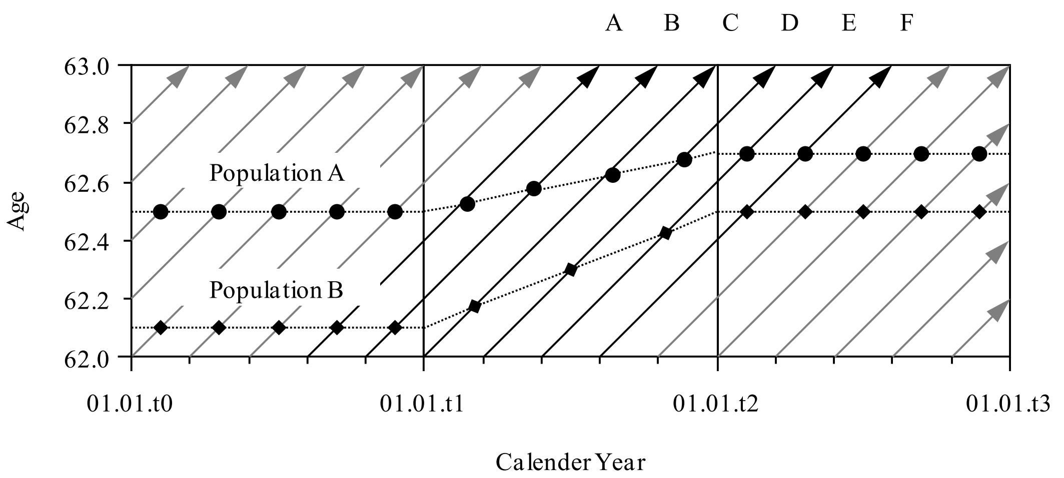
2. Introduction

During my first semester of studying demography at the University of Rostock, I was introduced to the concept of the period life table, a powerful tool in demographic analysis. Instead of analysing the single death rate of a year for at least 100 ages, the life table summarises all this information and provides only one important indicator: period life expectancy. Based on the simple assumption that period death rates will remain constant, life expectancy is the average life time or the mean age at death for an individual living in the current year. Thus, life expectancy is much easier to understand than a set of period death rates. Moreover, the level as well as the trend in period life expectancy is used as an indicator representing medical progress, improvements in socioeconomic conditions, and the successful implementation of social and health-behavioural policies (Riley 2001). Furthermore, life expectancy is an important indicator for cross-country comparisons of population health status and of social and lifestyle conditions (Wang et al. 2012). But the relevance of period life expectancy is much broader because of the extension of special methods that provide a detailed description of the changes in period mortality conditions. Thus, the level of life expectancy can be decomposed into the contributions of age and causes of death (Arriaga 1984, Pollard 1988). Other methods allow us to differentiate between life expectancy in good or bad health (Sullivan 1971, Salomon et al. 2012), or between the various levels of survival conditions caused by the heterogeneity of the population (Vaupel and Yashin 1985). Although the concept of the period life table is widely used in mortality research, the approach is not restricted to this field of demographic research. The application of the period life table techniques can also be found in fertility (Rallu and Toulemon 1994) or nuptiality research (Schoen and Nelson 1974).

Shortly before I finished my studies, I attended to a course that focused on special and new methods in mortality research. The lecturer presented a recent article by Bongaarts and Feeney (2002) which fundamentally criticised period life expectancy as an indicator for characterising period mortality conditions. The main conclusion of the authors was that period life expectancy is always distorted by a tempo effect, even when the mortality conditions in a period have been changed. In addition to providing theoretical evidence to support this claim, they argued that period life expectancy overestimates the survival conditions of recent periods by an average of two years in some developed countries today. Like many of other demographers, I was completely surprised by and sceptical of their results. If life expectancy is distorted by tempo effects, as Bongaarts and Feeney asserted, does that mean that all of the previous results and applications of period life tables over the last 100 years are distorted as well?

A simple example presented and published by the lecturer (Luy 2008) provides an initial introduction to the idea and meaning of tempo effects in period mortality. Figure 1 shows the ages at death for two different populations between ages 62 and 63 over three years. Each year and age group are illustrated by single rectangles. The diagonal arrows represent cohort life lines for a group of 20,000 individuals at age 62. As life events like birth, marriage or death occur, they can be illustrated along the life lines.

Figure 1: Two populations with different levels of mortality and differently decreasing mortality in year t1



Source: Luy (2008, p. 207)

In this example, each dot of both populations along the cohort lines in the first year t0 illustrate the deaths of 1000 individuals. Related to left age axis, 5000 deaths in Population A occur exactly at age 62.5 in the first year, while 5000 deaths in Population B happen at the lower age of 62.1. In year t0, no variation in mortality occurs, and the individuals in this age group in Population B die earlier than their counterparts in Population A. As a consequence, the death rate presented in the first column of Table 1 is lower for Population A than for the other population.

In the next year t1, better survival conditions lead to an increase in the age at death for Population A of 0.2 years and for Population B of 0.4 years. The easily identifiable consequence is the shifting of deaths outside the analysed year. In contrast to the earlier year, only 4000 deaths in age group 62 are observable in Population A, while 1000 deaths move along the life line E to the next year t2.

Table 1: Resulting death rates (*1000) for age 62 in Populations A and B experiencing differently decreasing mortality

	t0	t1	t2
Population A	51.3	***40.7***	50.8
Population B	52.4	***30.6***	51.3

Source: Luy (2008, pp. 206-209)

Moreover, due to the greater increase in the ages at death, the reduction in deaths is more significant for Population B. Thus only 3000 deaths occur in year t1 compared to the 5000 deaths of the earlier year. The remaining 2000 deaths move along the life lines E and F to the next year. In both populations, the death rates drop immediately due to the increase in life time. But what is curious is that the death rate is lower in Population B, even though the age at death as illustrated in Figure 1 is still lower than that of Population A.

Moreover, the observed death rates in year t1 become much more erratic when compared to the new constant rates in year t2. In the third year, the improvement in survival conditions stops and again leads to constant but higher ages of death for both populations. The deaths now start to decrease at age 62.7 in Population A and at age 62.5 in Population B. Since the ages at death are again constant, no further shift in deaths occurs, and again 5000 individuals in each population die in year t2. The resulting death rates in the third year are lower than in the initial year t0, and Population A again has a lower rate than Population B due to the higher age at death. But the comparison to the second year reveals another curious development: namely, that the current death rates in year t2 are higher, even though the age at death is higher than that of the preceding year.

These two unexpected results are caused by mortality variation and the inherent appearance of shifted deaths. Therefore, the temporary missing of deaths due to the shift in deaths leads to an undesired inflation or deflation of death rates. This undesired effect is interpreted as a **tempo effect** (Bongaarts and Feeney 2002, Bongaarts and Feeney 2008a, Bongaarts and Feeney 2008b). In both populations, the number of deaths temporarily deviates from the constant number of 5000 deaths, which leads to a deflation in the annual death rates. Moreover, because the number of shifted deaths is higher in Population B, the tempo effect is higher and causes more deflation than in Population A. Thus, the tempo effect distorts death rates and their interpretation of current mortality conditions. Without considering the tempo effect, we would conclude for the second year t1 (i) that the survival condition in Population B was better than in Population A, and (ii) that survival conditions had become worse in the third year t2 relative to the previous year. Both conclusions are completely wrong because the ages at death only increase, and Population A never experience a lower age at death than Population B. If we then assumed that at each age the death rates were distorted due to mortality change in the same way as in the example, the resulting period life table and their important indicator, the life expectancy, would be distorted as well.

After this seminar, it was completely obvious to me that the extent of tempo effects in death rates, and, consequently, in period life expectancy, have to be adjusted if we want to avoid mistakes in quantifying and interpreting current period mortality conditions. Thus, conventional life expectancy and other period mortality indicator are distorted by tempo effects. However, when I started to research in more detail the occurrence and impact of tempo effects, I encountered three main counterarguments which raised doubts about the appearance and impact of tempo effects in period mortality indicators.

The theoretical occurrence of tempo effects is only explained by one method which calculates the age-specific death rate based on the number of deaths and persons at risk within a one-year age and period interval (Horiuchi 2008, Feeney 2010). This set of deaths is also used in the previous example (Figure 1). Indeed, it has been shown that this kind of mortality rate is generally affected by a bias when mortality is changing (Hein 2001). Therefore, the tempo effect, as illustrated in the previous example, is only caused by an insufficient method for estimating period death rates.

The second counterargument refers to the definition of changes in period mortality. One definition is presented in the example in Figure 1. This definition was also accepted by Bongaarts and Feeney (Guillot 2008, Bongaarts and Feeney 2010). Instead of only producing mortality rates, the period mortality conditions produce a delay by a certain amount of time for all of the shifted deaths of the period. In the example, the delay due to changed mortality is 0.2 years per year for Population A (and 0.4 years per year for Population B). Consequently, the shifted deaths from year t1 will definitely occur in the next year, but they lead to a temporary missing of deaths and a deflation in the death rates in the period of improving survival conditions.

The definition of mortality changes proposed by Bongaarts and Feeney differs markedly from the traditional perspective. The conventional perspective assumes that current mortality conditions produce a precise set of age-specific death rates. Therefore, changes in the number of period deaths ─regardless of whether they are temporarily missing─ are naturally covered by the inherent decrease or increase in mortality rates (Wachter 2008). If mortality conditions become constant, the observed death rates will not change any further. In the example above, this assumption means that the death rates in year t1 already reflect a new constant mortality level, regardless of whether the deaths are shifted. This perspective is reflected in the period life table, which rests on the assumption that current death rates remain constant in the future. Thus, period life expectancy "is an unbiased indicator of period mortality conditions, and no adjustment is needed" (Guillot 2008, p. 140). But this conventional argument does not fit the further trend in mortality cited in the above example, which lead to an obvious distortion. However, the proponent of the traditional view mentioned only that this trend had never been observed in empirical data.

The third counterargument refers to the proposed methods for adjusting tempo effects in period life tables. Bongaarts and Feeney's basic assumption was that mortality rates have a constant shape which is shifted to higher or lower ages as a result of mortality variations (Bongaarts and Feeney 2002, Bongaarts and Feeney 2008a). However, a number of scholars have pointed out (Wilmoth 2005, Goldstein 2006) that this assumption leads to a weighted average of the life expectancies of those cohorts living at different ages in the analysed period. These critics then argued that the proposed tempo-adjusted life expectancy covers past mortality experiences and not current period conditions. The last two critiques are interrelated because the adjusted methods proposed by Bongaarts and Feeney refer to the description of how period mortality changes (Guillot 2008).

Despite these critiques ─or perhaps because of them─ I became increasingly interested in analysing the mechanisms of tempo effects in mortality. I noticed that there was a lack of basic research in this area, as the recent studies on the pros and cons of tempo effects started with a discussion of the proposed methods for tempo adjustment, without providing any proofing of the underlying mechanisms or an interpretation that went beyond idealised models. Therefore, the aim of my thesis is to provide basic research that seeks to explain the appearance of shifted deaths as a further mortality-related event in period mortality analysis. Based on this aim, I further want to show how shifted deaths can distort the interpretation of current mortality by applying period life table indicators. The structure of the thesis is based on three separate journal articles. Each of these articles includes a discussion of the literature, some specific research questions, a description of the data and the methods used, and a concluding section that describes the results obtained.

The first article deals with the following question: ***How can shifted deaths be adjusted through the use of other methods for calculating mortality rates?*** This question refers to the first critique that shifted deaths can only appear when applying a specific death rate, which is generally affected by a bias when mortality is changing. Therefore, the aim of this paper (Wegner 2010) in chapter three is to extend the idea of the appearance of tempo effects to other methods for calculating age-specific mortality rates. I hypothesise that tempo effects could be eliminated or at least minimised through the use of a special method for estimating period death rates. Based on three different sets of deaths resulting from the overlap of age, time and birth intervals, I attempt to explain the occurrence of tempo effects for three kinds of mortality rates. Instead of only considering the shift in deaths within an age interval, like in Figure 1 and other examples from previous research (Horiuchi 2008, Luy 2008, Feeney 2010), I also allow deaths to move outside the considered period, as well as outside the present age interval. By using one modelled trend of period mortality reduction, I am able to show that two theoretical kinds of tempo effects exist. The empirical validation of these theoretical findings is proofed through a comparison of the three different rates and the resulting tempo-adjusted life expectancy at age 50 for the year 2005 for more than 20 countries.

The second article (Luy and Wegner 2009) in chapter four asks the following question: ***What are the functions that have to be performed by period life expectancy?*** The discussion about tempo effects concluded that a distortion of life expectancy or other period indicators depends on different perceptions about the change in period mortality. However, the hypothesis is that period mortality measures should always reflect current mortality conditions, regardless of the extent to which period mortality varies. This objective can be broken down into a technical and a practical requirement for an indicator reflecting current period mortality. The technical function refers to the method for standardising the occurrence of shifted deaths within the standard life table method, while the practical function is related to the ability to cover all relevant information about current mortality conditions. By applying and comparing the traditional and the new views on mortality changes in the two exemplary populations, it can be shown that tempo-adjusted life expectancy is a more appropriate method for handling both kinds of tasks. Moreover, these results make it possible for the first time to provide a clear and easily understandable definition of tempo-adjusted life expectancy that goes beyond the technical aspects of this measure. In addition, the empirical estimates for tempo-adjusted life expectancy for the years 2001 and 2005 for more than 40 countries demonstrate the significant impact of tempo effects on the interpretation of recent period mortality conditions.

The third question is as follows: ***How should shifted deaths be represented and measured in a conventional life table?*** The question refers to the third article (Wegner-Siegmundt 2013) in chapter five, which is directly connected to the previous chapter. The limitations of the previous conclusions are accompanied by the modelling of shifted deaths and their indirect measure in empirical data. The tempo effect is then quantified in years by the difference between the proposed tempo-adjusted life expectancy by Bongaarts and Feeney and the conventional life expectancy. But most of the critiques of tempo effects refer to the assumption that age-specific death rates maintain a constant shape, and to the inherent assumption that the proportions of shifted deaths are independent of age in the tempo-adjusted life expectancy. In my opinion, Bongaarts and Feeney chose this assumption because they had not been able to indicate the age-specific differences in the proportion of shifted deaths in the empirical data. Measurements of age-specific shifted deaths have in fact been made, but only within a cohort perspective (Vaupel and Yashin 1986, Vaupel and Yashin 1987). I propose a method for translating this approach to a period perspective that can be used to characterise the intensity and timing of period-shifted deaths by age. This would make it possible to show the influence of these shifted deaths in the period life table, as it is still suggested in the previous article. Surprisingly, I find that the shifted deaths mainly make up the expectation portion of the period life table and their derived indicators. Related to the example in Figure 1, I find that the full period life table includes the remaining 4000 deaths of the year t1, and only shapes the expected ages at death for those 1000 individuals who experience a shift in their age at death in this year. But these expectations can be easily modified to fulfil the practical demand on the period mortality indicators, as it is proposed in the chapter 4.

At the end of the thesis, I summarise the results of all three articles which fill the existing lack of basic research on the origin and mechanisms of tempo effects. Thus, my research provides important and innovative insights regarding the appearance and the meaning of tempo effects, but it is also a statement about the need for additional research and empirical application. At the moment, only a handful of demographers are researching the tempo effect. John Bongaarts said to me after my presentation on the age-specific shifted deaths at the PAA 2012: “Welcome to the exclusive club of tempo researcher!” I hope, however, that the results of my thesis will lead to an expansion in the size of this club.

3. Tempo effects in different calculation types of period death rates

Wegner, Christian (2010) in Comparative Population Studies - Zeitschrift für Bevölkerungswissenschaft, Vol. 35(3), 543-568

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Abstract

The question as to whether or not tempo effects distort the measurement of period mortality is controversial in recent demographic research. Only few publications, however, illustrate the underlying phenomenon of tempo effects, namely that the period death rate may increase although the mortality of all cohorts living during the analyzed period has fallen. Moreover, related literature only focuses on one of three methods to derive the age-specific death rate. This article primarily deals with the questions whether other methods of age-specific death rate are also affected by tempo effects in the logic of Bongaarts and Feeney and whether the tempo effect can be minimised solely by applying a specific method. The results demonstrate that all types of death rates are influenced by tempo effects and that different methods do not eliminate the influence of tempo effects. Nevertheless, it is necessary to distinguish between two types of tempo effects, which can be revealed in theoretical as well as empirical perspective.

3.1. Introduction

The previous debate about tempo effects in period mortality has been essentially concerned with the question whether tempo effects influence period mortality indicators such as life expectancy at birth and whether current mortality conditions are therefore distorted (Bongaarts and Feeney 2002, Vaupel 2002, Wilmoth 2005, Bongaarts and Feeney 2008b, Guillot 2008, Luy 2008, Rodríguez 2008, Vaupel 2008, Wachter 2008, Luy 2009, Bongaarts and Feeney 2010). Only few publications explicitly deal with the unexpected and curious phenomenon that the trend in period death rates fluctuates despite a continuous improvement in survival conditions of all cohorts living during the analysed period (Horiuchi 2008, Luy and Wegner 2009, Feeney 2010). According to the logic of Bongaarts and Feeney (Bongaarts and Feeney 2002, Bongaarts and Feeney 2008a, Bongaarts and Feeney 2008b), these fluctuations in the death rates are caused by tempo effects. They are accompanied by a temporary change in the number of deaths within a period in which mortality conditions have changed. Although the unexpected trend in the period rates forms the basis of Bongaarts' and Feeney's methodical and empirical research, fundamental questions about tempo effects remain open until today.

A shortcoming in current research is the relation between the method to derive the death rate and the occurrence of tempo effects. Previous research merely analyses the cause of tempo effects by using the age-year-method, also known as type I rate. However, there are two further methods to compute the period death rate: the cohort-year-method (type II rate) and the age-cohort-method (type III rate). This leads to the question whether these two methods are also affected by tempo effects if period mortality has changed. All three methods are based on different numbers of deaths resulting from the overlap of age, time and birth intervals. Subsequently, changes in period mortality conditions have differing impacts on the respective method. Therefore, it can be assumed that the cause of tempo effects also differs. Due to the different characteristics of each method, a second important question can be raised: Does the extent of existing tempo effects depend on the selected type of death rate?

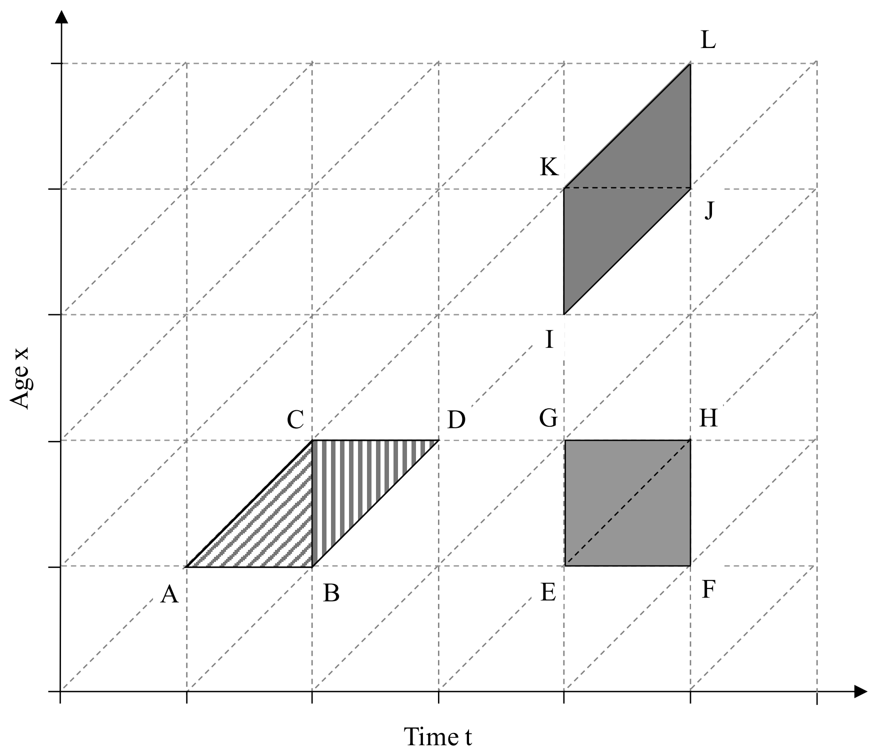
To answer these questions, the article is structured as follows: The first part of this paper introduces the different methods of deriving the death rate. The second part graphically illustrates the tempo effects in the logic of Bongaarts and Feeney by using the Lexis diagram. Their impact on age-specific mortality is explained by applying modelled numbers of living and deceased persons. A general classification of the tempo effects regardless of the model assumptions is carried out in the third section. The last part finally presents the effects of the different types of death rates and the resulting differences in tempo effects based on empirical data for 26 countries.

3.2. Methods to derive death rates

Mortality research distinguishes between three methods of deriving death rates. These methods are typified by the death counts emerging from the overlap of age, time and birth intervals. All three intervals can be presented graphically by using the Lexis diagram in Figure 2. The abscissa of the diagram shows the calendar time, whilst the age is levelled on the ordinate (Feichtinger 1973, pp. 18-25). All people born in a specific year and their demographically-relevant events can then be shown diagonally to age and calendar time. The overlap of age, time and birth intervals allows to extract two triangles of events (Becker 1874) which are marked in the Lexis diagram by right-angled triangles.

The 1st triangle of deaths includes the number of individuals of a birth cohort c who died at age x in year t. This area is shown in Figure 2 by the triangle ABC (cf. Table 2.1). The 2nd triangle of deaths also contains all persons of the cohort $c$who died at age x, but in the following year $t+1$. They are presented by the triangle BCD in Figure 2. The legs of each triangle present two different sets of living persons. Individuals who have reached age x in a year t are summarized as persons living of the same age (line AB in Figure 2). All persons aged x who lived exactly at the beginning of year $t+1$ are characterised as persons living at the same time (line BC in Figure 1). Both sets of living persons constitute the number of states whilst the triangles of death include number of events at a specific time or age (cf. Table 2.1).

Figure 2: Classification modes of deaths and living persons in the Lexis diagram



Source: based on Caselli and Vallin (2006)

The triangles of deaths and the sets of living persons define three methods for computing the death rate. The combination of both death triangles determines three standard ways of classifying the number of deaths which forms the numerator of each death rate (Becker 1874). The denominator contains the person-years which are estimated by the number of living persons (Feichtinger 1973, pp. 55-56).

The most common procedure used to derive the death rate in official statistics is the age-year-method; also referred to as the type I rate (Flaskämper 1962, pp. 342-391, Wunsch and Termote 1978, 85-87, Caselli and Vallin 2006, pp. 61-63). The German Federal Statistical Office has applied this method since the General Life Table of 1970/72 (Statistisches Bundesamt 2006). The method is based on the 3rd class of deaths resulting from the overlap of an age and year interval (square EFGH in Figure 2). This classification of deaths contains the 1st triangle of deaths of cohort c and of the 2nd triangle of the previous birth cohort $c-1$ at age x in year t (Table 2.2). The type I death rate ${}^{I}m(x,t)$ then is the quotient of the 3rd class of deaths to the person-years at age x in year t (Table 2.3). The average of the living persons at the same time at the beginning (line EG) and at the end (line FH) of year t is used as an approximation of the number of person-years.

Table 2: Surfaces of deaths and living persons and different methods for computing the period death rate

2.1. Events and states		Lexis diagram in Figure 2
${}^{I}D(x,c)$	1st triangle	Area ABC
${}^{II}D(x,c)$	2nd triangle	Area BCD
${}^{t}P(x)$	Persons living at the same age	Line AB
${}^{x}P(t+1)$	Persons living at the same time	Line BC
2.2. Classifications of deaths		
$D(x,t)$	3th class	Area EFGH
$D(c,t)$	2nd class	Area IJKL
$D(c,x)$	1st class	Area ABCD
2.3. Death rates		
${}^{I}m(x,t)$	$= \dfrac{D(x,t)}{0.5\cdot\left[{}^{x}P(t)+{}^{x}P(t+1)\right]}$	Type I death rate (age-year-method)
${}^{II}m(c,t)$	$= \dfrac{D(c,t)}{0.5\cdot\left[{}^{x-1}P(t)+{}^{x}P(t+1)\right]}$	Type II death rate (cohort-year-method)
${}^{III}m(c,x)$	$= \dfrac{D(c,x)}{{}^{x}P(t+1)}$	Type III death rate (age-cohort-method)

Source: based on Becker (1874), Caselli and Vallin (2006)

The National Institute of Statistic of France uses the type II rate to determine the period survival conditions, which is also known as the cohort-year-method (Flaskämper 1962, pp. 364-365, Wunsch and Termote 1978, pp. 85-87, Caselli and Vallin 2006, pp. 61-63). The type II death rate is based on the 2nd class of deaths determined by the overlap of a birth and a period interval (area IJKL in Figure 2). This class is composed of the 1st triangle of deaths of the cohort c at age x and the 2nd triangle of the same cohort at the previous age $x-1$ (Table 2.2). The comparison with the type I rate shows that the cohort-year-method includes all death counts of a cohort within a year t. However, a specific age classification is not possible because the deaths are stretched over two age groups. The number of person-years is estimated from the average of the individuals living at the beginning and the end of the observed period t. In contrast to the age-year-method, however, the living persons are aged $x-1$ at the beginning of the year (line IK in Figure 2), whilst the surviving persons are aged x at the end of the year (line JL in Figure 2). Accordingly, the death rate type II ${}^{II}m(c,t)$ is calculated from the quotients of the 2nd class of deaths to the approximated number of person- years (Table 2.3).

The last method is the type III death rate, which is also referred to as the age-cohort-method and was applied, for example, to calculate the first General Life Table of the German Reich 1871/81 (Becker 1874, pp. 38-45, Wunsch and Termote 1978, pp. 85-87, Caselli and Vallin 2006, pp. 61-63). Determining mortality by this method is an uncommon procedure in period analysis. Due to the characteristics of the method, it is mainly used in cohort analysis (Caselli and Vallin 2006). The type III death rate ${}^{III}m(c,x)$ takes into account the total number of deaths resulting from the overlap of a cohort and age interval. This area is referred to as the 1st class of deaths and comprises the 1st and 2nd triangle of a cohort c at age x (parallelogram ABCD in Figure 2). This class of deaths does not cover the number of deaths within one calendar year but adheres to two periods t and $t+1$. The number of living persons aged x at the beginning of year $t+1$ (line BC in Figure 2) is used as an approximation of the number of person-years to derive the death rate type III.

3.3. Tempo effects in different types of death rate

The presence and cause of tempo effects in each type of death rate are analysed by a modelled decline in mortality. The processes are graphically derived and explained in the Lexis diagram. The mortality model is based on the discrete models which are commonly used in literature for describing tempo effects (Luy 2008, Feeney 2010). Although all these models include simplified assumptions, they can be adjusted to a real population without modifying the underlying statements. Furthermore, other distortions, such as heterogeneity or selection effects, are excluded from the model.

The model illustrates a population in which no migration takes place and in which an annual constant number of births is distributed uniformly over the respective birth year. It is further assumed that individuals only die at a certain age x, whilst no mortality occurs in the preceding age $x - 1$ and the next age $x + 1$. Within age x, the number of deaths are distributed over five different dates at intervals of 0.2 years. The mortality conditions are assumed to be constant until the beginning of year t, so that the population is stationary. The decline of mortality is modelled by a linearly rising age at death at the rate of 0.2 years per year in period t. In the following year $t + 1$, age at death remains constant at the new, higher level. The new mortality conditions stay the same but have decreased in comparison to the base level. Hence, the model presents a population with constant mortality until the beginning of year t following by a decline of mortality in year t. From year $t + 1$ onwards mortality remains constant at a new level. There is never an observable contrary mortality-increasing effect. Further, with the help of modelled samples of living and deceased persons, the trend in mortality is illustrated. Under the constant mortality conditions, 1,000 persons alive precisely age x. Within the observed age group, 100 persons die uniformly distributed across the five dates of death.

3.3.1. The tempo effect in the age-year-method (type I rate method)

The Lexis diagram in Figure 3a shows the 3rd class of deaths at age x for the periods $t - 1$ to $t + 1$. Based on the model assumptions, constant mortality conditions are prevalent until the beginning of year t. Deaths in year $t - 1$ are spread over five times marked by vertical lines within the 3rd class of deaths at age x. In Figure 3a, they are labelled in as a1 to a5. The decline in mortality in year t goes hand in hand with a linear increase in age at death by 0.2 years per year. The increase in lifetime causes a postponement of deaths diagonally to the age and time axes. Consequently, two relevant effects occur:

(I) The first effect is an enlarged gap between the times of death (Feeney 2008, Horiuchi 2008). Under constant conditions, the range between the times a1 to a5 is 0.2 years. The gap (b1 to b4) increases to 0.25 years during the increase in age at death. Therefore, the space between times of death has widened in the year of the mortality change, which directly causes the second effect.

(II) As a result of the increased gap in the times of death, the last date b5 (and also all death counts belonging to it) is postponed into the following year $t + 1$. Consequently, the number of times of death in year t falls from five to four, and hence the number of deceased persons. Furthermore, the increasing age at death causes a shift of deaths into the next age $x + 1$ (area S1 in Figure 3a). Nevertheless, these deceased persons are still covered by the observed year t.

In the next period $t + 1$, the age at death remains constant at the new, higher level. The range between the times of death b5 to c4 has fallen from 0.25 to 0.2 years. Hence, the number of times of death has again risen to the old stationary level of five per year. However, the sequence of the times shows that the formerly constant number of deaths at age x of year $t - 1$ has been postponed both in time and age. In year $t + 1$, firstly those persons die who would have died in year t under the old mortality conditions (time of death b5), followed by the times c1 to c4. Moreover, time b5 as well as the subsequent time points are spread over two age groups. Within the age x, 80% of deaths are covered in year $t + 1$, whilst the remaining 20% take place early at age $x + 1$.

In a real population, the death rate would only consider those deaths which remain at age x. However, the shifted deaths from the previous age $x - 1$ during year t (area S2 in Figure 3a) and year $t + 1$ (area V in Figure 3a) would be included in the calculation of the death rate. This means that postponed deaths from previous ages during the mortality change may minimise or compensate for shifted deaths of the time b5. This is hypothetically the case if the number of deaths in area S2 is greater than the number of deaths in b5. Yet, in order to avoid the effect of postponed deaths of prior age on the derived death rate on the one hand and to analyse the overall impact of the postponed time of death b5 on the other hand, only the previously stationary number of deaths is included in the following calculation. Therefore, the age range must be extended from x to $x + 1.2$ in order to cover the age-shifted number of deaths. Only the net effect of the number of deaths due to the increasing age at death will be considered, because the model assumes no mortality at the preceding and following ages.[1]

The person-years (bordered) and the number of deaths (underlined) for the years $t - 1$ to $t + 1$ are shown in Figure 3b.

[1] An adequate model for the change in mortality over several age groups and the resulting trend in age-specific mortality rates was also simulated and led to identical tempo effects in the respective period mortality rate. Corresponding model calculations can be provided by the author by request.

Figure 3a: Mortality decline in the 3rd class of deaths and the resulting tempo effect

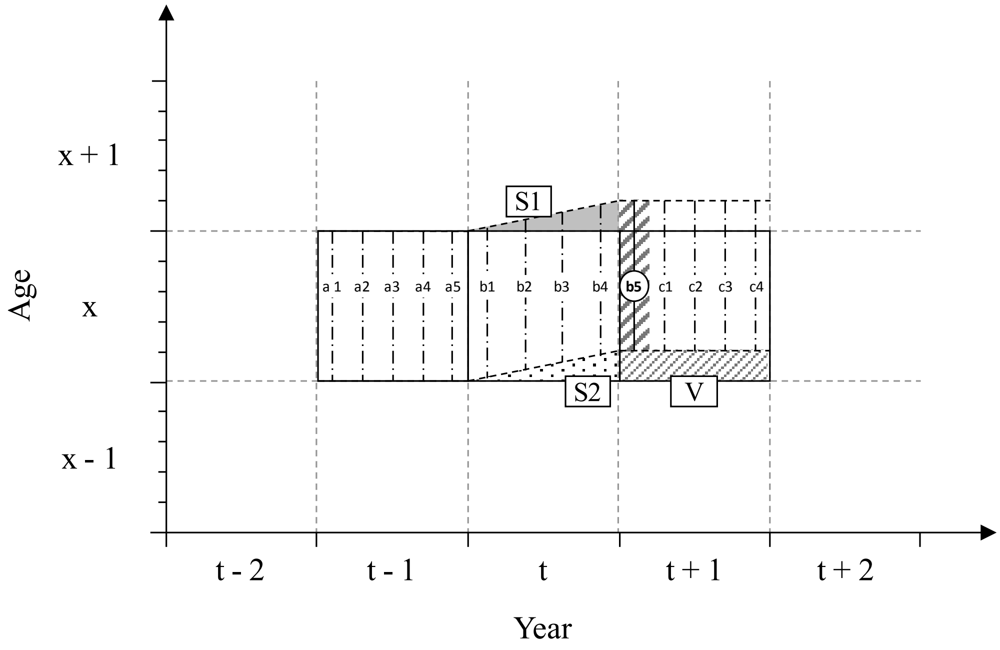
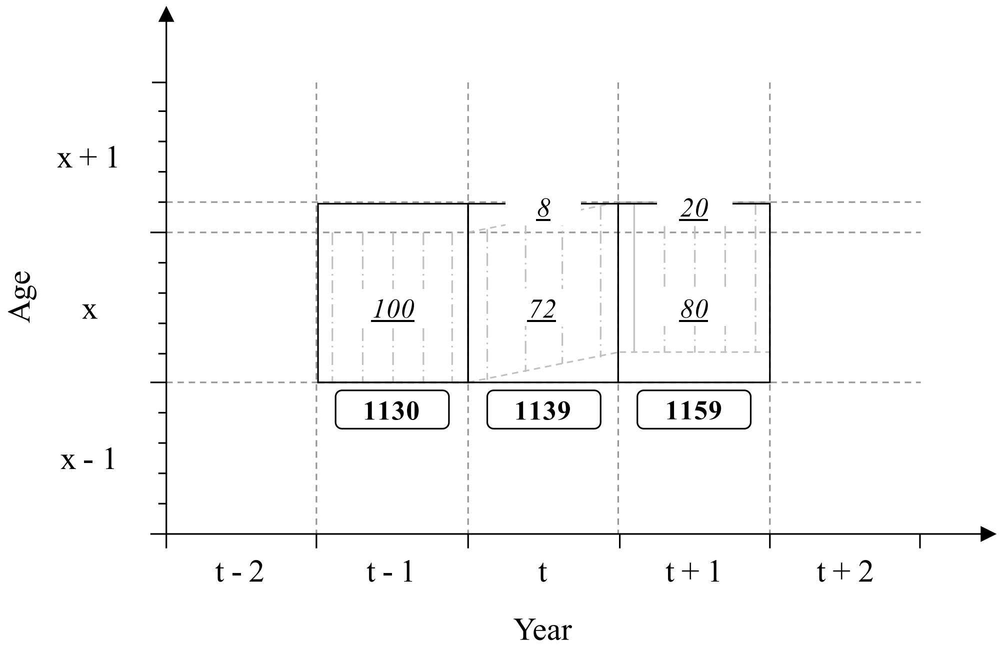


Figure 3b: Mortality decline in the 3rd class of deaths and the trend in number of person-years and deaths[a]



[a] The number of person-years are bordered and the number of deaths are underlined

Source: own design

The number of person-years in the initial year of the observation $t-1$ is 1,130.[2] 100 persons die during this year. The death rate for year $t-1$ is calculated as follows:

$$ {}^{I}m(t-1) = \frac{100}{1{,}130} = 0.0885 $$

The decline in mortality in year t reduces the number of deaths by 20% caused by the postponement of deaths into the subsequent year $t+1$. From 80 deaths in period t, 72 occur at age x and the remaining 8 at the next age $x+1$. At the same time, the number of person-years increases because both the number of persons living at the same time as well as the lifetime of deceased persons has increased slightly as a result of the rising age at death. The death rate in year t declines to:

$$ {}^{I}m(t) = \frac{80}{1{,}139} = 0.0702 $$

During the year $t+1$, the number of deaths reaches the stationary level of 100 persons because the times of death have increased to five again. However, the deaths occur 0.2 years later in age than at the initial level, so that the number of person-years increases slightly further. The death rate under the new constant level equals to:

$$ {}^{I}m(t+1) = \frac{100}{1{,}159} = 0.0863 $$

The increase in type I rate between year t and $t+1$ suggests an increase in mortality. However, Figure 3a illustrates that an increase in mortality did not actually occur for any observed person. Moreover, the number of person-years steadily increased over time. Only the number of deaths falls briefly in year t because of the decline in mortality and the resulting postponement of deaths. According to the argument of Bongaarts and Feeney, the decline and the subsequent unexpected increase in the death rate are caused by a tempo effect (Bongaarts and Feeney 2002, pp. 18-19, Bongaarts and Feeney 2008b, pp. 35-38). The tempo effect here primarily describes the disproportionate decline in the number of deaths in ratio to the person-years caused by a rising age at death. Hence, the increase in the death rate between year t and $t+1$ is not the consequence of an actual increase in mortality, but of the temporary strong decline and resurgence of death counts due to the mortality change.

[2] The person-years are calculated from the survivors and the age of those who died in the respective interval. In the year $t-1$, survivors contribute 925 person-years at age x and another 180 years until age $x+1.2$. The person-years of the deceased are 25.

3.3.2. Tempo effect in the cohort-year-method (type II rate method)

The question now arises as to whether the same tempo effect as with the age-year- method also occurs with the cohort-year-method. In both methods, death counts within a one-year interval form the basis for deriving death rate. Hence, a rising age at death also shifts deaths of the 2nd class out of the analysed period. The extent to which this process also causes tempo effects in the type II rate is analysed by using the simple mortality model same as in the previous section.

The Lexis diagram in Figure 4a again illustrates the five stationary times of death (a1 to a5) in the year $t-1$. In order to cover all deaths at age x, both cohorts $c-2$ and $c-1$ must be considered in this year. Although the shaded lines of the times of death contain only half of both 2nd classes of deaths, they can be conceptually expanded in order to consider all deaths of the cohorts in each year. The simplified model, however, does not influence the causes and impact of tempo effects in the type II rate.

As with the age-year-method, the increasing age at death in year t shifts the last time of death b5 into the following year $t+1$. Accordingly, the number of deaths in year t is again temporarily reduced. The mortality conditions in the model remain constant in the year $t+1$, whereas deaths occur 0.2 years later in age because of the risen age at death. Although the total number of deaths in year $t+1$ is identical to that of the previous stationary level, deaths are now stretched over three cohorts from $c-1$ to $c+1$. This expansion is caused by the postponed time point b5. The hatched area T1 in Figure 4a refers to the deaths of cohort c which under the former mortality conditions would have been covered in the age interval $[x-1/x]$. Due to the reduced mortality, these deaths now occur in the next age interval $[x/x+1]$. Furthermore, the shifted time of death b5 also contains deaths of the previous cohort $c-1$ (black area T2). These deaths now take place within the next age group $[x+1/x+2]$. The remaining times of death c1 to c4 are comparable with the old stationary times a1 to a4, whilst the deaths have shifted by 0.2 years over time and age.

The effect of the risen age at death on the type II rate is illustrated by using the model populations in Figure 4b. As with the type I rate, the focus of the model calculation lies on the trend in death rate, based on the constant number of deaths at the initial level.

Figure 4a: Mortality decline in the 2nd class of deaths and the resulting tempo effect[a]

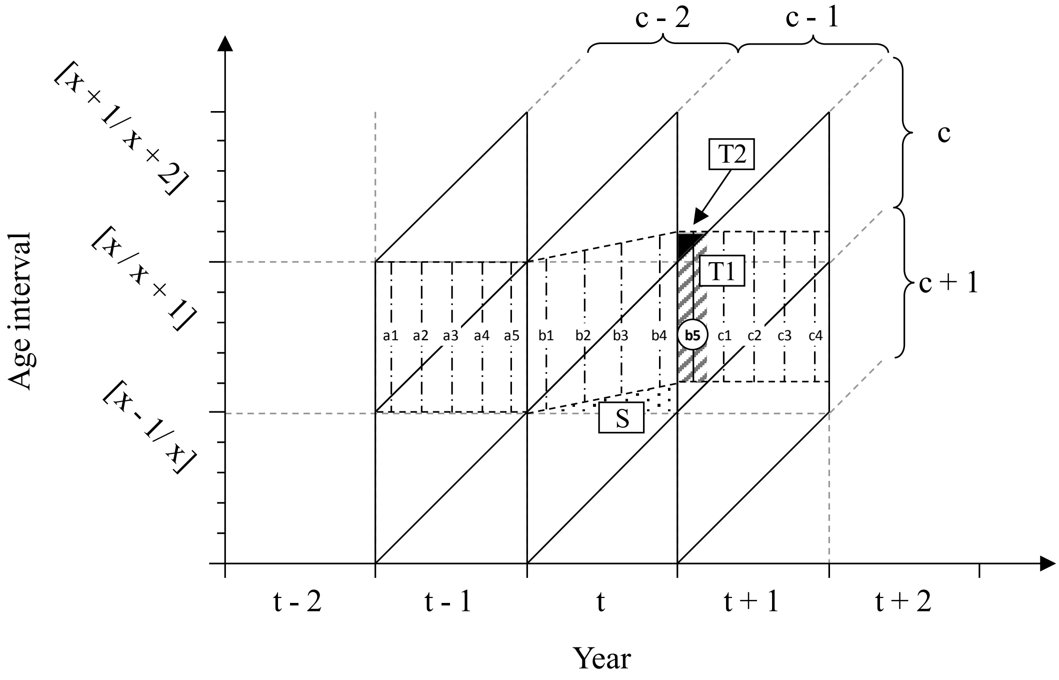
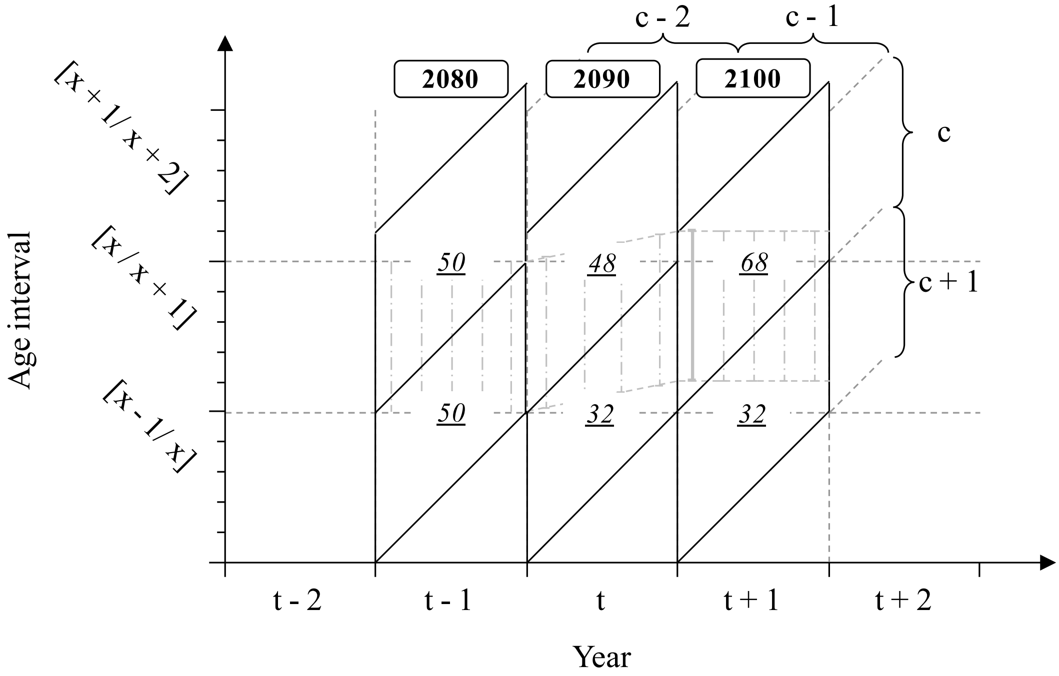


Figure 4b: Mortality decline in the 2nd class of deaths and the trend in number of person-years and deaths[a,b]



[a] The brackets indicate two considered age groups
[b] The number of person-years are bordered and the number of deaths are underlined

Source: own design

It is interesting here that in a real population the postponed deaths of the time b5 cannot be compensated for by shifted deaths from previous age groups in year t. The 2nd class of deaths covers two age groups. Therefore, deaths which are postponed to age x (area S) are still examined in the analysed class.

The following model considers the age range from $x-1$ to $x+2.2$ in order to take the shifted deaths of area T2 in Figure 4a into account. In comparison to the age-year-method, the number of person-years is higher in the cohort-year-method because in the 2nd class of deaths only half of the deaths are modelled.[3] Therefore, the individuals of cohort $c-1$ at the beginning of the year $t-1$ are not exposed to mortality risk until they have reached age x. The situation is similar for individuals born at $c-2$ who are not exposed to mortality after age $x+1$. The death rate in year $t-1$ then equals to:

$$^{II}m(t-1) = \frac{100}{2{,}080} = 0.0481$$

In year t, the number of deaths decreases by 20% due to the rising age at death and the postponement of deaths to the next period. The deaths of cohort c decline more strongly (from 50 to 32 deaths) in comparison to the older cohort $c-1$ (from 50 to 48). At the same time, the number of person-years increases slightly as a result of the shifted deaths. The death rate in year t is:

$$^{II}m(t) = \frac{80}{2{,}090} = 0.0383$$

The postponed deaths of b5 occur in the next year $t+1$ and hence increase the total number to the previously stationary base level of 100 deaths. At the same time, the number of person-years increases slightly because all deaths now occur at a higher age. The new stationary death rate is calculated as:

$$^{II}m(t+1) = \frac{100}{2{,}100} = 0.0476$$

The results show that the type II rate is also affected by tempo effects if period mortality changes. The rate also falls between year $t-1$ and t, whilst it rises again in the transition to the new stationary level in year $t+1$. In comparison to the base level, the rate is lower in the new constant level because the individuals survive 0.2 years longer. As in the case of

[3] In the first year, the survivors contribute to a total of 2,030, and the deceased 50 person-years, over the analysed age interval.

the type I rate, the increase in death rate during the transition to the new stationary level is the consequence of the tempo effect, caused by the temporary and disproportional fall in deaths in relation to the person-years in year t. Since both methods have identical period intervals, the tempo effects have a comparable effect on both death rates.

3.3.3. Tempo effect in the age-cohort-method (type III rate method)

The reduction in the number of deaths during the mortality transition leads to an increase in the death rate in both the age-year- and cohort-year-method, although mortality has continually fallen among the population. The significant reduction was caused by those deaths which were postponed from year t into the next year $t + 1$ because of the risen age at death. In the age-cohort-method, however, the number of the deaths is defined by age intervals and not by calendar years. A lower number of deaths in the 1st class can only occur via a postponement of deaths to the next age interval.

Figure 5a shows the 1st class of deaths at age x to $x + 1.2$ for the period from $t - 2$ to $t + 3$. Each class embraces two calendar years. To determine the period mortality in the period $[t - 2/t - 1]$, the death counts of cohort $c - 2$ at age x (area D in Figure 5a) and of birth cohort $c - 3$ at age $x + 1$ (area E in Figure 5a) are needed. In order to distinguish more easily between the periods, they were visualised in Figures 5a and 5b by alternate grey shading.

The death counts of cohort $c - 2$ at age x still experience the constant mortality level. Since the 1st class of deaths stretches over the years $t - 2$ and $t - 1$, a total of ten times of death (5 per year) are considered at intervals of 0.2 years. In the period $[t - 1/t]$, the age at death rises within the cohort $c - 1$ at age x. Whilst the times of death a1 to a5 are still characterised by the old mortality level, the range between the following times b1 to b5 expand, as it was already demonstrated in the previous sections. The number of deaths, however, no longer declines because of the postponement of the time b5 but because of the age increase in b1 to b5. All deceases of cohort $c - 1$ which are postponed to the following age level $x + 1$ (area T1 in Figure 5a) are not considered within the observed period $[t - 1/t]$.

Figure 5a: Mortality decline in the 1st class of deaths and the resulting tempo effect[a]

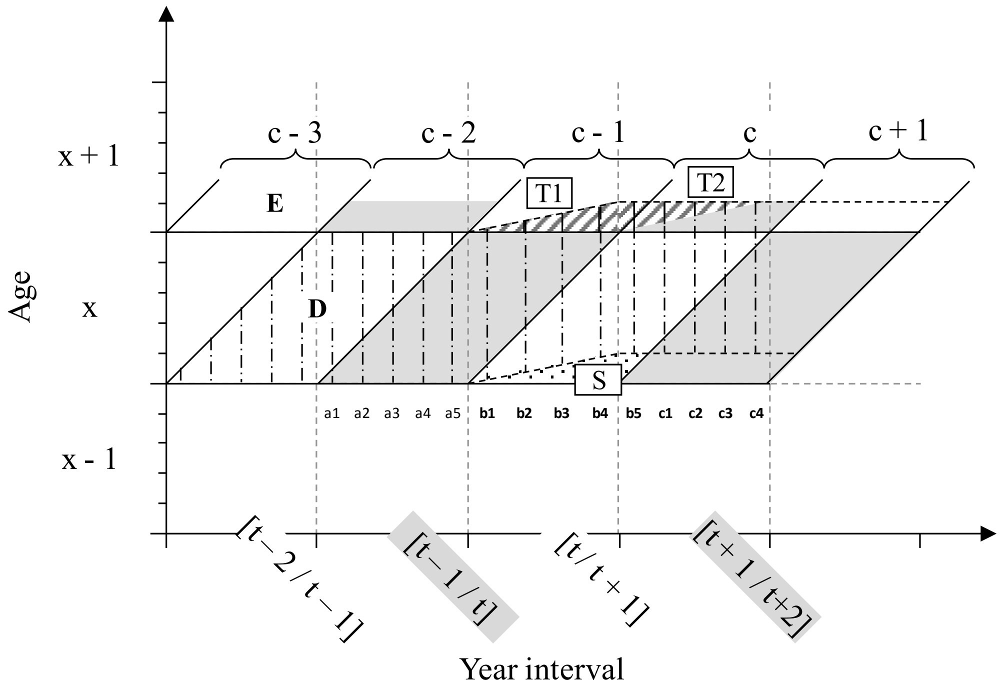
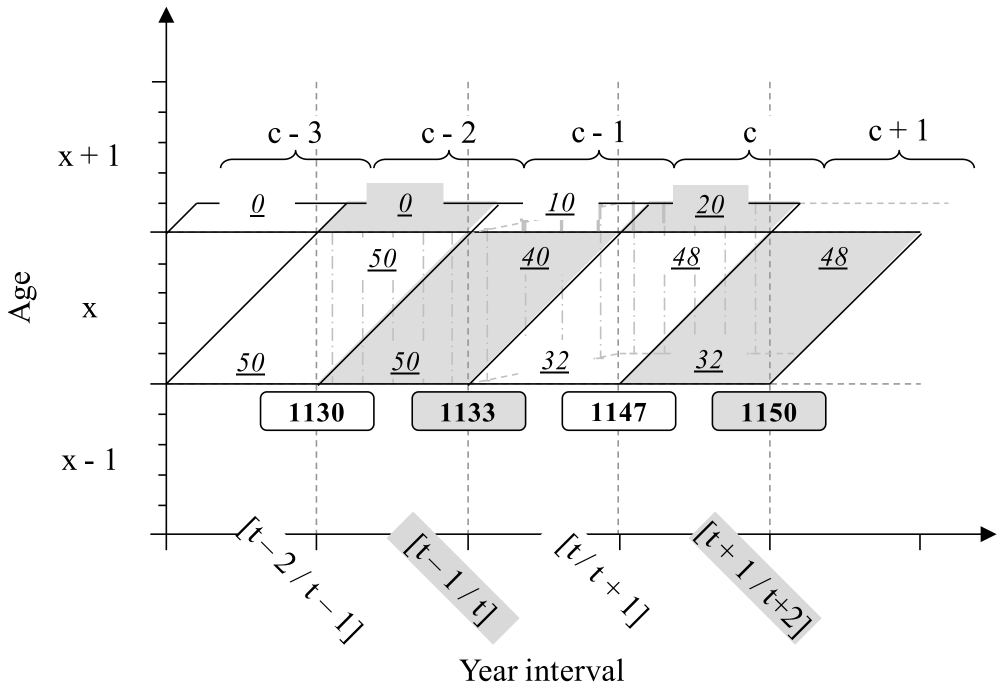


Figure 5b: Mortality decline in the 1st class of deaths and the trend in number of person-years and deaths[a,b]



[a] The brackets indicate two considered periods
[b] The number of person-years are bordered and the number of deaths are underlined

Source: own design

Consequently, the number of the remaining deaths at age x has hence fallen by 10% compared to the initial stationary level.

In the next period $[t/t+1]$, the age at death increases until the beginning of year $t+1$ and remains constant at the new level afterwards. The 1st triangle of cohort c at age x is still affected by the age-shifted deaths of b1 to b5. In contrast, the new stationary mortality level is already prevalent in the 2nd triangle. Although at the base level, the deaths only occurred at age x, the deaths of cohort c are now spread over age x and $x+1$.For quantifying the mortality conditions in period $[t/t+1]$, however, the deaths of cohort c at age x as well as the age-shifted deaths of the earlier cohort $c-1$ at age $x+1$ (area T1) are taken into account. The deaths of cohort c which have been postponed to the next age group $x+1$ (area T2) are not considered until the following period $[t+1/t+2]$. In a real population, the postponed deaths of cohort c from the previous age $x-1$ would additionally be accommodated in the measurement (area S). As in the age-year-method, the deaths from previous ages can therefore reduce or even compensate for the postponing effect (area T2). However, in the model used here – analogous to the approach in the other death rate types – these shifted deaths are ignored in order to illustrate the net effect of the rising age at death.

The new stationary mortality conditions can be observed in the period $[t+1/t+2]$ for the first time. The deaths are now spread over two age groups and come from both cohorts $c+1$ and c due to the improvement in survival conditions. The reduction in mortality in year t hence causes a postponement of deaths in the 1st class of deaths to the next age intervals.

Whether this delay causes a tempo effect in the death rate, as with the other two rate types, can be tested by using the model populations in Figure 5b. Again, the focus of the model lies on the impact of the risen age at death. 100 deaths occur in the stationary level of the period $[t-2/t-1]$. The number of person-years at age x to $x+1.2$ is 1,130.[4] The type III death rate is then calculated from:

$$^{III}m(t-2/t-1) = \frac{100}{1{,}130} = 0.0885$$

Caused by the rising age at death in the period $[t-1/t]$, the deaths at age x decrease by 10%. The formerly 50 deaths reduce to 40, whilst the 10 death counts are postponed to the

[4] The surviving persons together lived 1,080 person years, whilst the deceased contributed 50 person years to the total.

following age level $x + 1$, and hence not considered in the analysed period. The number of person-years increases slightly by the gained lifetime caused by the increased age at death. The death rate in the period $[t - 1/t]$ is:

$$^{III}m(t - 1/t) = \frac{90}{1{,}133} = 0.0794$$

The increase in the age at death also affects the death rate in the period $[t/t + 1]$. Although the age-shifted deaths of cohort $c - 1$ (area T1 in Figure 5a) are taken into account, however, the deaths of cohort c, which occur at the next age level (area T2 in Figure 5a) are missing. The number of person-years increases once more as a result of the rising age at death. The death rate then emerges from:

$$^{III}m(t/t + 1) = \frac{90}{1{,}147} = 0.0785$$

The new constant mortality conditions in the year $[t + 1/t + 2]$ include the number of deaths of cohorts $c + 1$ at age x and cohort c at age $x + 1$. Despite the different cohorts, the number of deceased is again 100, 80% of which occur at age x and 20% at age $x + 1$. The number of person-years is 1,150. Hence, the new constant death rate is derived as follows:

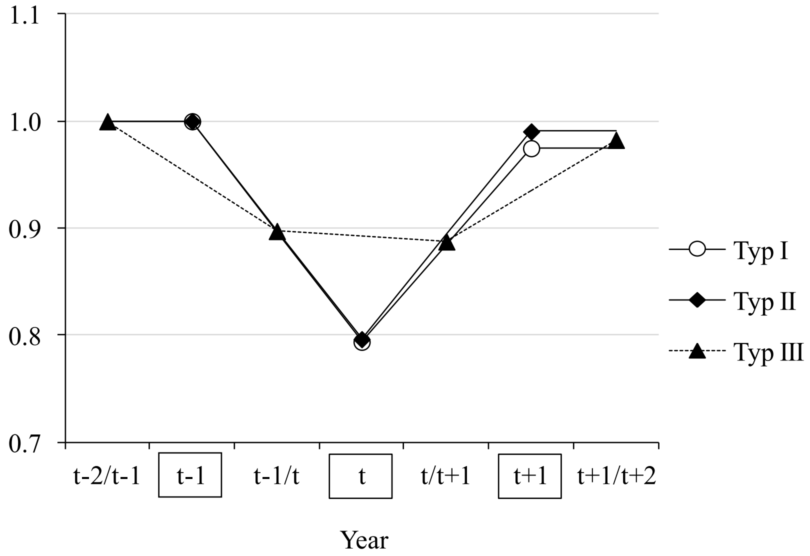
$$^{III}m(t + 1/t + 2) = \frac{100}{1{,}150} = 0.0870$$

In comparison to the previous period, the death rate increases slightly in the transition to the new stationary level. Therefore, the trend in the death rate type III is also influenced by tempo effects. However, the age-shifted and not period-shifted deaths influence the trend of the death rate by the disproportional decline in the number of deaths.

3.3.4. Comparison of the tempo effect in the three calculation procedures

The previous models have shown that tempo effects influence the trend in mortality in each of the methods of deriving the death rate. Figure 6 shows the relative change in all three death rates in the course of the linear increasing age at death compared to the constant level at the beginning. It is evident that the relative trend in the type I and type II death rates is almost identical. Both death rates decline by 20% in the year of the mortality reduction t although both classes of death and hence the levels of the rates differ. Even if all deaths of the 2nd class were considered in the death rate type II, the relative decline would also be identical because the number of times of death in year t also falls by 20%. The rate at the new constant level is less than two percent in comparison to the old stationary level. The relative level of the tempo effect in year t is consequently identical in both procedures.

Figure 6: Trend in modelled death rate by different computation methods in relation to a constant starting level[a]



[a] Type I and type II death rate relate to the number of deaths in a year (bordered time), and the type III death rate covers two periods

Source: own design

A different picture emerges following the trend of the type III death rate. The rate declines disproportionately quickly over two consecutive periods as a result of the tempo effect. As a result of the spread of the tempo effect over two periods, the impact is somewhat less pronounced in relative terms (10% in the period $[t-1/t]$ and 12% in the period $[t/t+1]$) than in the other two methods. The relative deviation at the new stationary level is reduced to less than two percent as in the type I and type II death rates.

3.4. Tempo effects of the 1st and 2nd kind

The differences in tempo effects between the types of death rate are partly dependent on the respective model, in particular on the linear increase in the age at death. A non-linear increase in the age at death in the model, by contrast, would lead to differing amounts of tempo effects, and thereby possibly cause a more pronounced tempo effect in the type III death rate. Hence, the model does not permit any universal statements regarding the degree of the tempo effect in all three methods. However, differences between the causes of the respective tempo effects can be demonstrated. Tempo effects accordingly result from two different effects. Based on different classes of deaths, the number of deaths is reduced by the postponement of deaths over either the period or age interval.

The postponement of deaths over the period interval can be referred to as a tempo effect of the 1st kind. As a result, the number of deaths of the 1st triangle (area ABC in Figure 2) decreases at a certain age if the age at death increases. The occurrence of this tempo effect influences the death rate type I and II. In both methods, the number of deaths in an analysed period can only decline if deaths have been shifted to the next calendar year. Consequently, the total number of postponed deaths is identical in both methods although the level of death rates differs due to the different classes of deaths.

The change in the 2nd triangle of deaths (area BCD in Figure 2) and the concomitant postponement of deaths beyond the age interval can also cause tempo effects, which are referred to as tempo effects of the 2nd kind. This kind is exclusively relevant for the age-cohort-method because only this method is influenced by age-shifted deaths. Within the type I and II death rate, the age-postponed deaths are still considered in the next age group within the observed year.

The two kinds of tempo effect differ according to whether the increase in the age at death – regardless of the nature of the increase – causes a postponement of deaths to the next period or age interval. When it comes to the practical application, the question arises whether the tempo effect can be minimised by applying a specific type of mortality. This question can only be answered by looking at two highly-simplified scenarios:

(I) In the mortality models which are commonly used in the literature (Feeney 2008, Horiuchi 2008, Luy 2008), the number of deaths of the 1st class remains constant during the mortality change. Therefore, the change in mortality only causes a postponement of deaths to the next period but not the next age interval. In these models, consequently, the resulting tempo effect of the 1st kind only influences the death rate derived from the age-year-method and the cohort-year-method. The trend of the type III death rate does not indicate any tempo effect.

The decline in this death rate is entirely caused by the in- crease in the number of person-years, whilst the number of deaths remains constant.

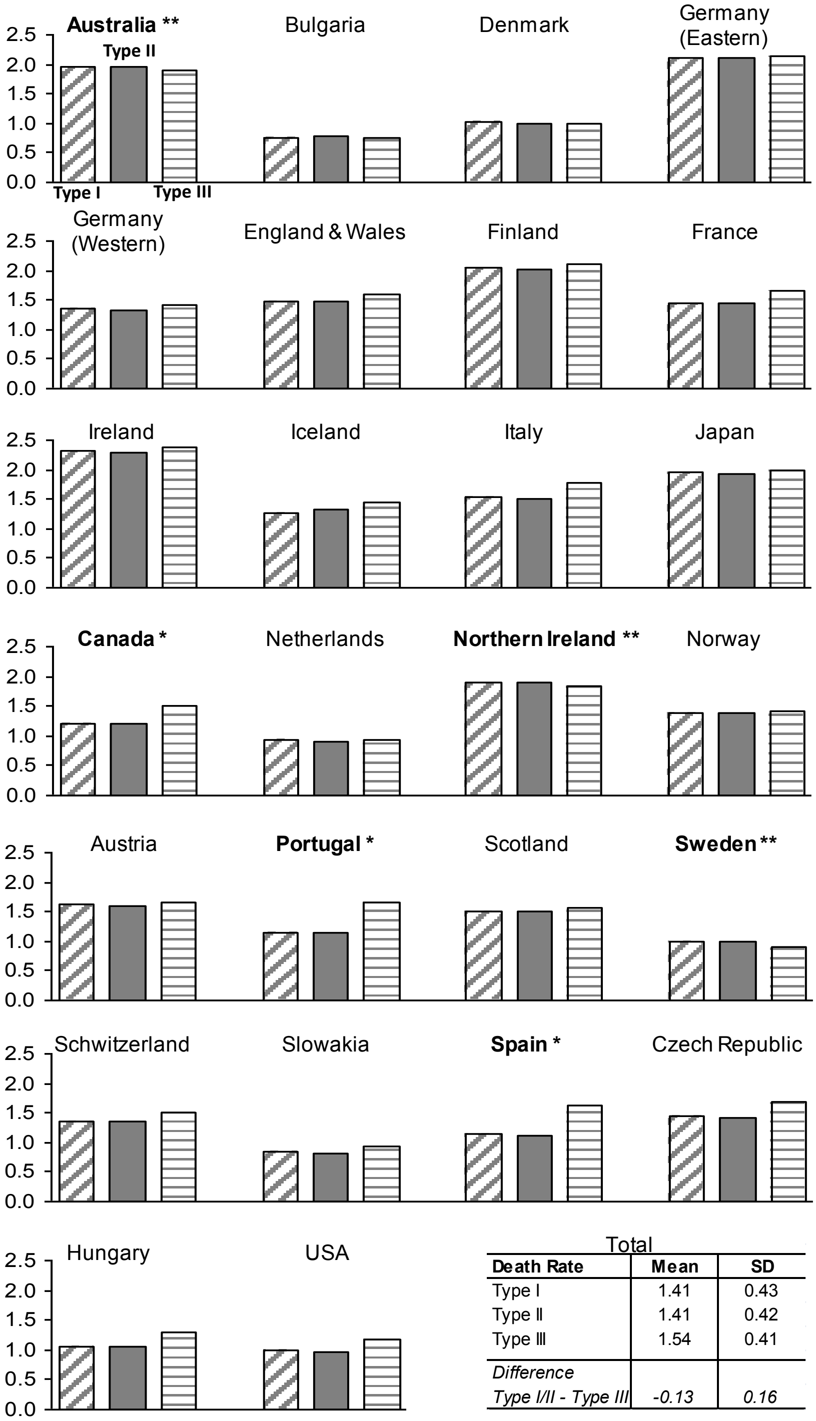
(II) In a second theoretically-conceivable scenario the 2nd class of deaths contains constant number of death during a change in mortality. In this scenario, reduced mortality can only occur through a postponement of deaths to the next age interval, but not by period-shifted deaths. In the cohort-year-method, the death rate decreases by the gain in person-years but the number of deaths remains unchanged. In the age-year-method, the number of deaths also changes at the respective age level, however, the age-postponed deaths still occur within the analysed period. Hence, the death rates of type I and of type II would not contain any tempo effect. Only the trend of the type III death rate fluctuates due to the tempo effect of the 2nd kind.

Finally, only these two extreme scenarios can demonstrate a significant difference between both kinds of tempo effect because only one kind of tempo effect can occur in each case. In all other cases (combinations of the two scenarios), tempo effects have a permanent influence on each type of death rate. The intensity depends on the increase in the age at death and the resulting shift of deaths to the next period and age interval. If more deaths would be postponed to the next age than to the next period, the tempo effect in the age-cohort-method would be greater than in the other two methods. The opposite is the case if more deaths are postponed in the following period than in the next age group.

3.5. The relevance of tempo effects in empirical data

The last section analyses the differences of tempo effects and their impact on life expectancy by using mortality data of the Human Mortality Database (HMD, access on October 14, 2010) for 26 countries. The HMD contains the two triangles of deaths as well as the number of persons living at the same time. Thus, all three types of death rate can be derived. The amount of the tempo effect corresponds to the difference between the conventional life expectancy and the tempo-adjusted life expectancy at age 50 for the year 2005. The reason for analysing life expectancy at age 50 is, firstly, that in developed countries, approximately 95% of mortality occurs above this age and, secondly, that the mortality conditions follow the methodical requirements of tempo adjustment.

Figure 7: Tempo effects in life expectancy at age 50 by different types of death rate, females, 2005



Death Rate	Mean	SD
Type I	1.41	0.43
Type II	1.41	0.42
Type III	1.54	0.41
Difference		
Type I/II - Type III	*-0.13*	*0.16*

* Difference between type I/II and type III death rate is lower than the mean - standard deviation

** Difference between type I/II and type III death rate is higher than the mean - standard deviation

Data: Human Mortality Database, own calculation

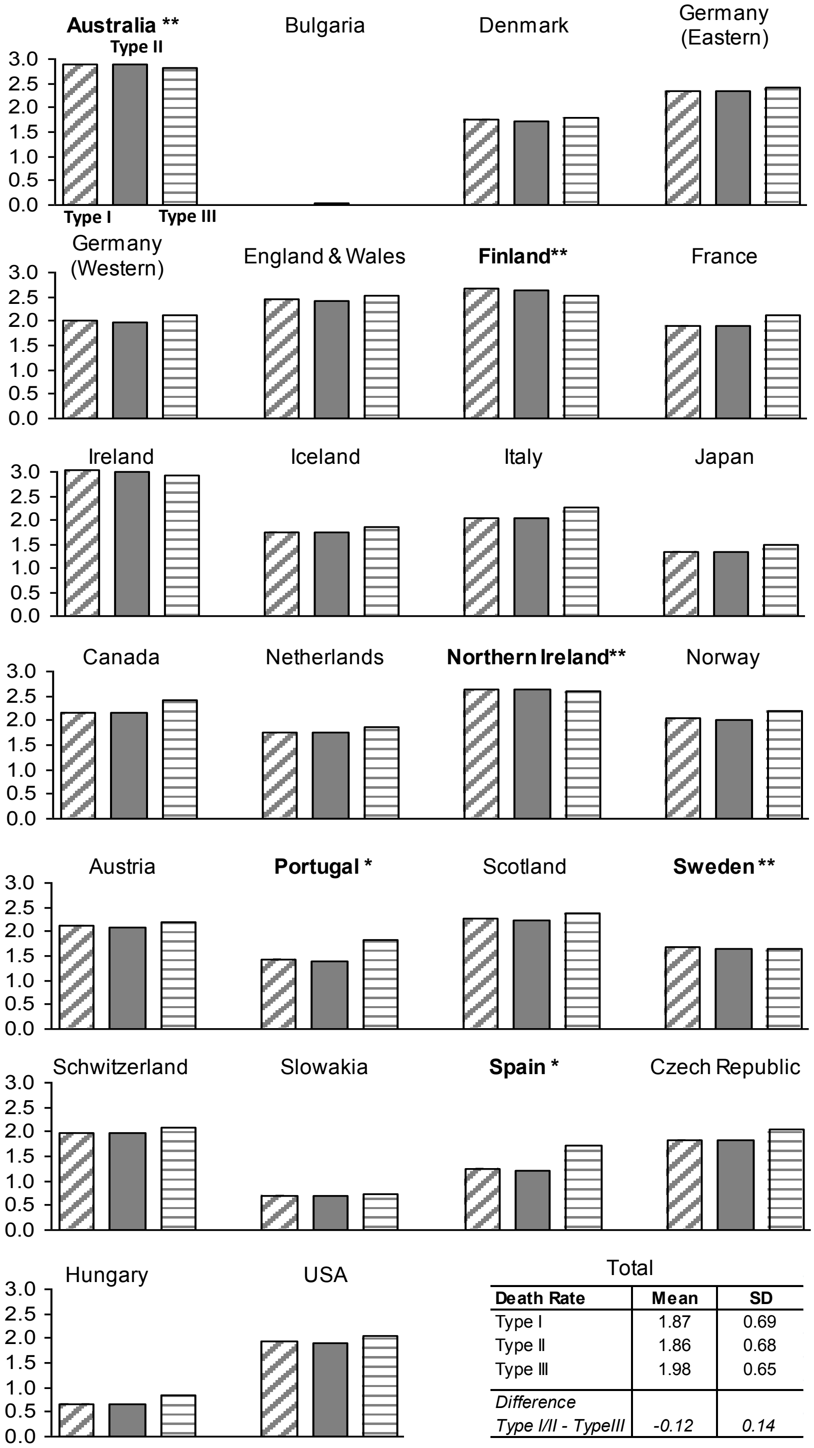
As proposed by Bongaarts and Feeney (Bongaarts 2008; Bongaarts/Feeney 2008b), tempo adjustment was carried out on the basis of the Total Mortality Rate (TMR) (see Appendix A for a more detailed description).

The degree of the tempo effect (measured in years) is shown in Figure 7 for women and in Figure 8 for men. The first two bars present the tempo effect of the 1st kind in the type I and type II death rate. For men as well as for women, there are either no or only marginal differences between both methods. The average tempo effect is 1.41 years among women (±0.43 years) and 1.87 years among men (±0.69 years). The lowest tempo effect is shown for Bulgaria, at 0.01 years among men and 0.76 years among women. This difference is presumably caused by the constant trend in life expectancy at age 50. Until 1996 the standardised death rate from cardiovascular disease continually increased in Bulgaria, whereas the death rate in e.g. Hungary, Poland and Romania either remained constant or declined (Meslé 2004). The low tempo effect in Bulgaria in 2005 suggests that life expectancy is only marginally increasing because mortality caused by cardiovascular disease could have reached a constant level.

In contrast, women in Eastern Germany with 2.11 years and men in Ireland with 3.01 years have the highest amount of tempo effect. The high tempo effect in Eastern Germany is the consequence of the rapid mortality decline since reunification, which can also be observed among East German men (Luy 2009). In Ireland, on the other hand, the significant increase in male life expectancy and the resulting high tempo effect can presumably be ascribed to the rapid drop in smoking-attributed mortality since the early 1990s (Peto et al. 2006).

The right-hand bar in Figures 7 and 8 presents the tempo effect according to the age-cohort-method. In the previous models, the tempo effects of the 1st and 2nd kinds differed considerably in quantitative terms. The empirical data, however, only show slight deviations. The tempo effect in the type III death rate is even slightly stronger than in the two other types of death rate. Among men and women, the average difference between the tempo effects of the 1st and 2nd kind is around -0.13 years (±0.16 years among women and ±0.14 years among men). Differences outside the standard error can be observed for Portugal (-0.51 years among women and -0.44 years among men) and Spain (-0.51 year among both genders), as well as among Canadian women (-0.31 years).

Figure 8: Tempo effects in life expectancy at age 50 by different types of death rate, males, 2005



Total

Death Rate	Mean	SD
Type I	1.87	0.69
Type II	1.86	0.68
Type III	1.98	0.65
Difference Type I/II - TypeIII	*-0.12*	*0.14*

* Difference between type I/II and type III death rate is lower than the mean - standard deviation

** Difference between type I/II and type III death rate is higher than the mean - standard deviation

Data: Human Mortality Database, own calculation

A lower tempo effect of the 2nd kind is shown for Australia (+0.07 years), Northern Ireland (+0.05 years) and Sweden (+0.10 years among women and +0.03 years among men), as well as among Finnish (+0.12 years) and Irish men (+0.07 years).

The empirical data, therefore, do not permit any unambiguous statements which type of death rate can minimise the degree of the tempo effect. The dynamics of the mortality change in each population rather determine the impact of the respective tempo effect as it was described in theoretic terms in the previous chapter. Consequently, the slightly higher tempo effect of the 2nd kind is ascribed to a higher proportion of age-shifted deaths, whilst a higher tempo effect of the 1st kind results from postponements of deaths into the next period.

3.6. Summary and discussion

In demographic research, the age-specific death rate is an important indicator to analyse the period mortality conditions. It also constitutes the key variable in the construction of life tables. Various recent publications have shown that the type I death rate is affected by tempo effects if the mortality conditions change during an analysed period. This paper demonstrates that all types of death rates are affected by tempo effects.

The modelled reduction in mortality causes a fall and subsequent increase in all death rate types during the transition to the new stationary level. This fluctuation is not caused by an increase in mortality, but by a temporary, disproportionate decline in the number of deaths if mortality changes in the respective period. Bongaarts and Feeney (1998, 2002) introduced the term "tempo effect" to describe this phenomenon.

The tempo effects that were revealed in all three methods can be divided in two different kinds according to their origin. The tempo effect of the 1st kind is caused by the postponement of deaths to the next period interval whereas the tempo effect of the 2nd kind is generated by the age-shifted deaths. All three methods of deriving the death rate are influenced in different ways by these kinds of tempo effect. Both the age-year-method and the cohort-year-method are affected by the tempo effect of the 1st kind. The age-cohort-method is influenced by the tempo effect of the 2nd kind if mortality changes within a period.

The modelled trends in the rates in chapter 3.3. present major differences in the scope of the tempo effect between each computation method. However, these differences resulted from the assumption that the age at death increases linearly and hence causes a higher tempo effect of the 1st kind. The decline in mortality in real populations, however, is taking place in period as well as in age. Thus, neither the tempo effect of the 1st kind nor of the 2nd kind only determines the mortality trend. This is finally shown in the empirical calculations for the 26 countries. The tempo effects in life expectancy at age 50 for the year 2005 only show marginal differences between the death rates of type I and II on the one hand and of type III on the other. Moreover, it is not possible to make any unambiguous statement whether the death rate type III or the other two methods minimise the impact of the tempo effect. The empirical data show higher as well as lower tempo effects for the age-cohort- method (type III rate).

This leads to several important questions which must be studied in greater detail in further research. The currently available methods to adjust the life expectancy by tempo effects are based on the assumption that the age-specific pattern of period mortality changes proportionally (Bongaarts 2008, Bongaarts and Feeney 2008b). It is assumed that tempo effects occur in all age-specific death rates and that these are more pronounced the higher the death rate is. To what degree the strict proportionality assumption is actually reflected in a real population, or whether different tempo effects are available in the age-specific death rates, are two methodically- and empirically-relevant questions for future research.

Despite the unresolved questions, the results of this paper confirm that it can certainly be expedient and helpful to adjust the death rates of all types by the tempo effect if period mortality changes. Without this adjustment, the trend in the death rates would suggest overestimated or nonexistent changes in period mortality. Moreover, other articles show that differences in period mortality between two populations may be heavily influenced by tempo effects and that the lack in using tempo-adjusted rates may lead to incorrect conclusions about the dynamic of mortality trends (Luy 2008, Luy 2009, Luy and Wegner 2009, Luy et al. 2011). Future analyses of period mortality can therefore only become more authoritative through an additional tempo adjustment of conventional indicators. Especially as period death rates and measures derived from them, such as life expectancy, are the most frequently used indicators to assess changes or differences in mortality between certain populations or sub-populations.

4. Conventional versus tempo-adjusted life expectancy – which is the more appropriate measure for period mortality?[5]

Luy, Marc & Wegner, Christian (2009) in Genus, Vol. 65(2), 1-28

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Abstract

This paper discusses which characteristics are appropriate for a measure of period mortality and how are these characteristics met in conventional and tempo-adjusted life expectancy. According to our perspective, a period mortality measure should include exclusively the current mortality and should allow to compare period-specific mortality conditions of two populations or to analyse changes between two periods without depending on past or future mortality trends. By using a simple population model, we show that conventional period life expectancy does not meet these demands since it includes specific assumptions regarding future mortality, which differ between different populations and can ultimately lead to paradoxes which disturb its practical purpose. Tempo-adjusted life expectancy, however, is free of such distortions and thus allows the analysis and comparison of pure period-specific mortality conditions. From these considerations we also derive an interpretable definition for tempo-adjusted life expectancy. We suspect that this lack of definition could be a major reason for the general rejection of mortality tempo-adjustment. Finally, we present estimates for tempo-adjusted life expectancy for the period 2001-2005 for 41 countries showing that tempo effects and their adjustment are not only a technical issue but can have significant impacts on the interpretation of period mortality.

4.1. Introduction

Life expectancy is still the most common measure for period mortality. Compared to other mortality measures life expectancy has the advantage of a distinct meaning with an easily understandable interpretation. For instance, a difference in life expectancy of 1.5 years between two populations in a certain year is much easier to assess than a difference in standardized death rates of, let's say, 0.0016. The same holds for showing the extent of mortality differences or the effects of changes in specific age groups or causes of death on overall mortality of a population. These specific characteristics make life expectancy the most important mortality measure for practical purposes, such as policy-making.

Recently, discussion of period life expectancy has taken a new turn among demographers. In a series of papers, Bongaarts and Feeney (2002, 2008a, 2008b) suggested the use of

[5] Contributors: No clear distinctions can be made about the single contribution of each author. In fact, the paper is the result of several discussions and comments among both authors. Based on a simple four-age-model by Wegner of the survival trend of two populations by Luy (2008), we wanted to illustrate the two assumptions of period mortality change. As a result, we derived together the interpretation of the tempo-adjusted life expectancy. The empirical analysis was done by Wegner due to the application of the tempo-adjustment method proposed by Luy (2006, Bongaarts and Feeney 2002).

tempo-adjusted life expectancy for the analysis of period mortality because conventional life expectancy is affected by tempo distortions. Unlike the discussion of mortality tempo adjustment in recent years (see the collection of papers in Barbi et al.(2008)) we want to focus on the question what characteristics a measure for period mortality should have and how these characteristics are met in conventional and tempo-adjusted life expectancy. In this context the most important question is how conventional and tempo-adjusted life expectancy reflect the period mortality of two populations experiencing different changes in (age-specific) mortality. We analyze these questions with a simple model population consisting of only four age groups. Nevertheless, the results are important for every kind of empirical mortality analysis comparing different populations, above all because the relations are represented in this paper in discrete time as they occur in practical mortality analysis. The reason for choosing such a simple population model is that it allows us to follow the future occurrence of postponed deaths more easily. As will be shown, this is the key for understanding the different assumptions behind conventional and tempo-adjusted life expectancy which lead to different consequences regarding the reflected period mortality conditions.

In the present paper we explain two main conclusions of our reflections: (i) we show a technical aspect behind period life table construction that has not been discussed so far and that shows why –according to our understanding of the technical purpose of a period measure– tempo-adjusted life expectancy is a more appropriate tool for standardizing period mortality than conventional life expectancy, and (ii) we show why –according to our understanding of the practical purpose of a period measure– conventional life expectancy can be misleading whereas tempo-adjusted life expectancy cannot. We are aware that other scholars might see the meaning of the technical and practical purposes of a period mortality measure in a different perspective. However, our considerations allow us to derive an interpretable definition for tempo-adjusted life expectancy. Such a definition is still missing in the demographic literature, which may be one of the main reasons for the general rejection of mortality tempo-adjustment. Finally, we present estimates for tempo-adjusted life expectancy for the period 2001-2005 for 41 countries. This empirical application demonstrates that tempo effects and their adjustment are not only a theoretical problem but can have significant impacts on the interpretation of levels and trends of period mortality.

4.2. Practical and technical purposes of a period measure: Demands on period life expectancy

Our perspective is determined by the requirement that –in order to fulfil the above-mentioned purposes– a period mortality measure should include only the current mortality conditions, i.e. the mortality conditions of the calendar year(s) analyzed. A period measure for mortality should enable us to compare exclusively the period-specific mortality conditions of two or more populations or the changes between two or more periods. From the demand "exclusively period-specific conditions" it follows that the calculated value itself is not expected to have a specific meaning for any cohort since period life expectancy contains the mortality of 100 different cohorts, each contributing approximately one percent to the overall mortality of a specific period. We know that no cohort will ever experience the age- specific mortality schedule of 100 different cohorts at 100 different ages at a certain moment of time. This is why period life expectancy refers to a "hypothetical" cohort of people. Nevertheless, according to our perspective, the mortality of the 100 real cohorts should be reflected in period life expectancy in the sense that an increase/decrease of period life expectancy must coincide with an increase/decrease of the life expectancy of (at least the majority of) the cohorts living during the period analyzed. The reason behind this demand is that the practical purpose of a period measure is to get information about the current mortality conditions of a population. This information should enable us to evaluate if the mortality of a population (meaning the real members of the population) decreases or increases (or is higher or lower than in other populations) so as to provide a basis for necessary or possible measures to improve survival conditions (for the real members of the population). Thus, period measures are calculated to get information about the real population – and this is why the real mortality of the currently living cohorts must be reflected in the hypothetical life expectancy based on period mortality conditions measured through age- specific death rates prevailing in a specific period.

The technical purpose of a period measure is to standardize the current demographic conditions for all compositional effects disturbing its practical purpose. In the following pages we show that both conventional and tempo-adjusted life expectancy standardize for such effects, but in a different manner. Since, as we have indicated, period measures are hypothetical by their very nature, it is not possible to conclude that one form of standardization is correct and the other incorrect. But it is possible to think about what consequences the two forms of standardization have for the parameter calculated and if these consequences meet the practical purpose of the measure. In order to do so, a period measure of mortality should include neither past mortality nor assumptions regarding (possible) future mortality since both refer to conditions outside the observation period. Measures, including the past mortality of the current living cohorts, should be separated from period measures, and might –in accordance with the analysis of fertility– be called "timing measures". In this understanding, the "cross-sectional average length of life" (CAL) as introduced by Brouard (1986) and Guillot (2003) would belong to the group of timing measures, as does the "average completed fertility" (Ward and Butz 1980). On the other hand, measures regarding the future mortality of the current living cohorts should be treated and seen as cohort projections. Both, timing and cohort measures should be strictly separated from period measures and not mixed with each other. This does not mean that period conditions cannot be affected by past trends. Former mortality conditions might indeed affect current conditions, e.g., through selection effects. Thus, past trends and conditions must be used for interpreting specific period conditions in the sense that they might explain higher or lower current mortality levels.

In the subsequent sections we show that conventional life expectancy does not meet our demands on a period mortality measure since it includes specific assumptions regarding future mortality that differ between different populations. These characteristics of conventional life expectancy can lead to paradoxes like decreasing period life expectancy, while all successive cohorts experience successive increasing life expectancy, or a situation in which period life expectancy indicates a higher level for one population as compared to another, while each cohort of the population with higher period life expectancy has a lower life expectancy than the corresponding cohort of the other population. Tempo-adjusted life expectancy, however, is free of these distorting effects and thus enables the analysis and comparison of pure period-specific mortality conditions.

4.3. A simple mortality model for comparing conventional and tempo-adjusted life expectancy

In order to demonstrate why we think that conventional life expectancy does not meet the practical and technical purposes of a demographic period measure we use a very simple population model consisting of four single age groups. The same simulations and calculations could be done with a more complex population containing 100 or 110 single ages. We prefer the simple model because it enables us to more easily follow the consequences of mortality changes for each age group and the total population. The starting point is a closed population with a constant number of annual births of 1,000 and constant age-specific mortality conditions (probabilities of dying). According to these mortality conditions, 200 individuals die at age 0, 100 at age 1, 500 at age 2, and the remaining 200 survivors die at age 3. "Constant conditions" means that these numbers occur identically for each cohort and in each single calendar year. Note that our calculations of the probabilities of dying $q(x)$ are based on the so-called "birth-year method" as proposed in the 19th century by Becker (1869, 1874) and Zeuner (1869, 1894, 1903). This is the intuitively correct way of calculating probabilities of dying which might be assumed to be free of tempo effects, unlike the typical estimation from age-specific death rates. Our models show, however, that the birth-year method contains tempo effects like any other method of $q(x)$ calculation. The age-specific number of survivors, deaths and probabilities of dying for our model are given in Table 3.

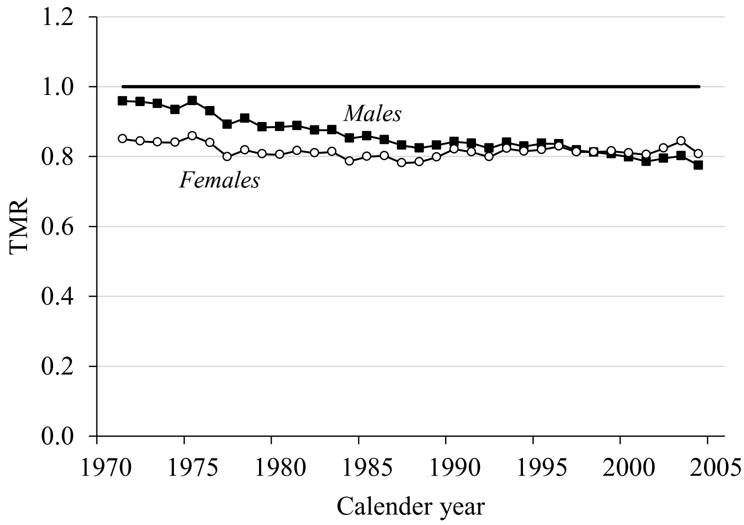
Table 3: Number of survivors at age x, deaths and resulting probabilities of dying of the model population

Age x	Survivors	Deaths	$q(x)$
0	1,000	200	0.200
1	800	100	0.125
2	700	500	0.714
3	200	200	1.000

Source: own calculation

Difficulties in calculating and interpreting period life expectancy arise only in situations of changing mortality conditions. In the development of human mortality, changes have mainly been characterized by improvements of mortality which lead younger cohorts to live longer and thus the members of younger cohorts to die later on average than their counterparts from older cohorts. A logical consequence of such changes is that the deaths of younger cohorts are postponed to a later moment in time (as compared to the survival of older cohorts). Compared to constant mortality conditions, this leads to a postponement of deaths (from a specific period) to a later moment. The consequences of this effect on period mortality –what Bongaarts and Feeney call the "tempo effect"– can be shown by the total mortality rate (TMR) as introduced by Sardon (1994a). As described by Guillot (2008, p. 131), "in a cohort (real or synthetic), the TMR is the number of lifetime deaths divided by the initial size of the cohort. In a life table with a radix of one, the TMR can be calculated by adding all age-specific life table deaths. Obviously, the TMR in a cohort, real or synthetic, is invariably one". The TMR can also be calculated cross-sectionally for a specific period. In this case, for each cohort alive in the observation period, the proportion of deaths occurring during that period (adjusted for all migrations until the observation period) is calculated and then these proportions are summed across all cohorts (for more details, see Guillot 2008). In principle, the TMR can be seen as the mortality equivalent to the fertility measure "timing index" (Ward and Butz 1980), reflecting the degree of completeness of the cross-sectional sum of cohort events. Like the timing index in the case of fertility, the TMR equals 1.0 when mortality remains unchanged. As soon as some or all currently living cohorts experience a change in mortality conditions, the TMR leaves unity and becomes higher than 1.0 in the case of increasing mortality and lower than 1.0 in the case of decreasing mortality.

Figure 9: Total Mortality Rate (TMR) for West German women and men, 1970-2005



Source: Statistisches Bundesamt (2006), own calculation

Figure 9 shows the TMR for West German women and men from 1970 to 2005. The TMR lies below 1.0 in all calendar years. This is the logical consequence of the improving survival conditions observable in almost every developed country for many decades.[6] These empirical values for the TMR show that some deaths are "missing" in the period perspective. However, in the period life table the quantum of mortality (and thus the TMR) is 1.0 since all members of the life table population die until the highest age. Consequently, the missing deaths from the empirical data must have been redistributed inside the life table before deriving the parameter life expectancy –this holds for both conventional and tempo-adjusted calculations. This is the starting point of an alternative view on the differences between conventional and tempo-adjusted life expectancy. Interestingly, this view reveals that both conventional and tempo-adjusted calculations standardize for the tempo effect- caused absence of period deaths. The difference between conventional and tempo-adjusted life expectancy can be seen as a consequence of the way the missing deaths are redistributed inside the life table, or, in other words, how tempo effects are standardized. What these differences look like and what consequences they have regarding the practical and technical purposes of a period measure can be followed in our model population. The modelling is driven by the idea of reconstructing the hypothetical cohort of the life table population as a result of the assumptions behind conventional and tempo-adjusted standardization. Note that the use of the birth-year method leads the age-specific estimates to always span two calendar years. For simplicity, in the following text only the first of these two years is given, i.e. "year 1" refers to birth-year type calculated probabilities of dying of years 1 and 2, "year 2" refers to birth-year type probabilities of dying of years 2 and 3, and so forth.

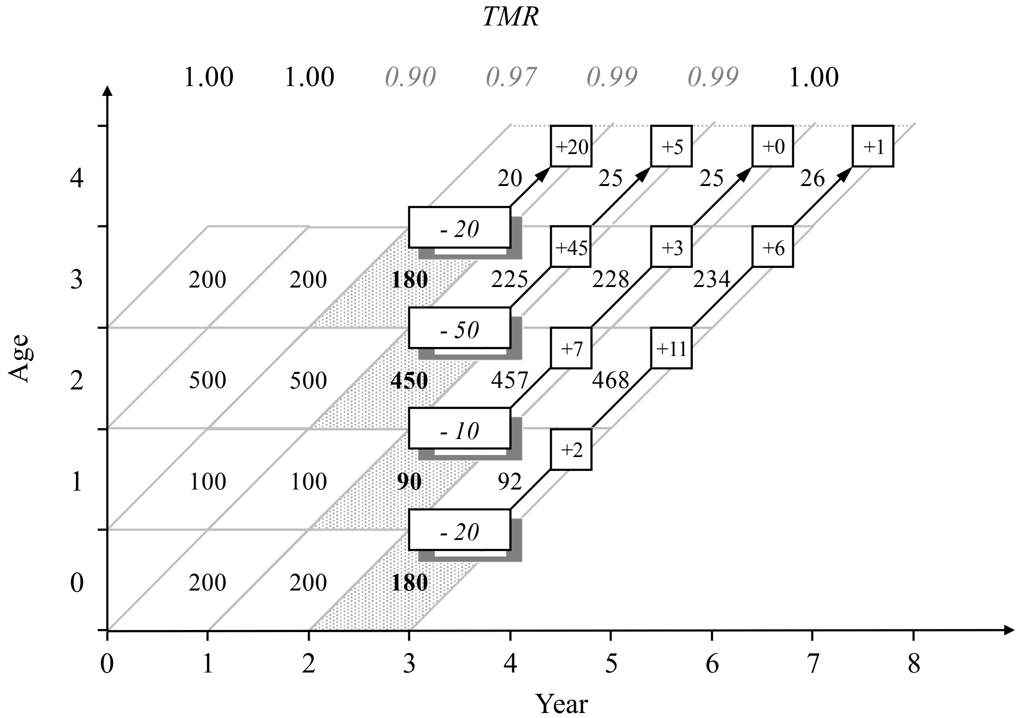
We assume that the constant, i.e. stationary, conditions as given in Table 3 remain unchanged until year 1. In year 2 we model an improvement of survival conditions in the population, leading to a reduction of deaths by 10 percent in each age group. Thus, in year 2 the corresponding numbers of deaths are 180 at age 0, 90 at age 1, 450 at age 2, and 180 at age 3. Compared to the situation before, 100 deaths (10 percent of 1,000) have been saved: 20 at age 0, 10 at age 1, 50 at age 2 and 20 at age 3.[7] This shift of deaths leads to an "incomplete" pattern of death numbers in year 2. Calculating the TMR for that year yields 0.9 (180/1,000 + 90/1,000 + 450/1,000 + 180/1,000), reflecting the relative amount of

[6] Exceptions are the eastern European countries from the former Soviet Union where life expectancy mainly decreased during the last decades.
[7] Here we assume a shift to the now reachable age of 4. Note that assuming a constant highest age of 3 would not affect the basic conclusions.

postponed deaths due to the survival improvement. As was shown in Figure 9, a TMR of 0.9 is a realistic representation of current mortality trends in developed countries.

Assume we are living in year 3 and we want to calculate life expectancy for year 2. From the modelled 10 percent reduction of the number of deaths in each age follows that the probabilities of dying $q(x)$ reduce by 10 percent as well. These $q(x)$ can be used to construct a period life table. Since we know that in this life table the TMR will equal 1.0 we can conclude that the 100 missing deaths in year 2 must have been redistributed within the corresponding period life table. In the following, we reconstruct this redistribution according to the conventional and the tempo-adjusted methodology, respectively. The goal is to visualize the consequences of the corresponding assumptions for the life table cohort born in year 2, i.e. the "hypothetical" cohort to which the estimated life expectancy refers, as well as for all other cohorts living in year 2 and how their life expectancy compares to the estimated period life expectancy.

Figure 10: Total Redistribution of postponed deaths according to the conventional life table assumption



Source: own calculation

Figure 10 shows the redistribution of postponed deaths according to the conventional life table method. Each parallelogram represents the age- specific number of deaths underlying the derived probabilities of dying according to the birth-year method. The deaths occurring in the period of changing mortality are highlighted by grey-shaded parallelograms. The numbers in the rectangles on the top of these parallelograms reflect the deaths postponed as compared to the preceding stationary conditions, i.e. 20 deaths at age 0, 10 deaths at age 1, 50 deaths at age 2 and 20 deaths at age 3. The basic assumption of the conventional life table is that the current probabilities of dying $q(x)$ derived from the deaths in the grey-shaded parallelograms remain constant in all future years. As a consequence, the hypothetical cohort of newborns will experience exactly these probabilities of dying during their complete life course. Moreover, from the assumption of constant $q(x)$ it follows that the 100 postponed deaths are redistributed into higher ages and thus into the following years according to the current (and from now on constant) $q(x)$ schedule. The small squares in Figure 10 illustrate this redistribution of postponed deaths into the subsequent ages. For example, 2 of the 20 postponed deaths at age 0 occurred at age 1, 11 at age 2, 6 at age 3 and 1 at age 4. It can be seen in Figure 10 that according to the conventional life table assumption this process takes the whole lifetime of the hypothetical cohorts. In other words, the standardization procedure of the conventional life table technique leads to a specific assumption regarding the future survival of the deaths saved. The exact pattern of their redistribution depends on the current age-specific mortality schedule. This mortality schedule includes both the age-specific probabilities of dying and the number of postponed deaths in the period analyzed. The latter follows from the fact that the probabilities of dying $q(x)$ are based on mortality conditions leading the TMR to being below 1.0. Furthermore, the TMR reflects the number of deaths that have to be redistributed (and thus the relative impact of this redistribution). Consequently, for populations with different TMR, different $q(x)$ and different tempo effects the conventional life table technique assumes different trends regarding the future mortality of the hypothetical cohorts constructed, as will be shown in the subsequent section. However, we can already conclude that changing mortality should be seen as a compositional effect that a period measure should adjust for.

As long as we assume that each person has to die, the effect of missing deaths is a temporary event since they must occur at some time in the future. The assumption of the conventional life table is one out of an infinite number of possibilities of what might happen to these postponed deaths. One might argue that this assumption is plausible given the current mortality changes. However, it is interesting that this assumption does not result in constant mortality conditions for the future years through which the hypothetical cohort born in year 2 runs during its life course. This can be seen by the values for the corresponding TMR as given at the top of Figure 10. Thus, according to the conventional life table assumption the TMR becomes 0.97 in year 3, 0.99 in years 4 and 5 and becomes 1.0 in year 6 when the last cohort affected by the mortality changes became extinct (TMRs calculated as described before). Since the desired interpretation of life expectancy is that it reflects the average age at death of a newborn under the assumption that the current mortality conditions remain constant, we can see that this desire is not fulfilled in conventional life expectancy for a period with changing mortality conditions. What remains constant are the age-specific probabilities of dying which are affected by tempo effects. The TMR shows that under the conventional life table assumptions the future period mortality conditions of the hypothetical population are not constant until all deaths postponed in the observation period are redistributed, i.e. until the youngest cohort alive in the observation period becomes extinct. The age distribution of survivors under the new constant conditions according to the conventional life table assumption, which apply from year 6 on, and the corresponding probabilities of dying, which are constant since year 2, can be found in Table 4.

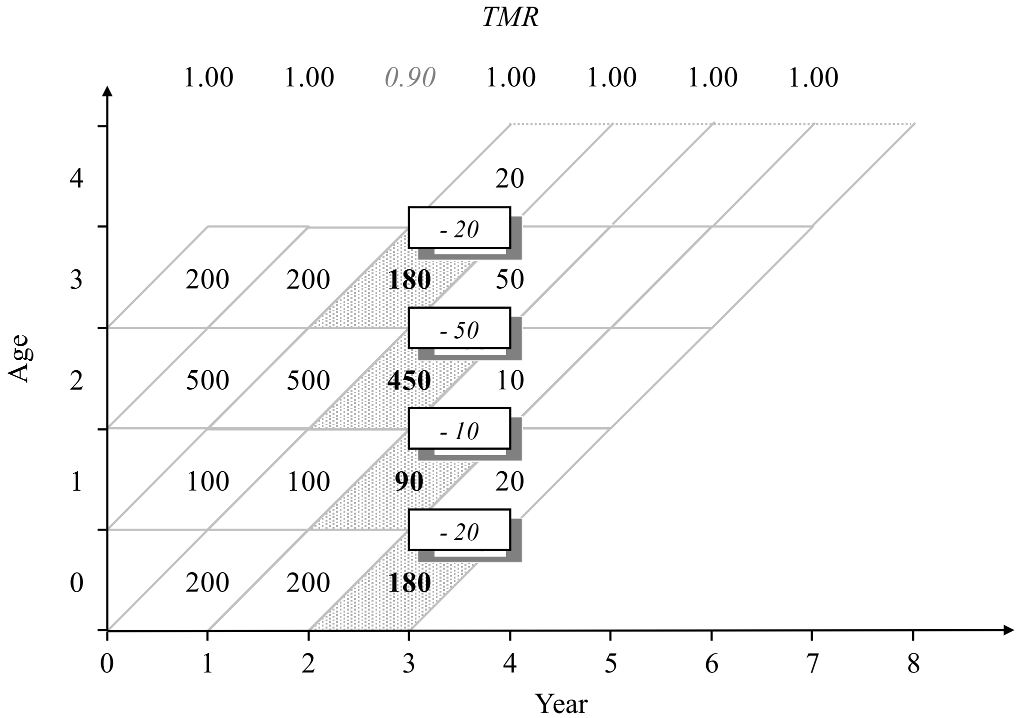
Table 4: Number of survivors at age x and probabilities of dying of the model population in the new constant mortality conditions according to the conventional life table assumption and the Bongaarts/Feeney assumption

Age x	Conv. life table assumption		Bongaarts/Feeney assumption	
	Survivors	$q(x)$	Survivors	$q(x)$
0	1,000	0.1800	1,000	0.1800
1	820	0.1125	820	0.1341
2	728	0.6429	710	0.6479
3	260	0.9000	250	0.9200
4	26	1.0000	20	1.0000

Source: own calculation

Up to this point, however, it is not clear if these consequences of the conventional life table assumption are a problem regarding the practical and technical purposes of a period measure. Before answering this question we have to look at the assumptions behind tempo-adjusted life expectancy in a similar manner. Tempo-adjusted life expectancy is based on a different scenario regarding the future outcome of the postponed deaths. The basic assumption here is that all postponed deaths occur in the next calendar year, as demonstrated in Figure 11.[8] This assumption could be seen as the most conservative, however, with the consequence that the assumed future trends immediately result in constant period conditions for the hypothetical population.

Figure 11: Redistribution of postponed deaths according to the Bongaarts/Feeney assumption



Source: own calculation

As can be seen in Figure 11, the age-specific number of deaths remains constant from year 3 on, as does the age-specific distribution of survivors. The latter can be found in Table 4. Table 2 also shows the corresponding probabilities of dying, which remain constant from year 3 on as well. That the Bongaarts and Feeney assumption immediately leads to new constant conditions can also be seen when the TMR is considered. According to the assumptions of tempo-adjusted life expectancy the TMR becomes 1.0 in year 3, the year following the changes in mortality, and remains constant for all future years (more details

[8] A similar illustration of the Bongaarts and Feeney assumption can be found in Guillot (2008).

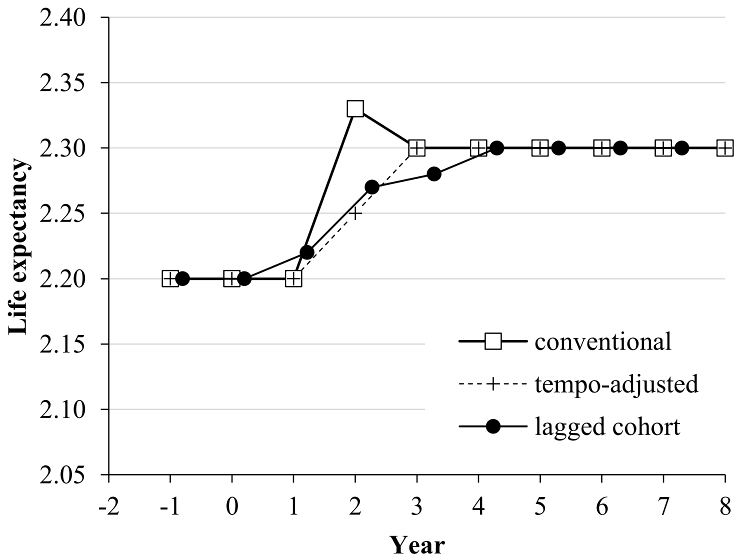
on the consequences of the Bongaarts/Feeney assumption are presented in the subsequent section).

In other words, tempo-adjusted life expectancy provides a way of standardizing current mortality changes that is identical for any population analyzed regardless the characteristics of tempo effects in the observation year. Any change in mortality conditions is standardized in such a way that the TMR is 1.0 for all future years.

4.4. A definition of tempo-adjusted life expectancy

When demographers analyze current period mortality conditions they do not know how mortality will develop and thus how the survival of postponed deaths will unfold. Let's assume first that the future will be as stated by the conservative assumption behind tempo-adjusted life expectancy (Bongaarts/Feeney assumption). Figure 12 shows that for this situation, conventional period life expectancy increases from the constant level of 2.20 years to 2.33 years in the time of mortality change (year 2) and declines directly after to the new constant level of 2.30 years.

Figure 12: Trends in period, tempo-adjusted and lagged cohort life expectancy assuming that postponed deaths occur in the next period (Bongaarts/Feeney assumption)



Source: own calculation

Figure 12 also shows the development of cohort life expectancy of all cohorts living during the years of changing mortality. Note that the cohort life expectancies are represented as lagged cohort life expectancies, i.e. the life expectancy of the cohorts is displayed at the average year of death of the members of the cohorts (birth year + life expectancy).[9] Two important aspects become visible: (i) no cohort ever reaches the level of conventional period life expectancy of year 2, and (ii) all successive cohorts experience successively higher life expectancies. There is no decline in life expectancy among cohorts as indicated by conventional life expectancies between years 2 and 3. If in an empirical application period life expectancy indicated such a decline of life expectancy this would probably be interpreted as an increase (or worsening) of mortality. Figure 12 shows, however, that in this example no cohort experiences an increase of mortality compared to the previous cohorts. On the other hand, tempo-adjusted period life expectancy of year 2 lies between the old and new constant levels of life expectancy. This makes sense since year 2 is the period of transformation between these two mortality levels.

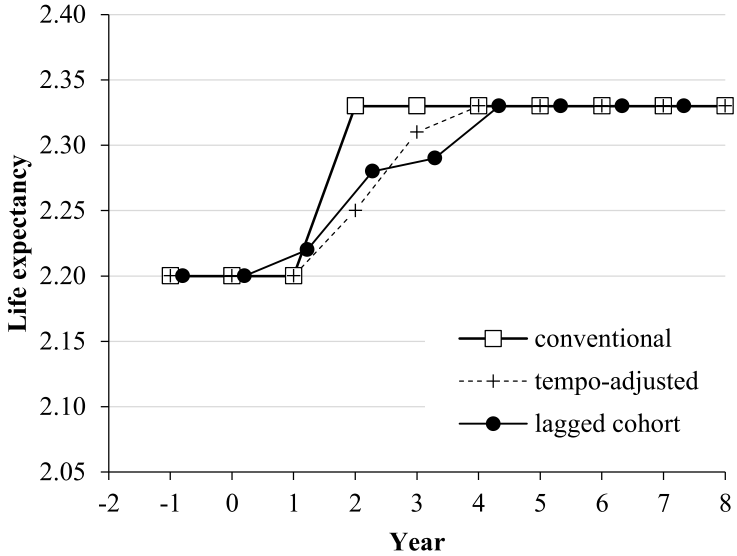
The example presented in Figure 12 provides a possibility to give tempo- adjusted period life expectancy an interpretable meaning. Thus, *tempo-adjusted life expectancy can be interpreted as the average of life expectancies of all hypothetical cohorts living during the observed period, assuming that all currently saved deaths occur instantly in the next period*. The cohorts alive during year 2 are the cohorts born in year 2 (life expectancy 2.30 years), year 1 (2.28 years), year 0 (2.27 years), and year -1 (2.22 years). Since we assumed that deaths postponed from the former highest reachable age 3 now occur in age 4 we also have to take into account the cohort born in year -2 (life expectancy 2.20 years) since this cohort would have reached age 4 in year 2. Thus, the average of cohort life expectancies is (2.30+2.28+2.27+2.22+2.20) / 5 = 2.25 years. As can be seen in Figure 12, this is the same value as provided by tempo-adjusted period life expectancy. Since the old mortality conditions resulted in a life expectancy of 2.20 years and the new mortality conditions resulted in a life expectancy of 2.30 years, a value of 2.25 years seems the appropriate description of period mortality conditions in the year of changing mortality.

It is easy to see that a similar definition is not possible for conventional period life expectancy even under the assumption that future mortality develops according to the assumptions of the conventional life table method. This can be seen in Figure 13 where the

[9] More detailed descriptions of the lagged cohort life expectancy and empirical estimates can be found in Bongaarts (2008), Goldstein (2006) and Rodríguez (2008).

same calculations are made for the case that mortality changes as assumed by the conventional way of determining life expectancy (conventional life table assumption).

Figure 13: Trends in period, tempo-adjusted and lagged cohort life expectancy assuming constant $q(x)$ (conventional life table assumption)



Source: own calculation

The graph shows that also in this case the trend of tempo-adjusted life expectancy is similar to the trend of cohort life expectancies. Furthermore, the interpretation of tempo-adjusted life expectancy as an average of hypothetical life expectancies of all cohorts living during the observed period, assuming that all currently postponed deaths occur in the subsequent period, holds true here as well. The trend of moderately increasing tempo-adjusted life expectancy as compared to the conventional period life expectancy also seems logical from the point of view that in year 2 only one cohort fully experiences the new mortality conditions whereas the majority of living cohorts experienced the old mortality conditions during most of their life courses. Conventional life expectancy, on the other hand, can only be interpreted as the average life expectancy of current newborns, assuming that the current age-specific $q(x)$ schedule remains constant.

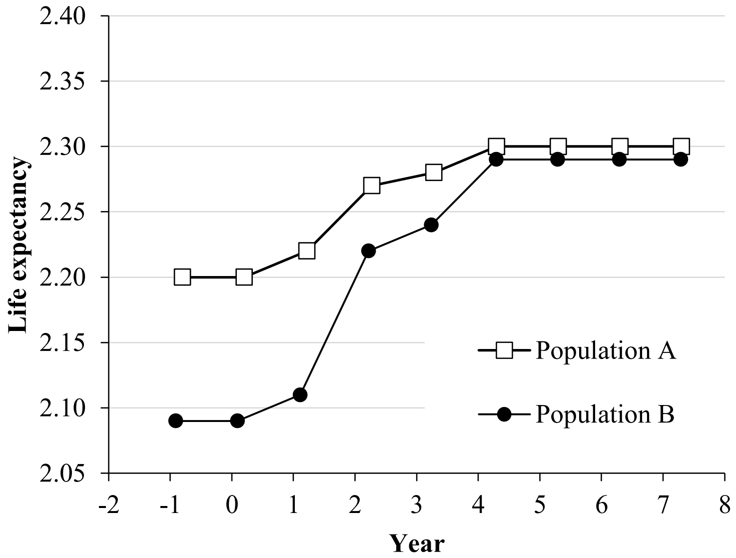
The examples presented in Figures 12 and 13 show that this assumption is not an appropriate way to standardize mortality conditions in a period of changing mortality. Note that in practical application the cohorts as shown in Figure 12 would be hypothetical cohorts constructed on the basis of current mortality conditions assuming that they belong to a stationary population until the year of mortality change (i.e. in practical application the year of observation) and assuming a specific future destiny of currently postponed deaths without any further or additional changes of mortality in the subsequent years. Thus, the aim of tempo-adjusted life expectancy must not be seen to produce an estimate for real cohort life expectancy. The hypothetical cohorts constructed for tempo-adjusted life expectancy are only an instrument for standardizing period mortality conditions to a new constant level. As was shown in the previous section, this does not hold for the hypothetical cohorts according to the conventional life table assumption. Instead, conventional life expectancy represents a cohort projection for the currently newborn including specific assumptions of changing future mortality, as can be seen clearly in Figure 13.

4.5. Conventional and tempo-adjusted life expectancy for populations with different changes of mortality conditions

The undesired consequences of the assumptions behind conventional life expectancy become most apparent when we consider two populations that experience different changes in their mortality conditions. This is the typical situation demographers are always faced with when they compare different populations by means of period life expectancy. To demonstrate this situation we add a second population to our model.

This population is called “population B” while the population used in the previous sections remains unchanged and is now called “population A”. As with population A, in population B the number of births remains constant at 1,000 and mortality remains unchanged until year 1. In the first case, in year 2 both populations experience a reduction in mortality conditions with all postponed deaths occurring in the next year 3 (Bongaarts/Feeney assumption). Thus, from year 3 on, mortality remains constant in both populations, as modelled for population A in the first example of the previous section. In our model, the assumed changes in mortality conditions occur in the same way in both populations. However, the two populations differ in the level of mortality and the pace of mortality reduction. Population B has higher mortality at any given time. Until year 1 the probabilities of dying in population B are 10 percent higher than in population A leading to a life expectancy of 2.09 years for population B, compared to 2.20 years for population A. During year 2, the probabilities of dying decrease by 10 percent in population A and by 20 percent in population B. Although the reduction in population B is twice the reduction in population A, the improvements are insufficient to reach the mortality level of population A. In the new constant conditions from year 3 on, population A's life expectancy is 2.30 years and the life expectancy of population B is 2.29 years. From these assumptions it follows that every single cohort of population A has a higher life expectancy than the corresponding cohort of population B (see Figure 14).

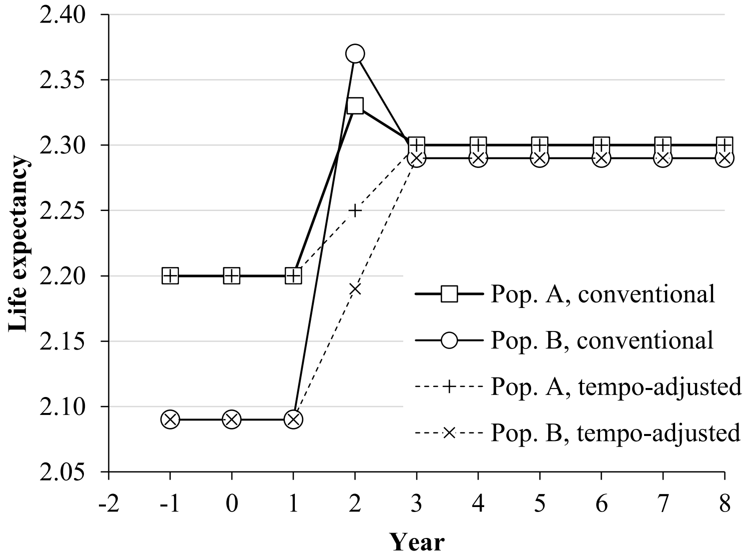
Figure 14: Lagged cohort life expectancies for the cohorts of populations A and B assuming that postponed deaths occur in the next period (Bongaarts/Feeney assumption)



Source: own calculation

However, as a consequence of the more intensive changes in population B during year 2, conventional life expectancy is higher for population B in that year. The conventional period life expectancy for population B is 2.37, whereas the conventional life expectancy of population A is 2.33 (see solid lines in Figure 15). Usually, every analysis based on such period results would conclude that current mortality conditions are lower in population B than in population A. In fact, from Figure 14 we know that no cohort in population B lives longer than the corresponding cohort of population A. Tempo-adjusted life expectancy, however, provides the desired results, indicating higher mortality conditions for population B, as can be seen from the broken lines in Figure 15. Furthermore, as was shown in the previous sections, this is yet another example where the conventional way of calculating period life expectancy yields values that no cohort of both populations ever reaches. On the other hand, tempo-adjusted life expectancy averages the life expectancies of the cohorts living during the period of changing mortality.

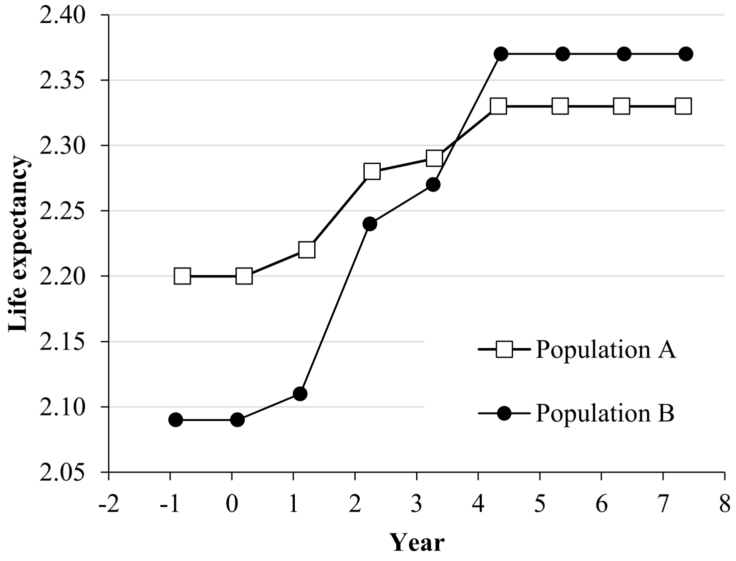
Figure 15: Conventional and tempo-adjusted period life expectancy for population A and population B assuming that postponed deaths occur in the next period (Bongaarts/Feeney assumption)



Source: own calculation

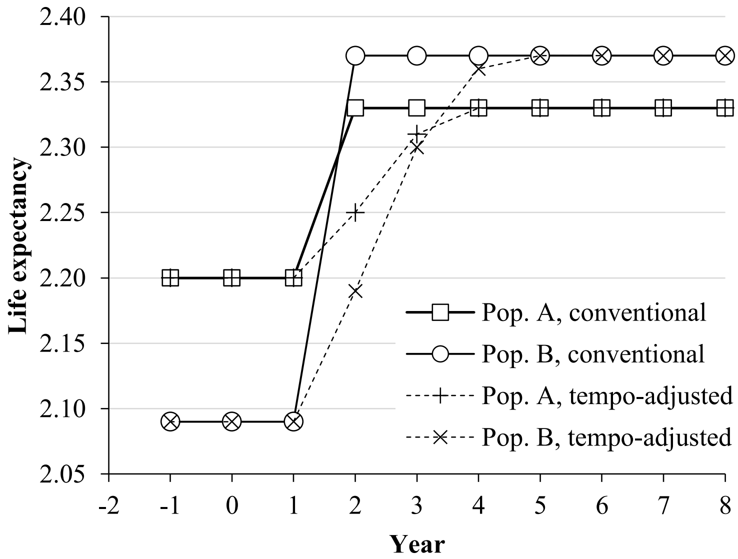
Let us now consider conventional and tempo-adjusted life expectancy for populations A and B for the case in which mortality changes according to the conventional life table assumption. Figure 16 shows the corresponding changes in cohort life expectancy in the two populations. Since, according to the conventional life table assumption, the $q(x)$ schedule predominant in year 2 remains constant for all subsequent years, the younger cohorts of population B experience a higher life expectancy than the corresponding cohorts of population A (see crossing-over of lagged cohort life expectancies in Figure 16). In this example, the crossing-over is visible in both period indicators, conventional and tempo-adjusted life expectancy (see Figure 17). However, tempo-adjusted life expectancy better reflects the trends of the real population, where in most cohorts alive in year 2 those in population A still experience a higher life expectancy than their counterparts in population B. From this point of view, the later crossing-over of tempo-adjusted life expectancy provides a more appropriate picture of the mortality conditions of the currently living cohorts than does the immediate crossing-over of conventional life expectancy.

Figure 16: Lagged cohort life expectancies for the cohorts of populations A and B assuming constant $q(x)$ (conventional life table assumption)



Source: own calculation

Figure 17: Conventional and tempo-adjusted period life expectancy for population A and population B assuming constant $q(x)$ (conventional life table assumption)



Source: own calculation

The last example undermines what was described and concluded in the previous section. First, it demonstrates that tempo-adjusted life expectancy can be interpreted as the average of hypothetical life expectancies of all cohorts living during the observed period, assuming that all currently saved deaths occur instantly in the next period. Second, the fact that tempo- adjusted life expectancy remains higher for population A in the first periods after the changes in mortality fits in with the mortality conditions of those cohorts alive in these periods. Thus, tempo-adjusted life expectancy seems to be the more appropriate indicator for period mortality conditions in light of the practical purpose of a period measure as described at the beginning of this paper. Third, it becomes clear again that conventional life expectancy must be seen as a specific projection of cohort life expectancy of those born in year 2 rather than being a valuable indicator for period mortality.

Consequently, this example shows that even in a situation in which mortality changes occur according to the conventional life table assumption, tempo-adjusted life expectancy provides not only more appropriate information on period mortality conditions, but, more importantly, does not lead to disturbing paradoxes such as those provided by conventional life expectancy in the case where mortality changes according to the Bongaarts/Feeney assumption. Comparing the two scenarios reveals that the Bongaarts/Feeney assumption can be considered as the most pessimistic extreme of what could happen to deaths that are postponed in a specific period. We could similarly think of an alternative, most optimistic scenario in which all postponed deaths survive until the highest possible age. Therefore, we modelled a scenario in which the mortality of populations A and B evolves according to this most optimistic case (results not shown here, the corresponding graphs are available from the authors). Even in this scenario, which produces opposite of the Bongaarts/Feeney assumption, tempo-adjusted life expectancy does not provide a reversed picture of the survival of cohorts as does conventional life expectancy in the case when mortality changes according to the Bongaarts/Feeney assumption. In this scenario, too, tempo-adjusted life expectancy yields the average life expectancy of all hypothetical cohorts alive in year 2. Furthermore, conventional life expectancy produces a picture of life expectancy trends that is even more favourable than the trends in lagged cohort life expectancies, i.e. a steeper increase in life expectancy and an earlier achievement of the new life expectancy level. This highlights the distortions tempo effects can create when conventional period life expectancy is used in order to track and evaluate trends in mortality.

4.6. Tempo-adjusted life expectancy 2001/2005 for 41 countries

In the previous sections we concluded that tempo-adjusted life expectancy is a more appropriate measure for period mortality than conventional life expectancy. In this section we show that mortality tempo- adjustment is not just a technical issue but can have severe impacts on the interpretation of period mortality, above all regarding the analysis of life expectancy differentials between populations or sub-populations. Luy (2006, 2008) has already shown this using mortality differences between eastern and western Germany. Once life expectancy is adjusted for tempo effects, the differences between eastern and western Germany do not decrease immediately after unification, and ten years later they are still higher when compared to the differences in conventional life expectancy. Thus, tempo- adjusted life expectancy can draw a very different picture of mortality differentials than conventional life expectancy. We extended the empirical application of mortality tempo-adjustment and estimated tempo-adjusted life expectancy for the years 2001-2005 (average of the estimates for these five calendar years) for 41 countries with sufficient mortality data. Most of the data used stem from the Human Mortality Database (HMD, access on July 31, 2009). Only the estimates for Greece and Romania are based on data from the Eurostat Database.[10]

Tempo-adjusted life expectancy was estimated by using the method proposed by Bongaarts and Feeney (2002), using a series of sex- and age-specific death rates from 1960 to 2005 and applying the shifting Gompertz mortality change model for estimating the tempo bias (a detailed description of this procedure can be found in Bongaarts and Feeney, 2002, and Luy, 2006)[11]. As proposed by Bongaarts and Feeney (2008b), estimates for tempo-adjusted life expectancy at birth assume no tempo effects below age 30. The resulting estimates for tempo-adjusted life expectancy differ only minimally from estimates of tempo effects based on annual changes in the TMR, which is an alternative way of estimating tempo-adjusted life expectancy (see Bongaarts and Feeney (2008a, 2008b)). Since the data necessary to determine the TMR is available for only a few countries, we used the method based on the shifting Gompertz mortality change model for all 41 countries.[12]

[10] Eurostat Database: http://epp.eurostat.ec.europa.eu/portal/page/portal/population/data/database.

[11] Exceptions regarding the used time series because of data availability are New Zealand Non- Maori (1960-2003), Australia (1960-2004), Greece (1961-2005), Romania (1968-2005), Taiwan (1970-2005), Israel (1983-2005) and Slovenia 1983-2005).

[12] Unlike the estimation procedures of Bongaarts and Feeney (2002) and Luy (2006), we applied the estimated tempo bias directly to the given conventional life expectancy. The iteration procedure of the method based on the shifting Gompertz mortality change model requires an initial assumption for the tempo bias in the first year of the used time series, i.e. in the case of our estimates the year 1960. In order to eliminate the sensitivity to the initial condition of the estimates for the tempo bias for the years 2001-2005, we used a tempo bias of 0, 1 or 2 years as the initial condition, depending on the trends in conventional period life expectancy between 1960 and 1970. That is, in the case of stalled life expectancy between 1960 and 1970, we assumed no tempo bias as the initial condition, in the case of steep rising life expectancy we

Tables 5 and 6 show the results for females and males, respectively. The first column presents the values for conventional life expectancy at birth, the second column the corresponding estimates for tempo-adjusted life expectancy. The next column gives the difference between conventional and tempo-adjusted life expectancy. In most cases this difference is positive, meaning that improvements in mortality conditions lead to tempo effects which bias conventional life expectancy upwards. However, there are some eastern European countries, such as Russia or Ukraine, where mortality increased during the last decades and thus tempo distortions caused the opposite effect. The last two columns contain the ranks of the countries according to conventional and tempo-adjusted life expectancy, respectively. The countries are ordered by the absolute amount of tempo effects, i.e. by the difference between conventional and tempo-adjusted life expectancy, with the country with the highest mortality tempo effects being on the top and the country with the lowest tempo effects being on the bottom of the table. The difference between the highest and lowest life expectancy and the standard deviation of the corresponding estimates for conventional and tempo-adjusted life expectancy reveal that among both sexes the differences between countries decrease once life expectancy is adjusted for tempo effects.[13]

Among females, Japan is the country with the highest conventional life expectancy (see Table 5). Tempo-adjusted life expectancy is three years lower than conventional life expectancy for Japanese females. But despite these significant tempo effects, Japanese women also show the highest tempo-adjusted life expectancy. However, the difference between Japan and the next country in the ranking of life expectancy decreases considerably. According to conventional life expectancy, Japanese females have an advantage of 1.97 years over France on rank 2. According to tempo-adjusted life expectancy, this advantage is only 0.66 years over Switzerland, which takes second place from France in the corresponding ranking. After Japan, France and Switzerland, Italy ranks fourth in conventional life expectancy, but in the ranking of tempo-adjusted life expectancy, Italy falls further behind Spain, Iceland and Sweden. There are some further cases showing that the effects of tempo-adjustment are more significant than just causing a change of the position of countries in the corresponding rankings of life expectancy.

assumed a tempo bias of 2 years, and for the cases in between we assumed a tempo bias of 1 year in 1960. In cases of decreasing life expectancy we used the equivalent negative values.

[13] Compared to conventional life expectancy the maximum differences decrease from 13.16 to 9.19 years among females and from 20.19 to 16.11 years among males, the standard deviation decreases from 2.91 to 2.34 among females and from 4.98 to 3.84 among males.

Table 5: Conventional life expectancy e_0 and tempo-adjusted life expectancy e_0^* for 41 countries, females 2001/2005, not tempo effects below age 30

	e_0	e_0*	Difference	Rank	
				e_0	e_0*
Japan	85.28	82.29	2.99	1	1
Eastern Germany	81.37	78.61	2.76	19	25
Taiwan	80.14	77.55	2.58	26	28
Italy	83.23	81.02	2.21	4	7
Australia	82.97	80.76	2.21	6	9
Ireland	80.62	78.55	2.07	23	26
Austria	81.85	79.78	2.06	13	17
Israel	81.60	79.63	1.98	14	19
Slovenia	80.55	78.62	1.93	25	24
France	83.31	81.40	1.91	2	3
Western Germany	81.58	79.73	1.86	16	18
Spain	83.21	81.36	1.85	5	4
Finland	81.90	80.04	1.85	12	15
New Zealand (Non-Maori)	81.93	80.09	1.84	11	13
Portugal	80.93	79.10	1.83	22	22
Poland	78.89	77.08	1.81	30	31
England & Wales	80.95	79.24	1.71	21	20
Czech Republic	78.89	77.19	1.69	31	30
Switzerland	83.31	81.63	1.67	3	2
Belgium	81.46	79.87	1.59	17	16
Iceland	82.82	81.23	1.59	7	5
Scotland	79.12	77.54	1.58	29	29
Hungary	76.89	75.35	1.54	35	36
Greece	81.59	80.07	1.51	15	14
Northern Ireland	80.60	79.10	1.50	24	21
Canada	82.23	80.83	1.41	9	8
Estonia	77.38	76.01	1.37	34	34
Norway	81.95	80.66	1.29	10	10
Denmark	79.76	78.50	1.26	28	27
Sweden	82.39	81.16	1.23	8	6
Slovakia	77.88	76.73	1.15	32	33
Luxembourg	81.43	80.31	1.13	18	11
USA	80.01	78.93	1.09	27	23
Romania	75.08	74.07	1.01	38	39
Russian Federation	72.12	73.09	-0.97	41	41
Latvia	76.28	75.38	0.90	36	35
Netherlands	81.07	80.22	0.85	20	12
Bulgaria	75.88	75.15	0.73	37	38
Lithuania	77.51	76.79	0.72	33	32
Belarus	74.69	75.33	-0.64	39	37
Ukraine	73.56	74.06	-0.49	40	40

Source: HMD and Eurostat, own calculation

For instance, according to the conventional values, eastern German females have a 1.36 years higher life expectancy than U.S. women. However, after tempo-adjustment the life expectancy of U.S. women exceeds that of eastern German women by 0.32 years. Thus, this example shows that paradoxes such as those demonstrated in the previous section with model populations A and B (where population B shows the higher conventional period life expectancy although each cohort of population A lives longer than the corresponding cohort of population B) can exist in empirical reality. Given the different histories and structural compositions of the U.S and the eastern German population, it becomes apparent that tempo- adjusted life expectancy can provide a completely different result regarding mortality differentials and consequently can lead to very different conclusions regarding the determinants of mortality. Besides Italy and eastern Germany, the women from Australia, Ireland, Austria, Israel and Finland are the "losers" in the ranking of tempo-adjusted life expectancy. On the other side, the "winners" among females are the Netherlands (moving up from rank 20 according to conventional life expectancy to rank 12 according to tempo-adjusted life expectancy) and Luxembourg (moving up from 18 to rank 11).

Among males the first two places in the life expectancy rankings remain unchanged: Iceland is ranked first, followed by Japan (see Table 6). Contrary to the situation among women, the difference between these two countries increases from 0.56 years to 1.27 years once life expectancy is adjusted for tempo effects. Among males, tempo-adjustment also provides a very different picture of mortality differentials. For instance, according to the conventional estimation method, life expectancy of New Zealand's males (Non-Maori) exceeds those of men from the Netherlands by 1.04 years. After tempo-adjustment, Dutch males show a slightly higher life expectancy with an advantage of 0.13 years. Also interesting are the effects of tempo- adjustment on life expectancy differences between East European countries. According to the conventional values, Latvia's life expectancy exceeds that of Russia by 6.69 years. According to tempo-adjusted life expectancy, however, the difference is more than three years smaller. Among males, the "losers" in the ranking of life expectancy after tempo-adjustment – falling three or more ranks – are Australia, New Zealand (Non-Maori), Austria, Italy, Ireland and England & Wales. The "winners" are Greece (moving up from rank 13 according to conventional life expectancy to rank 6 according to tempo-adjusted life expectancy), Luxembourg (moving up from 21 to rank 15), the Netherlands (moving up from 14 to rank 11) and Denmark (moving up from 22 to rank 19).

Table 6: Conventional life expectancy e_0 and tempo-adjusted life expectancy $e_0{}^*$ for 41 countries, males 2001/2005, not tempo effects below age 30

				Rank	
	e_0	e_0*	Difference	e_0	e_0*
Australia	78.00	74.74	3.27	4	10
New Zealand (Non-Maori)	77.46	74.58	2.88	7	13
Eastern Germany	74.89	72.10	2.79	24	26
Austria	76.09	73.32	2.77	17	21
Italy	77.50	74.75	2.75	6	9
Finland	75.08	72.34	2.74	23	24
Ireland	75.67	73.06	2.61	19	22
England & Wales	76.56	73.97	2.59	11	14
Russian Federation	58.75	61.32	-2.57	41	41
Slovenia	72.98	70.46	2.52	29	29
France	76.13	73.64	2.49	16	17
Canada	77.39	74.91	2.48	9	7
Switzerland	78.05	75.58	2.47	3	4
Western Germany	76.15	73.70	2.45	15	16
Belarus	62.72	65.09	-2.37	39	38
Northern Ireland	75.72	73.47	2.25	18	18
USA	74.82	72.58	2.24	25	23
Japan	78.38	76.16	2.22	2	2
Taiwan	74.29	72.09	2.20	26	27
Norway	76.98	74.82	2.16	10	8
Czech Republic	72.34	70.21	2.14	30	30
Belgium	75.51	73.38	2.12	20	20
Scotland	73.90	71.78	2.12	28	28
Ukraine	62.02	64.07	-2.05	40	40
Sweden	77.98	75.95	2.03	5	3
Portugal	74.28	72.28	2.00	27	25
Israel	77.43	75.57	1.86	8	5
Spain	76.47	74.61	1.86	12	12
Denmark	75.17	73.43	1.73	22	19
Netherlands	76.42	74.71	1.70	14	11
Luxembourg	75.39	73.79	1.60	21	15
Poland	70.43	68.83	1.60	31	31
Iceland	78.94	77.43	1.51	1	1
Greece	76.44	75.12	1.33	13	6
Hungary	68.47	67.17	1.30	34	35
Slovakia	69.92	68.74	1.18	32	32
Estonia	65.98	65.28	0.70	37	37
Latvia	65.44	64.77	0.68	38	39
Lithuania	66.02	66.43	-0.40	36	36
Romania	67.82	67.49	0.33	35	34
Bulgaria	68.86	68.66	0.20	33	33

Source: HMD and Eurostat, own calculation

4.7. Conclusions

Tempo effects exist and occur as do age composition effects. This was shown with the empirical TMR for West Germany from 1970 to 2005. We have shown that both conventional and tempo-adjusted life expectancy standardize for these tempo effects. However, the two measures differ in the way they standardize. Conventional life expectancy deals with tempo effect- caused postponed deaths as if there were no tempo effects, whereas tempo- adjusted life expectancy takes tempo effects explicitly into account. These preconditions raise the questions about the purposes of period measures and how these purposes are addressed by the two standardization procedures. In our opinion, period indicators should measure only period conditions including the effects of changes which are independent of past and future assumptions (technical purpose). Furthermore, a period measure of mortality should reflect the current mortality conditions of the real cohorts in order to allow conclusions for political or medical interventions (practical purpose).

In light of these demands, our theoretical (model) examples have shown that tempo effects can lead to severe distortions of information about the current mortality conditions of a population when conventional life expectancy is used as an indicator for period mortality: (i) conventional period life expectancy can reach a level that no cohort ever achieves, (ii) conventional period life expectancy can decrease although each subsequent cohort experiences an increase in life expectancy (thus, conventional period life expectancy indicates a mortality increase that is not experienced by any cohort), and (iii) conventional period life expectancy can provide a lower level for a population A as compared to a population B, although each cohort of population A has a higher life expectancy than the corresponding cohort of population B (thus, conventional period life expectancy indicates a higher mortality of a population in which every cohort lives longer than the corresponding cohort of the other population). Although the models where these paradoxes appeared are based on the assumption that mortality changes take place as stated by the (most conservative) Bongaarts/Feeney assumption, we think there should be no theoretical situation in which such paradoxes can occur. The examples where mortality changes have been modelled to follow the conventional life table assumption as well as the most optimistic scenario (all postponed deaths surviving until the highest possible age) have shown that tempo-adjusted life expectancy is free of such paradoxical and misleading results.

The examples revealed that paradoxes provided by conventional life expectancy arise when the mortality changes in the cohorts alive in the period of observation are less favourable than stated by the conventional life table assumption. Such paradoxes cannot happen to tempo-adjusted life expectancy since this measure is based on the most conservative assumption that all postponed deaths immediately die in the next period. Additionally, this assumption is automatically identical for all populations regardless of their level of mortality changes. Thus, tempo-adjusted life expectancy uniformly standardizes for tempo effects in all populations. This does not apply to the assumptions behind conventional period life expectancy.

From the findings presented in this paper we conclude that tempo- adjusted period life expectancy does fulfil our demands on a period measure and is an adequate way of standardizing period mortality conditions for the compositional effects of age and postponement of deaths. In Section 4.6. we showed with empirical data that mortality tempo effects can cause conventional life expectancy to being biased by more than three years. Thus, tempo effects can lead to distortions which are strong enough to severely influence the estimation of life expectancy differences between populations and sub-populations and consequently also the analysis of determinants of mortality differentials. These results suggest that we can expect tempo effects to similarly affect the empirical analysis of most mortality differentials, including the opening and the recent closing of the mortality gap between women and men in the developed world, the linear increase in record life expectancy at birth described by Oeppen and Vaupel (2002) (2002), the increasing mortality gap between eastern and western Europe, and other similar phenomena.

The discussion about tempo effects is mainly a discussion about the definition and interpretation of period indicators. The question is not whether tempo effects exist. The question is whether they have to be seen as distortions that have to be taken into account. We argue that period life expectancy as an indicator for period mortality conditions must have a meaning for the currently living cohorts. This is a necessary precondition since period life expectancy is used as an indicator for the current health conditions of a population, to evaluate the effectiveness of specific health measures, or to evaluate the impact of specific factors on mortality. If the measure we use does not reflect the mortality of the real population we cannot draw the desired conclusions. Most papers criticizing tempo- adjustment of life expectancy focus on aspects related to the specific adjustment formulae rather than discussing the practical importance of tempo distortions (see Luy 2006, Luy 2008). We hope that our alternative way of looking at the assumptions behind conventional and tempo-adjusted life expectancy might help lead this discussion in a direction that does justice to the tempo approach of Bongaarts and Feeney regarding its application in the analysis of period mortality.

4.8. Special acknowledgements

We thank the colleagues at the Vienna Institute of Demography for stimulating discussions, John Bongaarts and Mark Hayward for valuable comments on earlier versions of this paper that have been presented at the PAA 2009 Annual Meeting of the Population Association of America in Detroit, USA and at the XXVI IUSSP International Population Conference 2009 in Marrakech, Morocco, as well as Richard Bates for language editing. Furthermore, we are grateful for helpful comments and suggestions received from two anonymous reviewers. This work was supported through grant P20649-G14 from the FWF Austrian Science Fund.

5. The role of shifted deaths in period mortality analysis

Wegner-Siegmundt, Christian submitted to Demography

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Abstract

Period shifted deaths have become very popular due to the recent debate about tempo effects in period mortality indicators. However, the idea of shifted deaths was not born during this debate. Older studies showed that individuals can gain or loss life time caused by a shift of their age at death due to a variation in survival conditions. This paper extends already existing methods for presenting a simple approach for deriving the age-specific shifted deaths as another mortality-related event in a strict period perspective. Both events of shifted and occurred deaths can be characterized by their intensity and timing. As a consequence, the period life table is almost determined by those intensities while the timing of both events is composited by the largest portion of occurred deaths and a remaining modelled portion of the expected ages at death for period shifted deaths. The expectation part of the life table, however, is highly flexible and is not constrained to the classical assumption of constant death rate. Moreover, the flexibility in modelling ages at death for shifted deaths allows a much more detailed analysis of current mortality conditions than only using standard constant rate assumption.

5.1. Introduction

The subject of period shifted deaths has become very popular due to the recent debate about tempo effects in period mortality indicators (Bongaarts and Feeney 2002, Bongaarts and Feeney 2008b, Bongaarts and Feeney 2010). The lifespans of individuals may lengthen or shorten due to a shift in deaths as a consequence of changes in health behaviour or improvements in medical or social conditions. The on-going changes in lifespan means that, for the current period, some individuals will die or experience a shift in their age at death. However, a shift in deaths leads to some paradoxical situations in which period mortality indicators like life expectancy provide a distorted view of current mortality conditions (Bongaarts and Feeney 2002, Luy 2008, Luy 2009, Luy and Wegner 2009, Luy 2010). This unexpected effect is characterized as a tempo effect (Bongaarts and Feeney 2002). Although the occurrence and analysis of tempo effects have been hotly debated among demographers (see Barbi et al. 2008), the assumed origin of tempo effects, period shifted deaths, has been less researched.

There are several studies that have only shown the occurrence of shifted deaths and their impact on period mortality indicators as a consequence of a modelled transition from a constant to another constant but reduced mortality regime (Horiuchi 2008, Luy 2008, Feeney 2010, Wegner 2010). While these works are very important for our understanding of tempo effects, they do little to explain how period shifted deaths occur in populations that lack idealised constant survival conditions. Furthermore, the proportion of the shifted deaths over all ages has been indirectly derived for real populations (Bongaarts and Feeney 2008b, Guillot 2008, Luy and Wegner 2009). The proposed methods which control for shifted deaths as the cause of tempo effects (Bongaarts and Feeney 2002, Bongaarts and Feeney 2008b) assume that this total proportion also applies uniformly to the proportions of shifted deaths at each ages within a given year. There is no proof that this assumption of an age-invariant proportion of shifted deaths reflects the real situation because we have no method for deriving the period occurrence of shifted deaths by age. The critics of the tempo approach in period mortality mainly refer to the fact that these adjustments of tempo effects provide a reflection of past instead of current mortality conditions (Wilmoth 2005, Rodríguez 2008, Wachter 2008). However, tempo adjustment means a control of period shifted deaths. If the adjusted indicators refer to the past, then shifted deaths seem to be a relic of past survival conditions which are not related to the current progress of mortality?

The tempo discussion alone provides a lot of questions and explicitly shows the need for detailed research on the measurement and characterisation of period shifted deaths. Regardless of the tempo approach used, there are, however, a number of other important reasons why research about shifted deaths in period mortality is extremely interesting. First, the idea of shifted deaths is not new and was not started by Bongaarts and Feeney and their debate about tempo effects in period mortality. An analysis of cohort individuals who saw shifts in their age at death was previously conducted by Vaupel and Yashin (1986, 1987). As a result of survival improvements, individuals can be divided into two groups: people who would have survived under old mortality conditions, and "resuscitated" survivors whose lives were saved. The second category includes people who would have died under old and less favourable survival conditions, but who are still alive because of the on-going shift in their age at death. Because of two restrictions in the approach taken by Vaupel and Yashin, a more detailed look at shifted deaths is necessary. Their applied associations were based on the assumption that mortality is only declining (Eq. 4 in Vaupel and Yashin 1987). As a consequence, only postponements of deaths or lifesavings were considered in their approach, while preponed deaths were not recognised. However, mortality progress cannot be determined by a steady improvement in survival conditions only; we must also consider the possibility of an increase in mortality. The second limitation relates to the cohort perspective and the comparison of the mortality conditions of the two cohorts. But in the period perspective, at least 100 different cohorts and their survivors cross the period of interest. Therefore, the translation of the concept of shifted deaths into a period perspective without constructing hypothetical cohorts presents a new challenge in mortality research.

It is not surprising that some scholars have already addressed this challenge, although they tended to focus not on the shift in deaths, but on the intensity of period deaths. In general, the intensity of any demographic process is the average number of events per individual (Wunsch and Termote 1978, p. 14, Feichtinger 1979, pp. 15-20). The application of intensity is mostly related but not restricted to demographic events which are repeatable, like births per women or the number of marriages per individual. It seems trivial to examine intensity in a mortality analysis, because each individual will die once in his or her lifetime, and death is therefore a non-repeatable event. But this argument only applies to the real or hypothetical cohort which follows a group of individuals from birth to the age at death.

Sardon (1994a) showed rather convincingly that mortality intensity based only on period occurred deaths and not on period rates deviates from the expected, natural value of one under changed mortality conditions. His results showed exactly the same translation divergence as Ryder's (1956, 1964, 1983) in his fundamental work on the association between cohort and period fertility intensity under changed fertility conditions. Since mortality is always accompanied with a fixed and natural intensity of one, the change in the intensity in the period perspective can only be caused from a change in the timing when deaths occur (Bongaarts and Feeney 2008b). Therefore, a period mortality intensity of one can only be indicated if the timing of deaths is constant. But a change in the average age at death caused by a variation in survival conditions yields a difference in the period intensity that deviates from one. In the words of Sardon, deaths in a period perspective are "non-renewable, but not inescapable" (Sardon 1994a, p. 131). Thus, this difference can only be caused by a shift in deaths.

Although the variation of the period intensity from one indicates the appearance of shifted deaths, it says nothing about the age distribution and the age-specific magnitude of the shifted deaths in the analysed period. But what happens if we combine the cohort approach by Vaupel and Yashin with the period intensity approach by Sardon? The answer is provided in the first part by applying a simple discrete approach. The result interestingly shows that beside the observed proportion of deaths, the proportion of shifted deaths can be characterised as another mortality-related event. Moreover, both events can be indicated at each age as long as mortality is changing. The second part then focuses on the age distribution of occurred and shifted deaths by their intensity and timing. While the timing and intensity of occurred death are quite simple to estimate, the characterization of period shifted deaths have to consider whether deaths are preponed or postponed. However, the analysis of different directions of shifted deaths allows a detailed and very interesting insight into the dynamic of current mortality progress.

The third part refers to the question: How does the period life table handle the occurrence of shifted death? The period life table is a standard and powerful tool in demographic research and provides important indicators like the life expectancy (Preston et al. 2001). As we will see, the largest portion of the life table already deals with the intensity and the timing of period observed proportions of deaths. The remaining portion applies the period intensity of shifted deaths, while only the expected age of death is modelled by the life table. The last part discusses the high flexibility of the life table model in implementing different assumptions for modelling the expected ages at death for the proportion of shifted deaths. In addition to the conventional approaches, other assumptions, like the death-delay model (Guillot 2008, Luy and Wegner 2009), also create a standardised set of death rates which consequently determine the level of life expectancy and the current mortality conditions. Thus, the life table exactly reflects the current period conditions, plus an additional and flexible value of the expected life time for the shifted deaths.

5.2. Measuring period shifted deaths by age

The initial point for deriving period shifted deaths is the period intensity of mortality. The intensity or "total mortality rate" (TMR) is technically defined as the sum of the cohort proportion of deaths occurring in the year $[t, t+1)$ from age group 0 to the maximal attainable age w (Sardon 1994a). The TMR then shows the expected average number of deaths per newborn if the cohort proportions of death in the analysed year remain constant in the future. Moreover, the period intensity of deaths can also be defined as the "timing index" (Ward and Butz 1980), which reflects "the degree of completeness of the cross-sectional sum of cohort events" (Luy 2010, p. 428). The difference of the TMR and the expected value of one defines a further intensity for a period: the intensity of the shifted deaths or the “total shifted death rate” ($TSDR$). Since each individual will definitely die in the future, the $TSDR$ represents the intensity of the total number of deaths of a cohort which do not occur during the analysed year.

$$TSDR(t) = 1 - TMR(t) = 1 - \sum_{x=0}^{w} i(x,t) \quad (1)$$

The proportion of deaths in age x and year t based on the number of births born $t-x$ years ago is defined as the unconditional death rate $i(x,t)$[14]. One possibility for estimating the unconditional death rate is shown by the survival proportion $s(x,t)$ multiplied by the death probability $q(x,t)$ of the age interval $[x, x+1)$ in the year $[t, t+1)$:

$$i(x,t) = s_c(x,t) \cdot q(x,t) \quad (2)$$

At first glance, the easiest method for estimating the survival proportion is to compare the proportion of living persons at some exact age to the number of births from which those persons were descended. However, the proportion is biased because of past migration, which increases or decreases the number of living persons, regardless of the original number of births. Hence, immigration can lead to an overestimation of the survival proportion because the immigrants and their additional number of deaths are not accounted for in the initial number of births. Conversely, the proportion may be underestimated due to outmigration, since individuals who emigrated are still counted in the number of births.

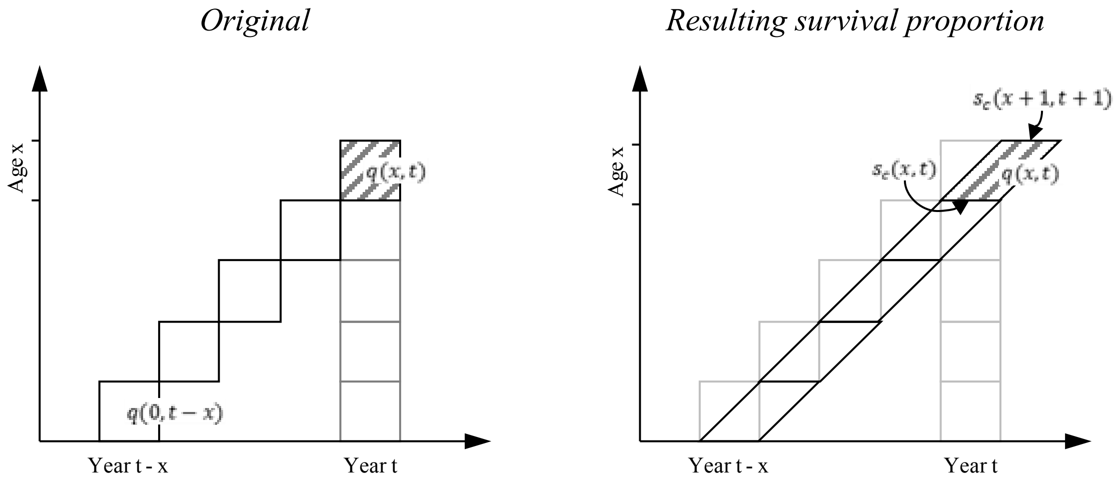
In order to minimise the migration bias, we can use an alternative method that relies on past series of age-specific mortality rates to reconstruct survival proportions. Rates are still affected by migration, but only within the considered age interval, because migration flows before this age interval are included in the current number of living and deceased persons[15]. The migration bias cannot be fully eliminated using this method, but it is significantly smaller than it is when the first method is used. For many countries, mortality rates and approximate death probabilities (Preston et al. 2001, Wunsch 2006) are calculated using the Type-I method (Caselli and Vallin 2006). This method is based on the third set of deaths (Keiding 1990), which covers the deaths of two successive cohorts within a one-year age and period interval. As a consequence, the approximated death

[14] It has also been called frequency, reduced rate (Sardon 1994b, Wunsch and Termote 1978, p. 15) or mortality rate of the second kind (Bongaarts & Feeney 2008b)

[15] If detailed migration data are available, the age-specific death rate and inherent probability can be fully adjusted under specific assumptions about the distribution of migration and possible deaths by migrants within the considered age and time interval (Batten 1978, pp. 2-15)

probability $q(x,t)$ is also based on the third set of deaths, but the number of persons who are exposed to the risk of dying is a mixture of the people from two successive cohorts who survived. Thus, the application of this kind of probability generates a survival proportion made up of the contributions of two successive cohorts.

Figure 18: Estimating survival proportions based on the third set of deaths



Source: own design

To illustrate this feature, the left Lexis diagram in Figure 18 shows the third set of deaths by rectangles for different ages within one year. For each rectangle, the age-specific death probabilities can be approximated by the inherent age-specific death rates (Preston et al. 2001, pp. 44-47). The survival proportion at some age can then be derived by applying the death probabilities from the diagonally arranged rectangles in the past. The right Lexis diagram in Figure 18 illustrates this transformation to a quasi-cohort by displaying the rectangles as parallelograms. But the proportion of deaths and applied rates or probabilities within each parallelogram still depend on the third set of deaths. The survival proportion at age x for a year $[t, t+1)$ can then be calculated by the product of all of the age-specific survival probabilities (one minus death probability) since birth:

$$s_c(x,t) = \prod_{a=0}^{x-1}[1 - q(a, t-x+a)] \qquad \text{with } s(0) = 1 \tag{3}$$

The application of the survival proportion instead of real numbers of living persons generates a population distribution that depends only on mortality. Interferences caused by different birth numbers or migration in the past are adjusted so that the derived indicators are based on mortality progress only (Guillot 2003).

After a normalisation of births (and a partial adjustment for migration), we can now apply Eq. 1 in indicating the period mortality intensity of a newborn. But it is also possible to examine the mortality intensity of an individual at some ages greater than zero. Based on the definition of the demographic intensity, the TMR and the inherent $TSDR$ at some age x can be defined as:

$$TSDR(x,t) = 1 - \frac{1}{s_c(x,t)} \cdot \sum_{a=x}^{w} i(a,t) \tag{4}$$

Thus, the age-specific *TMR* on the right side of Eq. 4 shows the expected average number of events, or, more technically, the proportions of cohort deaths above age x per individual survived to age x in the year t. Indicating the age-specific proportion of shifted deaths $g(x,t)$, Eq. 4 can be multiplied by the survival proportion. The result on the left side is just the sum of the proportions of the shifted deaths between age x and the highest attainable age w in the analysed year:

$$\sum_{a=x}^{w} g(a,t) = s_c(x,t) - \sum_{a=x}^{w} i(a,t) \tag{5}$$

Thus, the sum of the shifted deaths results from the difference in the survival proportion at age x and the sum of the period deaths above age x. Solving Eq. 5 with respect to the proportion of shifted death in age group $[x, x+1)$ results in

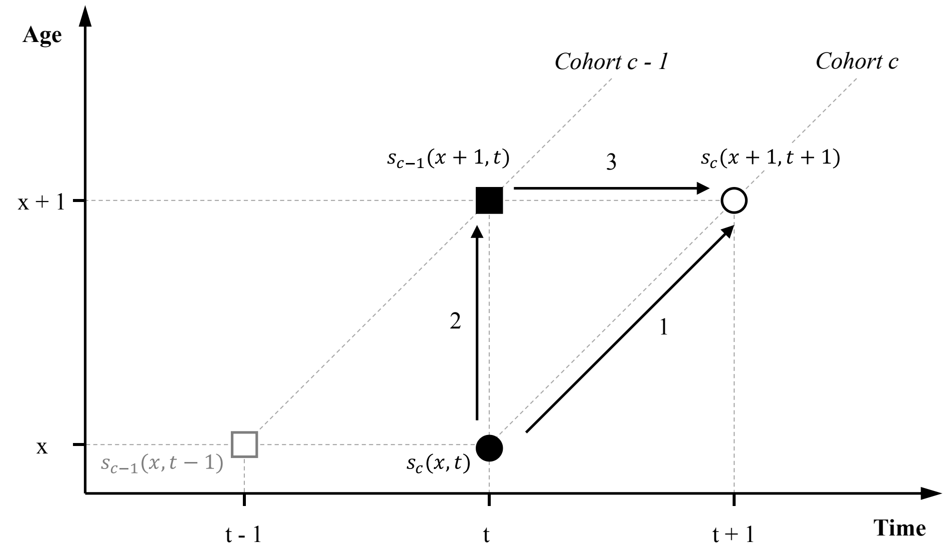
$$g(x,t) = i(x,t)^* - i(x,t) \tag{6a}$$

$$\text{with } i(x,t)^* = s_c(x,t) - s_{c-1}(x+1,t)$$

Expression 6 is of special interest because it shows a discrete conversion of the McKendrick (1925) and von Förster (1959) equation in a population with constant births. The expression shows in total three mortality-related events which characterise the dynamic of population change in the analysed period after excluding other demographic factors, like variations in fertility and migration.

The decrease in the survival proportion along the cohort is represented by the unconditional death rate, or those (birth-normalised) deaths occurring in the analysed year.

Figure 19: Changes in the survival proportion by three period mortality-related events



Source: own design

This change is illustrated in Figure 19 by Arrow 1, and indicates the natural attrition of survivorship due to mortality. The change in the survival proportion between two subsequent ages within the analysed year (Arrow 2 in Figure 19)—namely, between the current cohort at age x and the preceding cohort at age $x+1$—leads to the stationary unconditional death rate $i(x,t)^*$ of a population with the age distribution based on the period survival proportions (Bongaarts and Feeney 2002, pp. 26-27). Implementing these two changes in Eq. 6a results in:

$$g(x,t) = s_c(x,t) - s_{c-1}(x+1,t) - s_c(x,t) + s_c(x+1,t+1)$$

$$g(x,t) = s_c(x+1,t+1) - s_{c-1}(x+1,t) \tag{6b}$$

Consequently, the unconditional rate of shifted deaths shows the third kind of change (Arrow 3 in Figure 19) by the absolute growth in the survival proportion between the current and the following year. However, this change does not occur at exact age x, but as a consequence of a shift during age x at the beginning of the next age group. Therefore, the comparison of the survival proportion at the beginning of the next age group shows the basic characteristics of the shifted deaths. A higher survival proportion at the beginning of the subsequent age group and the next year compared to the current year can only be caused by better survival conditions in the past and the current periods, which lead to a postponement of the deaths of some individuals. As a consequence, the positive unconditional rate of shifted deaths $g(x,t)^{+}$ can be differentiated from all shifted deaths by:

$$g(x,t)^{+} = \begin{cases} g(x,t), if\ i(x,t)^{*} - i(x,t) \geq 0 \\ 0, if\ i(x,t)^{*} - i(x,t) < 0 \end{cases} \tag{7a}$$

A negative shift is, however, shown by a lower survival proportion as a result of worse survival conditions for the cohort in the past and the current periods. In this case, higher mortality had caused deaths to occur earlier than they had under the previous and more favourable survival conditions experienced by the survivors in the previous cohort. In line with the definition of positive shifted deaths, a negative unconditional rate of shifted deaths is then defined as:

$$g(x,t)^{-} = \begin{cases} g(x,t), if\ i(x,t)^{*} - i(x,t) < 0 \\ 0, if\ i(x,t)^{*} - i(x,t) \geq 0 \end{cases} \tag{7b}$$

The associations in Eq. 6 and 7 precisely follow the definition of lifesaving advanced by Vaupel and Yashin (1986, 1987, Vaupel 2008). Hence, a survival advantage or disadvantage can only result from a change in past and current mortality conditions, and the inherent lengthening or shortening of the lifespan can only be due to a shift in deaths. As an extension of the approach by Vaupel and Yashin, the approach proposed here considers the survival differences of all cohorts crossing the analysed period.

A further differentiation of the shifted deaths can be made based on their origin. The postponement or the preponement of deaths reflects mortality changes from both the past and the current period. If we reformulate Eq. 6b based on the number of survivors at the beginning of age group $[x, x+1)$ and the age-specific survival probabilities

$$g(x,t) = s_c(x,t) \cdot p(x,t) - s_{c-1}(x,t-1) \cdot p(x,t-1)$$

$$\text{with } p(x,t) = 1 - q(x,t)$$

results again in a difference in the survival proportions between two successive cohorts, and therefore in an occurrence of earlier shifted deaths at the beginning of the age interval. Under the condition of homogenous mortality risks, the survival proportion $s_c(x,t)$ can be substituted by survivors under the old condition, or those survivors of the earlier cohort $s_{c-1}(x,t-1)$ and the one-year age and period earlier shifted deaths of $g(x-1,t-1)$:

$$g(x,t) = g(x-1, t-1) \cdot p(x,t) + \quad \textit{Survivors of earlier shifted deaths}$$

$$s_{c-1}(x, t-1) \cdot [p(x,t) - p(x, t-1] \quad \textit{Appearance of shifted deaths originated in age group [x, x+1)} \tag{8}$$

The first component in Eq. 8 involves all shifted deaths from the past cohort which are further shifted under the current mortality risks. The second group contains the new shifted deaths in the current age interval, and thus indicates the age-origin of the shifted deaths. The number of survivors at the beginning of the age is standardised by the old mortality experience of the previous cohort. It is assumed that this survived proportion reflects the proportion of the younger cohort c at age x in year t who never experienced a shift in the age of their death in their lifetime. The change in the age-specific survival risk between the current and the previous period then determines the proportion of the new occurrence of shifted deaths in age group x.

The same expression, but in relationship to the composition of the age-specific growth rate in a specific year, was derived by Horiuchi and Preston (1988). Hence, the survival difference at the end of an age and period interval is a consequence of past changes in mortality conditions and the current mortality conditions. Moreover, it shows that the transition to individuals surviving because of shifted deaths moves in only one direction. Once individuals have experienced a shift in their age at death, they retain this status until they die, assuming their death has been postponed. In the case of negative shifted deaths, more individuals died in the current period or in earlier times compared to the preceding cohort. Although these individuals died in the past, their ages at death related to the previous and better cohort survival conditions have not yet been reached in the analysed period. Hence, if those people had not died earlier, their ages at death would not have been in the analysed age group of the actual current period.

Since the $TSDR$ characterised the proportion of individuals who experienced a shift in their age at death initiated in the current period, as well as additional shifted deaths from before the current period, the intensity of shifted death is also a timing index like the TMR. The period intensities of occurred and shifted deaths refer to the survival status of those cohorts living in the period of interest. They are pure period indicators of the current mortality conditions reflecting the deaths of individuals and the movement of deaths.

5.3. Timing of mortality-related events under current period intensities

The derived proportion or unconditional rate of shifted deaths shows that the mortality conditions in each period can be characterised by a total of three period mortality-related events and their related intensities. Thus, the period intensity of deaths can be determined by the difference between the prevalent intensity of the shifted deaths and the intensity of the hypothetical stationary deaths, which is always equal to one. Because of the distinction between postponed and preponed deaths, the intensity of the shifted deaths can be further differentiated based on the kind of movement:

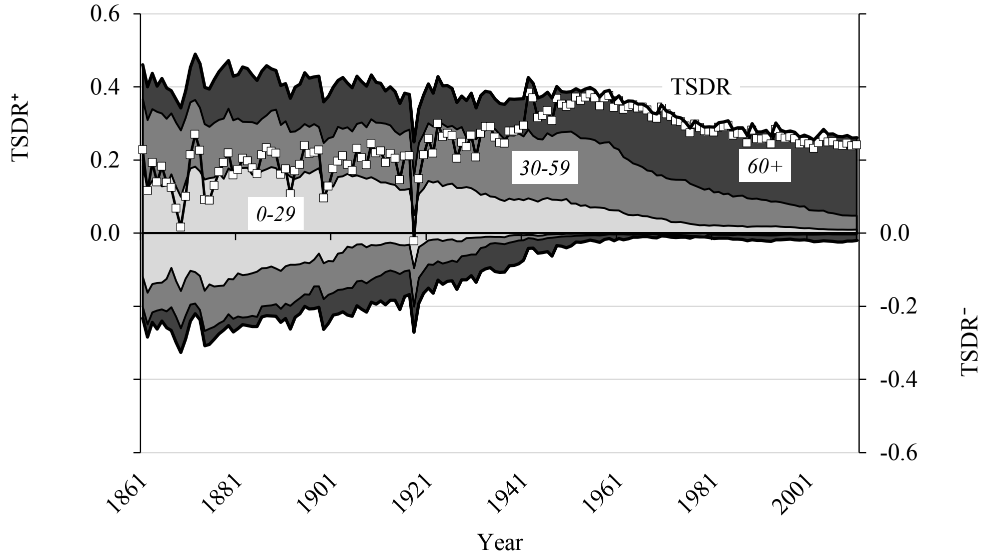
$$TSDR(x,t) = \frac{1}{s_c(x,t)} \cdot \left[\sum_{a=x}^{w} g(a,t)^{+} + \sum_{a=x}^{w} g(a,t)^{-} \right]$$

$$TSDR(x,t) = TSDR(x,t)^{+} + TSDR(x,t)^{-} \qquad (9)$$

The differentiation in Eq. 9 now allows us to take a detailed look at the dynamic of shifted deaths within each period. Figure 20 presents the trend in the period intensity of shifted deaths and the differentiated trend for positive and negative shifted deaths for the Swedish population between 1861 and 2011 (HMD, access on February 22, 2013). The white-dotted line shows the overall trend in the intensity of shifted deaths for age zero. Related to Eq. 4, the intensity of period deaths is just the difference between the $TSDR$ and one. Until 1920, the trend of shifted deaths is accompanied by strong fluctuations with an almost constant average value of 18% shifted deaths. Thus, 82% of the natural amount of mortality intensity had already occurred in the respective periods. A significant decrease in the $TSDR$ is observable for the year 1918, when the Spanish flu epidemic hit the Swedish population. The epidemic was so strong that the intensity even turned into a negative value. Therefore, the year 1918 was the only year in which the sum of the proportions of occurred deaths exceeded the natural intensity of deaths. Between 1919 and 1955, the trend was characterised by a steady increase in the $TSDR$ to the maximum value of 37% shifted deaths. After 1955, the intensity of shifted deaths steadily declined to reach the current value of 24% in 2011. Hence, only four-fifths of the natural mortality intensity occurred in the recent period, while the rest is still moving.

The decomposition of the $TSDR$ shows that the almost constant trend until 1918 was caused by the parallel on-going trend in the intensities of the positive (42%) and negative shifted deaths (24%).

Figure 20: Intensities of shifted deaths differentiated by postponed and preponed deaths for Sweden between 1861 and 2011



Source: Human Mortality Database, own calculation based on Equations 6 to 9

However, the increase in the $TSDR$ between 1919 and 1955 was mainly determined by the decrease in the proportion of preponed deaths, especially in the middle age group of 30-59 and the oldest age group of 60+. On the other hand, the unconditional rates of postponed deaths among people of younger ages (0-29) also started to decline after 1918, while people in the middle age groups benefited from a higher proportion of postponed deaths until 1950. The decline in the $TSDR$ after 1955 was mostly influenced by the reduction in postponed deaths among individuals under age 60. At the same time, the proportion of positive shifted deaths at higher ages started to increase. Although on a significantly lower level, the slight increase in the intensity of preponed deaths is also determined by the oldest age group. In the year 2011, 81% of shifted deaths occurred at older ages, 16% at ages 30-59 and only 3% at the youngest ages. This little exercise shows that the trend in the $TSDR$ precisely reflects the three typical stages of survival improvement found in contemporary countries (Meslé and Vallin 2006). The dominant pattern of the decline of infectious diseases might explain the marked reduction in preponed deaths prior to the 1940s. The second stage, which is characterised by an increase in postponed deaths at middle and older ages, is mainly related to the population-wide availability of antibiotics and new vaccines. The third stage is dominated by an increase in the proportion of postponed deaths at older ages. Improvements in medical treatments for cardiovascular diseases and the progressive adoption of healthy life styles have led to a further shift in deaths and to the potential for a further postponement of deaths at ages 60+.

While the intensities provide detailed insights into mortality progress, the timing of each period event further intensifies the analysis of period mortality-related events. The timing of an event is the average age at which the events occur. Based on the Eq. 6a, the period timing can be estimated for all three kinds of mortality-related events. The period timing of occurred deaths is defined by the standardised mean age at death (Sardon 1994b):

$$MAD(x,t) = \frac{\sum_{a=x}^{w}[a+0.5]\cdot i(a,t)}{\sum_{z=x}^{w} i(a,t)} - x \tag{10}$$

This measure reflects the average age at death only, based on the prevailing death distribution in the period. However, the same measure can be applied for the postponed and the preponed deaths of the period. Thus, the mean age of shifted deaths, or the $MASD$, is then characterised as:

$$MASD(x,t)^{+} = \frac{\sum_{a=x}^{w}[a+0.5]\cdot g(a,t)^{+}}{\sum_{z=x}^{w} g(a,t)^{+}} - x \tag{11a}$$

$$MASD(x,t)^{-} = \frac{\sum_{a=x}^{w}[a+0.5]\cdot g(a,t)^{-}}{\sum_{z=x}^{w} g(a,t)^{-}} - x \tag{11b}$$

The $MASD$ shows the average age at which either positive or negative shifted deaths occur in the analysed period. In contrast to the MAD, this timing of shifted deaths refers not to the death of individuals, but to the saving or the loss of life time as a result of the improvement or the deterioration of current and past survival conditions. The timing of the hypothetical stationary distribution of deaths is, consequently, the weighted average of the timing indicators for deaths and shifted deaths. Since the intensity of the stationary number of deaths is always one generated from the sum of the intensities of deaths and shifted deaths (Eq. 4), the weights are given by the TMR and the $TSDR$ only. Therefore, the mean age of the stationary death distribution, or the $MAHD$, results from

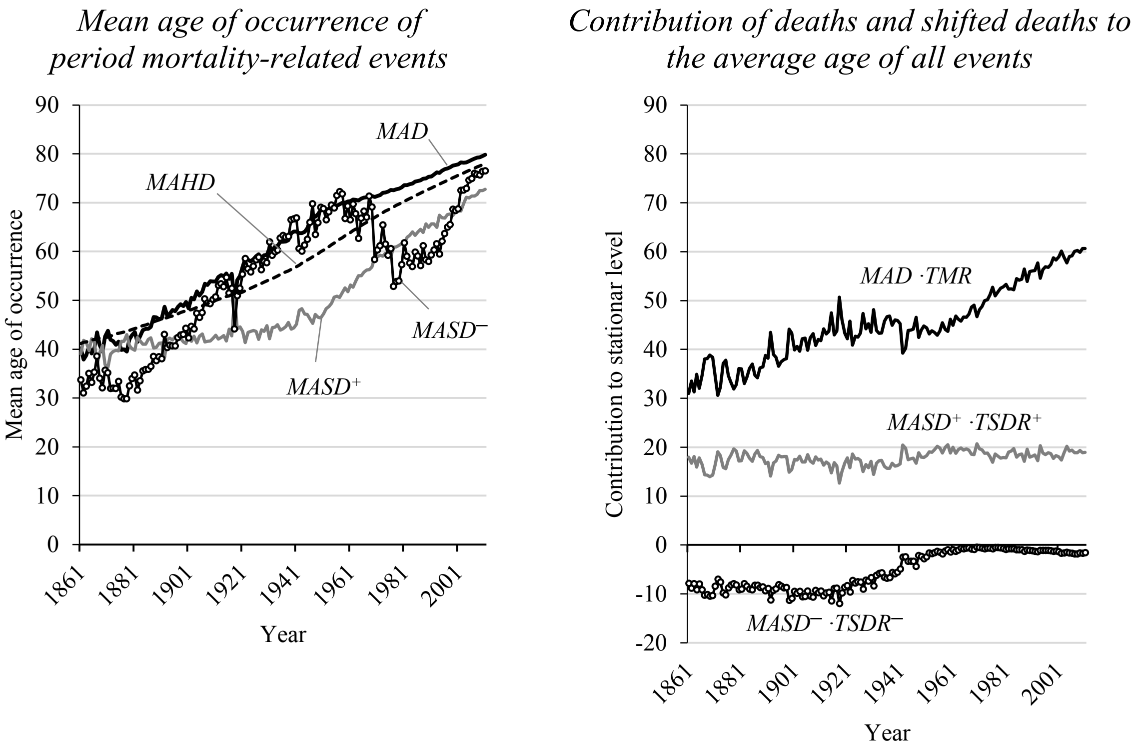
$$\begin{aligned} MAHD(x,t) = {} & TMR(x,t)\cdot MAD(x,t) + \\ & TSDR(x,t)^{+}\cdot MASD(x,t)^{+} + \\ & TSDR(x,t)^{-}\cdot MASD(x,t)^{-} \end{aligned} \tag{12}$$

Two possible interpretations of the $MAHD$ can be proposed based on the discrete solution of the McKendrick and von Förster equation in expression 6a. First, the $MAHD$ shows just the combined timing of the occurrence of the shifted and the occurred deaths in the analysed period. Thus, the combined timing is based on the intensities of both the deceased individuals and the individuals who experienced a shift in their age at death. The second interpretation is that the $MAHD$ provides a mean age at death based on the assumption that the survival proportions of those cohorts who crossed the period remained stable. As a consequence, the $MAHD$ refers to a hypothetical situation in which mortality becomes immediately constant (Bongaarts and Feeney 2008a). Based on the survival proportion, this hypothetical mean age at death considers only the cohort's past history (Sardon 1994b), and does not consider a further improvement or deterioration of survival conditions in the analysed period.

Based on the Swedish data, the left part of Figure 21 shows the trends in the mean ages at death, the shifted deaths and the weighted average of both events. The black line shows the trend of the MAD, which increases continuously from 40.2 years in 1861 to 69.1 years in 1955. Between 1955 and the beginning of the 1980s, the slope was much less steep than in the preceding years. But since 1985, the mean age at death has been increasing strongly, reaching to a level of 79.8 years in 2011. A completely different but still very interesting trend is that of the mean ages of shifted deaths. The average age of the positive shifted deaths was almost constant at around age 42 until the mid-1940s. A marked increase in the mean age started after 1950, with a linear increase of 0.4 years per year. The trend in the mean age for negative shifted deaths is mainly characterised by strong fluctuations. While it started at a lower level compared to the mean age of positive shifted deaths and of occurred deaths, the timing increased strongly from 33.7 to 72.2 years until the beginning of the 1960s. The next two decades were characterised by a strong decline in the average age of negative shifted deaths. Even after the year 1994, the trend was again determined by a strong increase from around 60 years to 76.5 years in 2011.

The combined timing trend of occurred and shifted deaths, or the mean age at death for the hypothetical stationary level, is significantly smoother than it is for the other trend. Between 1861 and 1925, the trend increased linearly by 0.2 years per year. During the period with the strongest increase in the $TSDR$, the combined trend also increased sharply, from 52.7 years in 1925 to 62 years in the mid-1950s.

Figure 21: Timing at age zero of period mortality-related events for Sweden between 1861 and 2011



Source: Human Mortality Database, own calculation based on Equations 10 to 12

Since 1960, the yearly increase in the combined mean age was still positive, but the degree of the slope declined from year to year. The right part of Figure 21 allows us to take a closer look at the weighted contribution of the occurred and the shifted deaths for the combined mean age of occurrence. We immediately notice the almost stable contribution of the postponed deaths. On the other hand, the weighted contribution of the negative shifted deaths was stable until 1920, and it fell to almost zero thereafter. Although the level is very low, it has been increasing in the last 20 years. The contribution of the occurred deaths, which is the group with the highest proportion of all mortality-related events, took place in two stages: one of increase, and, interestingly, one of decrease. Between 1861 and 1920, and again from 1955 to 2011, the weighted contribution increased steadily. The second period in particular was characterised by a strong increase of 0.3 years per year. Between 1921 and 1954, however, there was a reduction in the weighted contributions. One explanation for this reduction is that there was a strong increase in the $TSDR$ and an inherent fall in the TMR. During this period, the availability of antibiotics and the beginning of the cardiovascular revolution had led to a significant reduction in negative shifted deaths at all ages, and an increase in postponed deaths in the age group 30-59. As a result, the combined trend of deaths and shifted deaths was mainly dominated by a reduction in the contribution of preponed deaths and a slight increase in the weighted timing of positive shifted deaths.

5.4. Timing of mortality-related events under expected intensity

The examination of a mortality-related event that unfolds over several periods raises the question of to what extent the life table, which is a standard tool in mortality research, reflects the appearance of shifted deaths. In general, the period life table describes the dying off of a hypothetical cohort based on the age-specific death rates of the analysed period (Preston et al. 2001). Since all three mortality-related events observed in the period combine current as well as past mortality conditions due to the composition of the shifted deaths, the life table must provide a snapshot of the current period survival conditions only, independent of past mortality experiences. However, this requirement is only partly met when we examine in more detail how the life table death distribution and the timing are composed.

Related to the period information, the definition of the life table can be refined so that it shows the expected future deaths among those cohorts living in the analysed period at different ages. Hence, each cohort living in the period can be separately described using a period life table starting at the specific age x which the cohort members have reached in the period. The survival proportion of this remaining "cohort-specific" period life table $l_x(a,t)$ is then based on the radix presented by the period survival proportion $s(x,t)$ and the observed death probabilities of the year:

$$l_x(a,t) = s(x,t) \cdot \prod_{z=x}^{a-1} [1 - q(z,t)] \tag{13}$$

with $a > x \geq 0$ and $l_x(x,t) = s(x,t)$

The life table unconditional rate of deaths can then be calculated by the multiplication of the life table survival proportion with the period death probability at some age:

$$d_x(a,t) = l_x(a,t) \cdot q(a,t) \tag{14}$$

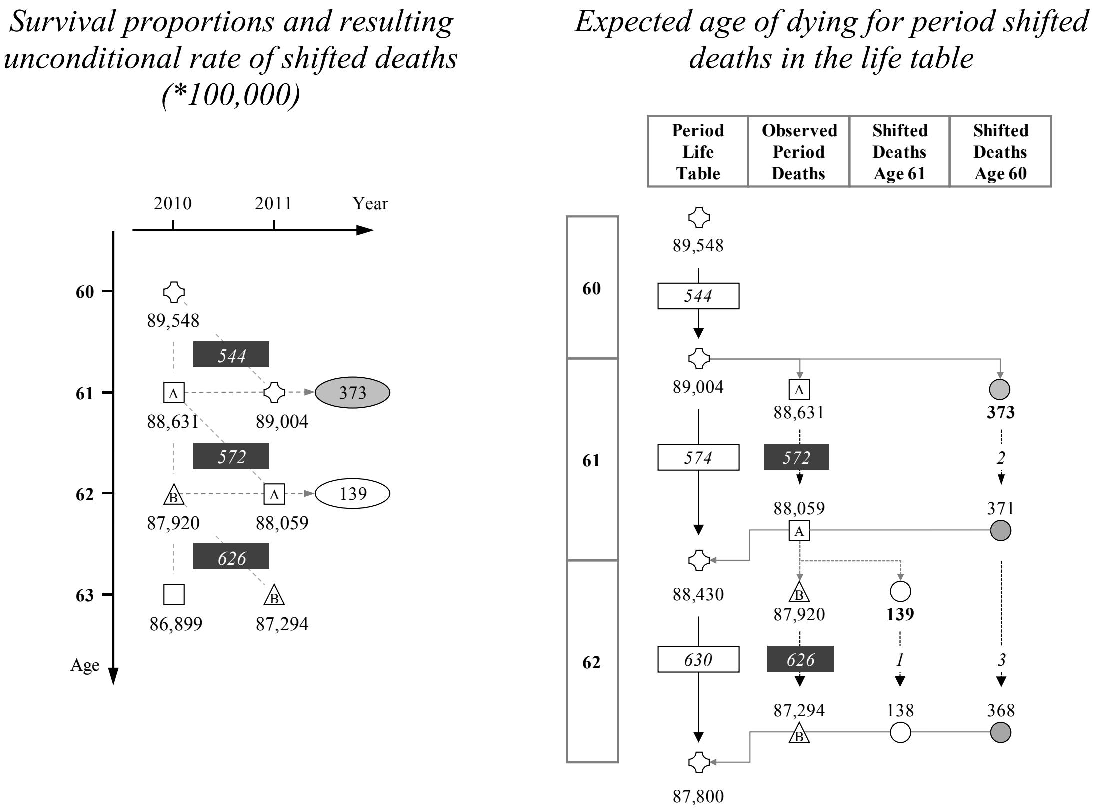
Since the life table shows the expected dying out of the studied cohort under the current period rates, the expected mortality intensity must always be one. Thus, the difference between the expected intensity in the life table and the current mortality intensity of the period can only be caused by the modelling of the age at death for those individuals who experienced a shift in their age at death in year t. This difference can be formulated using a simple term, whereas the difference in the unconditional death rate between the life table and the period equals the expected age distribution of deaths ${}^{g}d_a(x,t)$ for the currently shifted deaths that occurred in the period:

$$d_x(a,t) - i(a,t) = {}^{g}d_x(a,t) \tag{15}$$

Before presenting a solution for the transversal arrangement for the expected death distribution of the period shifted deaths in Eq. 15, I will provide an empirical example that should illustrate the assumed relation. After a birth normalisation of 100,000 births per year (multiplying Eq. 3 by 100,000), the age-specific numbers of survivors and deaths from age 60 to 62 for the year 2010 in Sweden are illustrated in the left Lexis diagram in Figure 22. The diagram shows that 544 individuals died in age group 60. In the next age group, age 61, 572 died out of 88,631 survivors; while in the last age group shown, age 62, 626 people died. Applying Eq. 6b enables us to determine the number of shifted deaths from the difference in the number of survivors at the end of the age group considered. Thus, 373 deaths from age 60 and 139 deaths from age 61 were positively shifted out from the year 2010.

Based on period death probabilities, we can now construct a life table or hypothetical dying out of the survivors at age 60 with a radix of 89,548 surviving individuals. The first column in the right diagram in Figure 22 shows the survival structure and number of life table deaths. The first row displays the life table survivors at ages 60 (89,548 survivors) and 61 (89,004 survivors). It becomes apparent from the left Lexis diagram that the period number of deaths at age 60 is equal to the number of life table deaths. This is not surprising because the survival proportion and the mortality risks are equal in the life table and the period. The period mortality conditions cause a shift of 373 deaths in age group 60. Using this information for the further construction of the life table allows us to break down the life table survivors into the 88,631 survivors under the old mortality condition (second column in the right diagram of Figure 22, labelled by A), and 373 postponed deaths from age 60 (fourth column).

Figure 22: Example for expected age at death for period shifted deaths in the life table, Sweden 2010



Source: Human Mortality Database, own calculation

It is particularly exciting that we already know from the period data that 572 survivors would have died in the age group 61 under the old conditions. However, the period data cannot show at which ages the 373 shifted deaths from age 60 will occur. But the life table model assumes that these shifted deaths will die based on the age-specific mortality rates derived from the period data. In the example, two shifted deaths from age 60 are expected to occur at age 61. The balance of the remaining 371 shifted deaths and the 88,059 survivors under the old mortality conditions results in exactly the number of life table survivors at the beginning of age 62. We can continue to decompose the life table survivors at the next age by again combining the period information and the life table death distribution of the shifted deaths. The group of life table survivors at age 62 is composed of

371 people who survived due to previously shifted deaths (originally from age 60), the survivors under old conditions from the period data (87,920 survivors marked with triangle B) and 139 shifted deaths from age 61. As in the previous age group, the life table shows the survivors of the period, or the survivors under the old conditions. Additionally, the table further provides an expectation about the ages at death of the shifted deaths from age 60, as well as from age 61 based on the period mortality risk. In the example, the remaining shifted deaths from age 60 lose three and the shifted deaths from age 61 lose one earlier rescued death in age group 62.

Consequently, one part of the distribution of the unconditional death rates in the life table exactly reflects the period distribution of the unconditional death rates. The other part shows the age-by-age inclusion of the shifted deaths and their expected survival status at some specific age, as was shown, for example, by Luy and Wegner (2009). This last association is precisely illustrated by the solution of Eq. 15 (Appendix B):

$$ {}^{g}d_x(a,t) = q(a,t) \cdot \sum_{z=x}^{a-1} \left[g(z,t) - {}^{g}d_x(z,t) \right] \tag{16} $$

with $a > x \geq 0$ and ${}^{g}d_x(x,t) = 0$

However, Eq. 16 can be applied separately for the group of positive (Eq. 7a) or of negative (Eq. 7b) shifted deaths. Due to the fact that the survival status of the period shifted deaths is not directly assignable from the period data, the life table unconditional death rates of the shifted deaths characterise the expectation part of the life table. On the other hand, the inclusion of the period observed proportions of death might be defined as the period-based part. As a consequence, the expected life table intensity of one is just the sum of the period intensities of the occurred and the shifted deaths. But, in contrast to Eq. 4, the $TSDR$ in the life table refers to the expected deaths of the period shifted deaths, and no longer to the event of the movement.

The division of the life table deaths into period-based and expected deaths allows us now to estimate the mean age at death for the period observed deaths and for the expected deaths from the period shifted deaths. The central tendency for the period deaths is known from the MAD. Thus, as part of the life table, the MAD shows the conditional remaining life expectancy for those individuals who died in the period considered, but were at least aged x or older.

The expected death distribution for the period proportion of the shifted deaths represents the second part of the life table. Based on the unconditional rate of the positive shifted deaths and the constant observed death rates, the timing of death is given by:

$$MAD_g(x,t)^+ = \frac{\sum_{a=x}^{w}[a+0.5] \cdot {}^{g}d_x(a,t)^+}{\sum_{a=x}^{w} {}^{g}d_x(a,t)^+} - x \tag{17a}$$

$MAD_g(x,t)^+$ then shows the conditional and remaining expected mean age at death for all of the individuals who experienced a positive shift in their ages at death above the chosen initial age x in the period t, if the period death rates and inherent approximated probabilities stay constant. The same timing indicator can also be derived for the proportion of preponed deaths:

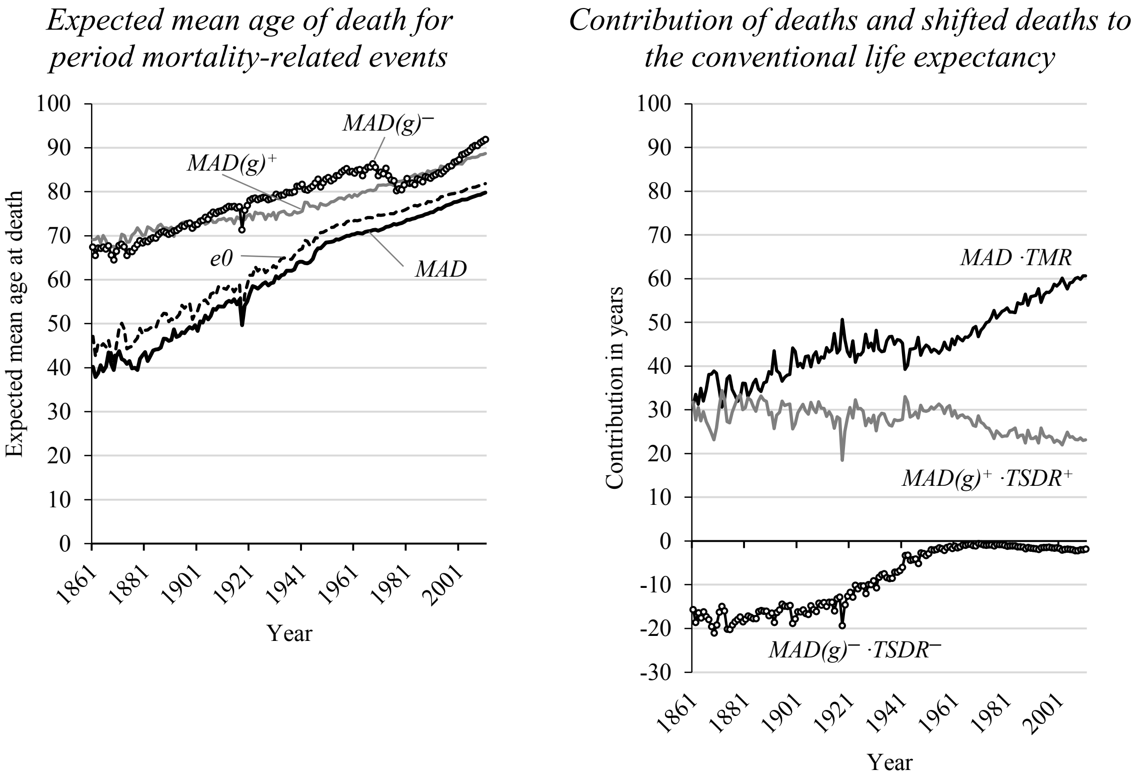
$$MAD_g(x,t)^- = \frac{\sum_{a=x}^{w}[a+0.5] \cdot {}^{g}d_x(a,t)^-}{\sum_{a=x}^{w} {}^{g}d_x(a,t)^-} - x \tag{17b}$$

But this indicator characterises the expected average loss of life years due to negative shifted deaths if the death rates remain constant. Furthermore, the indicator shows the expected age at death if those individuals had not died at earlier ages due to the worse survival conditions in the past or in the current period, compared to the mortality experiences of the preceding cohort. The combined mean age at death or the conventional period life expectancy at age x is then the average of the conditional mean ages weighted by the period intensity of the observed and the shifted deaths.

$$\begin{aligned} e(x,t) &= TMR(x,t) \cdot MAD(x,t) + \\ &\quad TSDR(x,t)^+ \cdot MAD_g(x,t)^+ + TSDR(x,t)^- \cdot MAD_g(x,t)^- \end{aligned} \tag{18}$$

Equation 18 interestingly shows that the period life expectancy aggregates both the contributions related to the mean age at death of the period deaths, and the expected mean age of the shifted deaths. The composition of life expectancy shows very clearly that the conventional life expectancy is not simply a result of constant mortality rates, but also of the period intensity and the timing of the period unconditional rate of deaths. The assumption of constant rates is reflected in the period-based and expected part of the life table, but with their impact on the life table death distribution clearly distinguished. Thus, the largest portion of the life table death distributions and their timing are made up of the period distribution and the timing of the unconditional death rates. The remaining portion refers to the dying out of the period intensity of the shifted deaths. However, the period cannot anticipate at which ages these shifted deaths will occur. At this point, the life table provides a model for deriving the expected mean age at death for the period intensity of shifted deaths based on specific set of death probabilities, as seen in Eq. 16.

Figure 23: Timing at age zero of expected mortality-related events for Sweden between 1861 and 2011



Source: Human Mortality Database, own calculation based on Equations 16 to 18

The left side of Figure 23 presents the trend of the mean ages at death at age zero for Sweden, again for the period 1861 to 2011, based on the period unconditional rates of death (black line); as well as the expected age at death of the positive shifted deaths (grey line) and of the negative shifted deaths (white-dotted point line) for the years 1861 to 2011. As expressed in Eq. 18, the expected mean age at death for the occurred period deaths is exactly equal to the indicator for describing the timing of the current period mortality intensity (Eq. 10). The main differences are found for the expected ages at death for the postponed and the preponed period deaths compared to the period timing of occurrence. This is, however, not surprising because the trend in the expected mean age at death must be different from the average age of the occurrence of shifted deaths, or of those saved period deaths. At first glance, the dynamic of the expected mean age at death follows the dynamic of the mean age of occurrence, but on a significantly higher level. Therefore, the expected mean age at death for the postponed deaths started to increase slightly from 69.1 years in 1861 to 75.3 years at the beginning of the 1940s. In the following years, the mean age also increased linearly by 0.2 years per year. As we saw in the occurrence of the preponed deaths (Figure 21), the expected loss of life years for the negative shifted deaths was also characterised by a stronger increase until the end of the 1960s than the positive shifted deaths. The rise did not, however, exceed the increase in the mean age of period deaths. Nevertheless, between 1861 and 1969, the expected loss increased from 67.4 years to more than 85 years in 1969. In contrast to the mean age of occurrence, the drop in the expected loss of life time was only observable for one decade. After reaching the lowest value of 80.5 years in 1980, the expected loss again began to increase, and currently stands at around 92 years.

The conventional life expectancy (dashed black line on the left side of Figure 23) as the weighted measure of period death timing and the expected age at death for the shifted deaths was always above the period mean age at death. While the difference was greater at the beginning of the period studied (around seven years in 1861), it had decreased to two years in 2011. In addition to the closing of the gap between the two timing indicators, there are at least two interesting findings in the comparison of the two trends. First, short-term fluctuations in the conventional life expectancy are determined by the intensities of the shifted deaths. An example is the impact of an influx of Jewish refugees from Norway in 1942[16] and from Denmark in 1943[17] (marked by arrows in Figure 6). The increase in the life expectancy for both periods is mainly determined by the increase in the expected age at death and the intensity of the postponed deaths. Both effects could have been influenced by the immigration of middle-aged and older Jews, which led to a brief increase in the number of people living in Sweden, and therefore to a reduction in Swedish death rates. Another interesting question can be answered by analysing the contribution of the shifted and the

[16] http://www.jewishvirtuallibrary.org/jsource/Holocaust/Chronology_1942.html (last access 04/23/2013)

[17] http://www.jewishvirtuallibrary.org/jsource/Holocaust/Chronology_1943.html (last access 04/23/2013)

occurred period deaths in life expectancy in the right side of Figure 23. What determines the convergence of life expectancy and the mean age of the period occurred deaths? Since the impact of preponed deaths was marginal after 1960, the convergence was mainly characterised by a decreasing contribution of postponed deaths, while the impact of period occurred deaths was increasing. In fact, this result is directly related to the trend in the observed intensities (see Figure 20). Since the 1960s, the trend in the $TSDR$ has been declining, while the TMR has been progressively increasing.

5.5. Current conditions and modelled shifted deaths

The decomposition of the life table shows that the major portion of the table death distribution is already determined by the period occurred deaths. The remaining, modelled portion is based on the intensity of the period shifted deaths and their expected dying-out based on a set of constant death rates. The question is now whether the life table constrains the use of period death rates for modelling the expected timing of death for the period shifted deaths. This question refers to the assumption that the period rates reflect current mortality conditions. There is no doubt that they do so because the death rate shows the ratio between the period proportion of cohort deaths and the period proportion of living persons of the same cohort. Precisely this portion of period mortality is considered in the period-based portion of the life table. Hence, the reproduction of the period observed death distribution in the life table is determined by the cohort survivors and the underlying age-specific rate, as illustrated by the column of observed deaths in the right side of Figure 22. Therefore, the period life table merely considers rates as a reflection of current mortality conditions. The expected portion of the life table thus only assumes that the period shifted deaths could experience a dying out based on the same set of mortality rates. But this set is a feature of the model, because applying different sets of rates results in different mean ages at death for the period shifted deaths.

The conventional life table model applies the observed period rate and the approximate probabilities for estimating the expected timing of death for the postponed and the preponed deaths. The average expected life time for the shifted deaths $ALT(x,t)$ can then be derived from the combination of the timing of the occurrence of mortality-related events (Eq. 12) and the timing of expected deaths (Eq. 18):

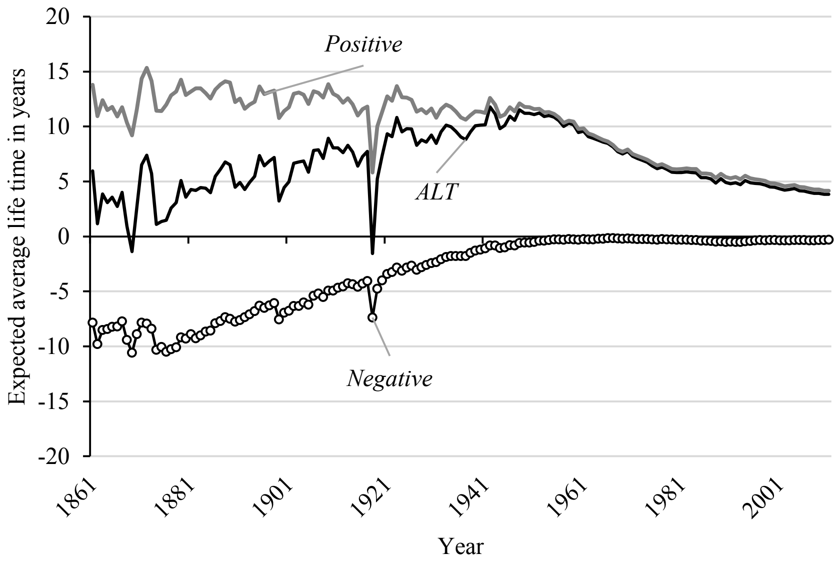
$$ALT(x,t) = e(x,t) - MAHD(x,t) \tag{19}$$

$$= TSDR(x,t)^{+} \cdot \left[MAD_g(x,t)^{+} - MASD(x,t)^{+}\right] +$$

$$TSDR(x,t)^{-} \cdot \left[MAD_g(x,t)^{-} - MASD(x,t)^{-}\right]$$

Since the contribution of shifted deaths is framed by their intensities, *ALT* results from the sum of the weighted absolute difference in the mean ages at death of the period shifted deaths that had occurred and the expected deaths in the life table. The indicator *ALT* then shows the expected gain or loss of years of life if the individuals experience a shift in their age at death in the analysed period. Therefore, the period life expectancy is based only on the period timing of mortality-related events: namely, the current conditions, plus the modelled average gain or loss of life time for those survivors who experienced a shift in the age at death in the considered period.

Figure 24: Expected average life time at age 0 for period shifted deaths based on constant death rate assumption for Sweden between 1861 and 2011



Source: Human Mortality Database, own calculation based on Equation 19

Figure 24 shows the trend in *ALT* based on an assumption of constant period death rates, which are further separated into positive and negative shifted deaths for Sweden. At first glance, we can see that the trend looks similar to the dynamic of the period intensity of the shifted deaths. This is, however, not surprising because the contribution of the shifted deaths to the mean age of the occurred mortality-related events (Eq. 12) and to the life expectancy (Eq. 18) is weighted by the period intensity of the shifted deaths. The expected average life time for the positive shifted deaths started at a level of almost 14 years in 1861, and increased to the recent value of 4.2 years. Particularly remarkable is the trend of the last 50 years, in which the expected life time for positive shifted deaths was reduced by more than the half. In fact, the expected average life time has recently been determined by the trend in positive shifted deaths because of the very low expected average loss of life time for the negative shifted deaths. The decrease in ALT from 9.6 years in 1961 to 3.8 years in 2011 represents a reduction of 60% over the last 50 years. Therefore, the gap between the mean age of the occurrence of period mortality-related events and conventional life expectancy has closed significantly in recent decades.

Another option for modelling the expected mean age at death comes from the discussion about tempo effects. The death-delay model (Guillot 2008, Luy and Wegner 2009) assumes that shifted deaths only experience a one-year age and time shift. Although this assumption seems unusual at first glance, it is based on all of the information that can be gleaned from a strict period perspective. When we look at the period mortality-related events, all we can know is that the share of people whose ages at death were shifted definitely will not die within the analysed year in the case of postponed deaths, or will not reach their anticipated age at death under the old and more favourable survival conditions in the case of preponed deaths. Without making any further assumptions about the future, we can make the simplified assumption that those shifted deaths can only experience a one-year period of survival within the prevalent period interval. However, this assumption does not negate the possibility of a further shift in the following years. In fact, the assumption completely ignores for the moment any additional progress in survival conditions that may occur in the future.

The implementation of the death-delay assumption in Eq. 16 is simply the replacement of all age-specific deaths probabilities by one:

$$ {}^{g}d_x(a,t)^{*} = \mathbf{1.00} \cdot \sum_{z=x}^{a-1} \left[g(z,t) - {}^{g}d_x(z,t) \right] \tag{20} $$

$$ \text{with } a > x \geq 0 \text{ and } {}^{g}d_x(x,t) = 0 $$

Thus, the expected deaths of the shifted deaths in the life table are just the one-year age shift of the period distribution of the unconditional rates of the shifted deaths. Hence, the advantage of the death-delay model is the application of a universal and simple standard (Luy and Wegner 2009), which is always equal for different periods or populations. The resulting expected timings of positive and shifted deaths are just the increase of their period timings of the occurrence by one year. The resulting average expected life time for shifted deaths is then only determined by the period intensity of the shifted deaths:

$$ALT(x,t)^* = TSDR(x,t)^+ \cdot [MASD(x,t)^+ + 1 - MASD(x,t)^+] + TSDR(x,t)^- \cdot [MASD(x,t)^- + 1 - MASD(x,t)^-] \tag{21}$$

$$ALT(x,t)^* = TSDR(x,t)$$

The comparison of the expected average life time for the shifted deaths shows that the value based on the death-delay assumption (see Figure 20) is significantly lower than the constant period rate assumption. But this result is just caused by the most conservative one-year shift assumption. However, the decrease in the *TSDR* over the last 50 years indicates a decline of only 33%. Compared to the 60% reduction from the constant period rate assumption, the rate of decrease in $ALT*$ is significantly lower.

Since the shifted deaths are assumed to gain a year-by-year increase in life time until they will die, the resulted weighted average of the observed and expected timing leads to:

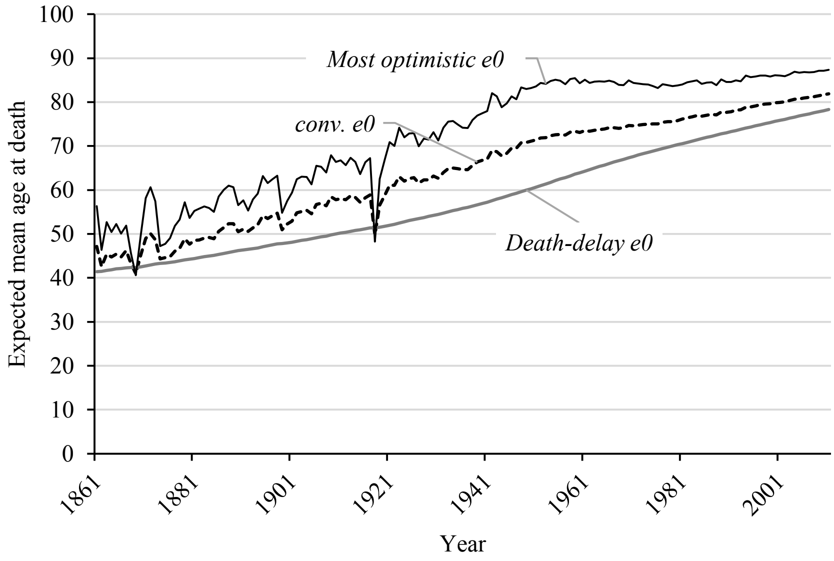
$$\begin{aligned} e(x,t)^* = {} & TMR(x,t) \cdot MAD(x,t) + \\ & TSDR(x,t)^+ \cdot [MASD(x,t)^+ + 1] + \\ & TSDR(x,t)^- \cdot [MASD(x,t)^- + 1] \\ = {} & MAHD(x,t) + ALT(x,t)^* \end{aligned} \tag{22}$$

Since $e(x,t)^*$ is based on the death-delay assumption, this measure refers to those indicators which are adjusted for tempo effects. Instead of the methods proposed by Bongaarts and Feeney (Bongaarts 2008, 2008b), this measure recognises the age-specific proportions of the shifted deaths. Hence, the age variation of the shifted deaths is fully included in $e(x,t)^*$. The resulted tempo-adjusted indicator is then decomposed into the mean age of the occurrence of mortality-related events and the additional contribution of the shifted deaths by their one-year shift. The difference between the conventional life expectancy and $e(x,t)^*$ is not caused by any distortion, but by different assumptions of the modelling of the shifted deaths. Thus, the difference between Eq. 19 and 22 is only presented by the different expected average life times for the shifted deaths, because the period mean ages of the mortality-related events are similar in both assumptions. Hence, the derived associations are not based on any tempo adjustment of distorted mortality rates. Moreover, the observed death rates are still included in this measure through an indication of the period unconditional death rates from the cohort survival proportion in the period-based part of the life table. Therefore, their timing is defined by the *MAD* and is still included in the mean ages of period mortality-related events and in both model assumptions.

In principle, it is possible to make several other assumptions related to a specific research question. For example, we can apply the period death rates of one country as a standard set for estimating the expected mortality timing of shifted deaths for the comparison of life expectancies among different countries. But this kind of standardisation is comparable to the death-delay model, while the later models are more restricted in their perceptions of the expected average life time for shifted deaths. Another very optimistic assumption is presented by a set of death probabilities, which are zero for every age group except for the highest age group of 110, with a death probability of one (related to the highest age group in the life tables from the Human Mortality Database). This model assumes that all of the postponed deaths will die at age 110, while the preponed deaths would have died at age 110 if they had not died earlier.

Together with the most optimistic model, Figure 25 presents the weighted average of the timing of period mortality-related events and the expected mean age at death for the period shifted deaths for the conventional and the death-delay models. The dotted line shows the trend in conventional life expectancy, as was explained in Figure 23. The lowest grey line shows the resulting trend by assuming a one-year and age shift for the shifted deaths. The trend is similar to that of the *MAHD* from Figure 21, but it is slightly higher due to the contribution of the current *TSDR*. The most optimistic trend is illustrated by the black line in Figure 25. Due to the optimistic assumption that the highest age group will be reached, the resulting average timing is high compared to the other models. Moreover, fluctuations from conventional life expectancy are more pronounced, with a decreasing trend seen in the 1960s. Despite the different levels and dynamics of each trend, all of the models are based on the mean ages of period mortality-related events. The differences are only based on the different models for indicating the expected ages at death for the shifted deaths.

Figure 25: Trend of period life expectancies under different assumption about the average expected life time for period shifted deaths, Sweden 1861-2011



Source: Human Mortality Database, own calculation

5.6. Conclusions

The role of period shifted deaths is important in period mortality analysis because they are just mortality-related events, like the deaths of individuals. The appearance of shifted deaths results from a survival advantage or disadvantage in the period proportions of cohort survivors living in the current period at different ages. These observed survival differences are caused by variations in past and current mortality conditions. Hence, individuals can experience continuous shifts in their age at death before and during the analysed period, or they can start to gain life time due to the death postponements in the current year. The period mortality is then characterised by a total of three mortality-related events: shifted and occurred deaths, as well as the hypothetical constant deaths as a combination of both previous events. Under constant mortality conditions, the number of mortality-related events will be reduced to only the deaths of individuals. Unlike in other studies, the proposed discrete methods for deriving shifted deaths presented here are very simple, and do not require any assumptions. The major limitation of this approach is the amount of data needed. The estimation of the three mortality-related events for one year must be conducted using data for at least the last 100 years. Based on the data in the Human Mortality Database (HMD), the identification of shifted deaths is only possible for a small group of countries: Denmark, England & Wales, Finland, France, Iceland, Italy, Netherlands, Norway, Scotland, Sweden and Switzerland.

The second part showed that each of these events can be characterised by the intensity and the timing of their occurrence based on their period distributions alone. Since individuals can experience a postponement or preponement of their deaths, it absolutely necessary to differentiate between the intensity and the timing related to each kind of movement. Although shifted deaths result from past and current conditions, they can also be used to detect the stages of major improvements in past survival conditions, as illustrated by using Swedish data.

Although the period intensity and the timing characterise the current appearance of mortality-related events, they have the disadvantage of combining current and past changes in mortality conditions. Therefore, standard methods for describing and explaining current mortality conditions use death rates which only provide the current ratio of deceased and living individuals, regardless of past conditions. The derived period life table is then assumed to be a model that describes the expected dying out of a hypothetical cohort based only on current mortality conditions. However, the question of how the period life table implements the period occurrence of the shifted deaths has led to two new and unexpected results. First, the largest proportion of the life table death distributions and their timing are influenced by the intensity and the timing of the period occurred deaths only. Moreover, the period intensity of the shifted deaths explain the remaining proportion of the life table death distributions, while their expected age at death is modelled based on the assumption of a constant death rate. Therefore, the life table also builds on the intensity of period events, which are affected by past and current mortality conditions. The second result shows that period life expectancy is dominated by the weighted timings of the period occurred and shifted deaths. The difference between the life expectancy and the period timing of mortality-related events is simply given by the expected average life time of the period shifted deaths. However, the remaining expected life time for the shifted deaths depends on a set of constant death rates, which can be modified independently of the observed period death rates. In principle, it is possible to apply any set of constant death rates when modelling the expected age at death of the shifted deaths. Although each different set of constant death rates results in different period life expectancies, all of these models are still based on the same initial timing presented by the period average age of the occurred and the shifted deaths. Thus, current mortality conditions are characterised by the occurred and the shifted deaths of the period. The last finding in particular further highlights the importance of shifted deaths in period mortality analysis. The period life table and in special the period life expectancy is not just a model/ indicator based on the assumption of constant age-specific death rates but rather a reflection of current period mortality conditions plus an additional expectation of the age at death for the period shifted deaths.

The high flexibility of the expectation part provides an exceptional variability of analysing current mortality conditions influenced by the appearance of shifted deaths. As a result of the discussion about tempo effects in period mortality, it was shown that the most conservative assumption of a one-year gain of life time for shifted deaths provides a simple and unique standardisation for the expected age at death of the shifted deaths. Furthermore, the derived tempo-adjusted life expectancy shows immediately the steady progress of mortality changes, and could further show the specific mortality transition level of one or several populations (Luy and Wegner 2009). Other assumptions, like the most optimistic perception, are highly sensitive with respect to the current age schedule of shifted deaths. The Swedish example presented an unexpected stagnation over more than three decades which was maybe caused by the reduction in the proportion of period shifted deaths and the ongoing increase in the mean age of period occurred deaths. Both extreme assumptions show that the analysis of period shifted death is a helpful and absolutely needful extension of the period mortality conditions instead only assuming constant period death rates.

6. Summary

My goal in writing this thesis was to gain a basic understanding of the tempo effects caused by changes in period mortality conditions and the inherent appearance of shifted deaths. In the past 10 years, only 20 articles or book chapters have focused on the impact of tempo effects on mortality. Most of these contributions discussed the technical pro and cons of the tempo-adjusted method proposed by Bongaarts and Feeney (Luy 2008, Luy 2010). But there was a lack of basic research on the origins and underlying mechanisms of tempo effects, as well as their potential for distorting conventional period mortality indicators. Thus, a number of questions related to this topic have yet to be settled, such as whether period mortality conditions can be reflected by mortality rates, or whether they are distorted by tempo effects, as the example in the introduction chapter shows. The need for more detailed research (Bongaarts and Feeney 2010, p. 11) on the appearance of tempo effects in mortality is therefore acute. The results of my thesis help to fill this gap in the previous research by addressing basic questions about the characteristics of tempo effects in period mortality indicators. Each of the presented articles refers to a specific question based on the counterarguments of tempo critics. My results show that these counterarguments vanish when we go one step back and focus on basic research on the appearance and the meaning of tempo effects in period mortality analysis.

The first part looked at the question of whether the appearance of shifted deaths and inherent tempo effects depends on the kind of mortality rate. The results in chapter 3 showed that all three types of mortality rate are affected by tempo effects. However, I was surprised to find that not one but two kinds of tempo effects could be derived depending on the origin of the shifted deaths. The first kind of tempo effect is caused by the postponement of deaths to the next year, whereas the second kind of tempo effect is generated by the shift of deaths via the age interval to the next group, either in the same or in the following period. The modelled trend of shifted deaths suggests that lower tempo effects of the second kind are caused by applying the death rate, which is based on the set of deaths within a one year of the age and cohort interval. However, the empirical comparison of the tempo effects in life expectancy at age 50 showed only marginal differences between both kinds of tempo effects. Therefore, it was impossible to reduce or dismiss the tempo effects by choosing one of three types of period death rates.

The answer to the first research question is that tempo effects are not statistical artefacts that depend simply on the kind of calculated death rate. Thus, a shift in deaths and a temporal deflation or inflation of death rates always occur when mortality is changing. But the question of whether the deflation or inflation affects the interpretation of conventional life expectancy has been a matter of debate, and the discussion has mainly centred on the definition of period mortality change.

Therefore, chapter four of the thesis called for the implementation of tempo effects based on a different definition of period mortality change and asked for the function that have to be performed by period life expectancy. The first partial results produced several contradictory trends in conventional life expectancy following a change in period mortality. Conventional life expectancy was projected to reach a level that no cohorts crossing the period of interest had ever achieved. Moreover, life expectancy could decrease or increase, even though each subsequent cohort would never experience such fluctuations in their mean life time. As a consequence, the comparison of conventional life expectancy between two populations was strongly distorted by such paradoxes, and led to a misinterpretation of the current period mortality conditions. Applying the tempo-adjusted life expectancy based on the conservative assumption that shifted deaths only survived to the next age group did not result in such misleading results. Moreover, the chapter also presented a comparison of the conventional and tempo-adjusted life expectancy under the traditional assumption of constant death rates. However, the conventional indicator immediately reached the expected stationary level, but it completely ignored important changes during the transition of all cohorts to this new constant mortality level. But the tempo-adjusted life expectancy precisely reflected the steady progress of mortality changes, and could further show the transition level of each population. Therefore, applying the conservative assumption of a one-year age shift was found to be a simple and standardised way to analyse period mortality conditions, regardless of the assumptions about how period mortality conditions were changing.

The second partial result of chapter four showed that the empirical analysis of the gender-specific life expectancy adjusted for tempo effects supports these theoretical findings. In addition to changing the ranking of countries with the highest and lowest life expectancies, the results provide important and unexpected details in the cross-country comparison of life expectancy. The tempo-adjusted life expectancy shows, for example, that period mortality conditions in Eastern countries are even more homogenous than conventional indicators have found. Therefore, tempo effects distort conventional life expectancy and the inherent interpretation of current mortality, even if period mortality condition has changed. This misleading interpretation cannot happen when tempo-adjusted life expectancy is applied, because this indicator only reflects the mortality changes of the period, without any expectations about the future. Thus, tempo-adjusted life expectancy precisely meets the practical and technical requirements of a period mortality indicator. However, the assumption of constant proportions of shifted deaths is a limitation. Therefore, the results of the third chapter are necessary for explaining tempo effects, but the adjustment of tempo effects is still controversial in the absence of knowledge about the age distribution of period shifted deaths.

This was the starting point for chapter five which asked for the age schedule of shifted deaths and their implementation in conventional period life table. While each of the previous chapters referred to the appearance of shifted deaths as the origin of tempo effects, these shifted deaths were only derived within a model or in a restricted assumption within the adjustment method. By translating the concept of shifted deaths from a cohort to a period perspective, it was, however, possible to identify different age-specific proportions of period shifted deaths in the empirical data. Interestingly, the derivation showed that the shifted deaths depended not only on the difference in the number of deaths, but also on the survival proportions of the two successive cohorts living in the analysed year. More important was, however, the finding that the estimation of shifted deaths classified them as mortality-related events. Thus, the analysis showed that changed period mortality conditions affected not only deceased individuals, but also individuals who experienced a shift in their age at death. The Swedish data showed that only analysing the period proportions of shifted deaths exactly reflects the relevant stages of past period survival improvements. The second result referred to fact that the conventional life table is based only on current period death rates, but that the table is, surprisingly, already able to show the period intensity of occurred and shifted deaths and the period mean age of death. Thus, the life table includes all mortality-related events of the period, even though these events are influenced by past and current mortality conditions. The unique feature of the life table is that it provides a projection of the ages at which the shifted deaths might occur. However, the distinction between the period determined deaths and the expected death distribution for the shifted deaths shows that the period life table is not limited to modelling the ages at death for the shifted deaths by applying the period death rates. Any assumption can be made to estimate the expected average life time for the shifted deaths. However, the previous chapter four found that only the conservative assumption of one-year age and period shifts produces the most accurate reflection of current mortality conditions, and is applicable as a standard assumption for every population in every period. Thus, the application of the conservative assumption in the life table model prevents the presence of paradox trends in period mortality indicators caused by a tempo effect.

All of these results explicitly showed that even when period mortality is changing, (i) shifted deaths appear in any method for estimating death rates, and can be differently distributed over ages, (ii) shifted deaths cause tempo effects which distort current mortality conditions, and (iii) the tempo effect can be easily adjusted by an anticipated one-year increase in life for the period-shifted deaths in the life table model. All three findings contradict the counterarguments of tempo critics and highlight the importance of considering tempo effects in period mortality analysis.

Given my results on the occurrence and the interpretation of shifted deaths and the inherent occurrence of tempo effects, I have some suggestions for further research. In the future, researchers must find solutions for the data required in estimating tempo-adjusted period indicators. In particular, the estimation of the age-specific proportion of shifted deaths for one year requires us to have age-specific mortality rates for at least the last 100 years. Based on the Human Mortality Database, it appears that there are only 11 countries that have the long time series needed in my proposed method for estimating age-specific shifted deaths. The challenge for future research is therefore to develop direct or indirect methods for bypassing these data limitations. Another interesting question refers to the shift in the deaths differentiated by causes of deaths or other factors, like health status or socioeconomic factors. In principle, an analysis of the different causes of deaths and of the shift in cause-specific deaths can be done using the approach presented in chapter five. Again, however, the demand for data limits these options. Moreover, the researcher would have to analyse whether the special methods related to the life table, like the age and cause decomposition of mortality, could also be used for tempo-adjusted indicators or the period timing of the occurrence of period mortality-related events. After these are proofed, there would no barriers to applying these special methods to the detailed analysis of factors in tempo-adjusted indicators.

The third and most important task for the future is the application of the tempo approach in answering existing or new research questions about the trend in period mortality conditions. Only few studies have already applied the tempo approach in explaining period trends in mortality. For example, we used the tempo approach for analysing whether tempo effects distort the geographical variation in European mortality among 34 countries in 2001/05 (Luy et al. 2011). The results showed that, based on the tempo-adjusted life expectancy found at age 15, Western, Central and Eastern Europe are different areas with unique mortality patterns. But the level and ranking of conventional and tempo-adjusted life expectancy differ significantly based on the adjustment controls for the period improvement or impairment of survival conditions. For example, males in Russia, Belarus and Ukraine were found to have a life expectancy that was two years higher than was estimated using conventional life expectancy estimation methods. In this ranking, for example, males in Austria fell four places and males in Denmark fell three places. On the other hand, Greek males moved from seventh to fourth place, and were ranked in the top five European countries with the highest male life expectancies after adjusting for tempo effects. In another publication, Luy (2005, 2008) showed that tempo effects mainly explain the huge increase in life expectancy and the inherent convergence to the Western mortality pattern among eastern Germans after the fall of the communist regime. More recently, Mackenbach (2012) considered tempo effects as plausible distortions in his analysis of the convergence and divergence of the European mortality pattern. These few examples show that tempo effects play an important role in analysing and explaining period mortality patterns. At the moment, the potential of tempo adjustment is still being explored, and it is likely that it will prove to be a unique tool in demographic analysis. I am confident that my thesis contributes to the realisation of this potential.

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Appendix A - Total mortality rate and tempo adjustment

The total mortality rate (TMR) is a rarely used measure to determine the mortality condition of a period (Sardon 1994a, Bongaarts and Feeney 2008b). The calculation is based on the age-specific death rates of the 2nd kind. These rates state the proportion of persons of a birth cohort who died at age x at time t.

(1) Death rate 2nd kind

$$i(x,t) = \frac{D(x,t)}{B(t-x)} \qquad \text{A.1}$$

$D(x,t)$ Number of deaths at age x of year t
$B(t-x)$ Number of persons born t-x years ago

The TMR is then calculated from the sum of the age-specific death rates of the 2nd kind.

(2) Total Mortality Rate (TMR)

$$TMR(t) = \sum_{x=0}^{w} i(x,t) \qquad \text{A.2}$$

Similar to the total fertility rate (TFR), the TMR is a "quantum measure" as it states the average number of events per individual for a hypothetical cohort of a year t. Since each person can only die once, the expected value of the TMR is always one (see Chapter 5). However, the TMR is below one if the average age at death increases during an analysed period or is bigger than one if the age at death declines (Bongaarts and Feeney 2008b, Luy and Wegner 2009). The difference of the TMR of one is regarded as being the indicator of the presence of tempo effects. In order to adjust the conventional death rates by the tempo effect, Bongaarts and Feeney work on the assumption that the effect is constant at all ages. The tempo-adjusted death rate $m(x,t)^*$ is then determined from the ratio between the conventional death rate and the TMR.

(3) Tempo-adjusted death rate

$$m(x,t)^* = \frac{m(x,t)}{TMR(t)} \qquad \text{A.3}$$

Appendix B - Deriving the life table death distribution for period shifted deaths

Initial assumption is expressed by

$$d_x(a,t) - i(a,t) = {}^{g}d_x(a,t) \qquad \text{B.1}$$

with $a > x \geq 0$ and ${}^{g}d_x(x,t) = 0$

Expressing the unconditional rates by survival proportion and death probabilities

$$l_x(a,t) \cdot q(a,t) = s(a,t) \cdot q(a,t) + {}^{g}d_x(a,t) \qquad \text{B.2}$$

Using Eq. 13 (p. 80) leads to

$$s(x,t) \cdot \prod_{z=x}^{a-1} [1 - q(z,t)] \cdot q(a,t) = s(a,t) \cdot q(a,t) + {}^{g}d_x(a,t)$$

$$s(x,t) \cdot \prod_{z=x}^{a-1} [1 - q(z,t)] = s(a,t) + \frac{{}^{g}d_x(a,t)}{q(a,t)} \qquad \text{B.3}$$

Substitute the product term in the left side of B.3 by the sum of all life table death before age a

$$s(x,t) - \sum_{z=x}^{a-1} d_x(z,t) = s(a,t) + \frac{{}^{g}d_x(a,t)}{q(a,t)} \qquad \text{B.4}$$

The period survival proportion $s(a,t)$ in the right side of B.4 can be expressed by the initial survival proportion at age x minus the sum of the period stationary unconditional rate of deaths $i(x,t)^*$ between initial age x and before age a

$$s(x,t) - \sum_{z=x}^{a-1} d_x(z,t) = s(x,t) - \sum_{z=x}^{a-1} i(z,t)^* + \frac{{}^{g}d_x(a,t)}{q(a,t)} \qquad \text{B.5}$$

Using Eq. 6a (p. 71) for substituting $i(a,t)^*$ leads to

$$s(x,t) - \sum_{z=x}^{a-1} d_x(z,t) = s(x,t) - \sum_{z=x}^{a-1} i(z,t) - \sum_{z=x}^{a-1} g(z,t) + \frac{{}^{g}d_x(a,t)}{q(a,t)}$$

$$\frac{{}^{g}d_x(a,t)}{q(a,t)} = \sum_{z=x}^{a-1} i(z,t) + \sum_{z=x}^{a-1} g(z,t) - \sum_{z=x}^{a-1} d_x(z,t) \qquad \text{B.6}$$

The sum of the life table deaths $d_x(a,t)$ can be substituted by Eq. B.1

$$\frac{{}^{g}d_x(a,t)}{q(a,t)} = \sum_{z=x}^{a-1} i(z,t) + \sum_{z=x}^{a-1} g(z,t) - \sum_{z=x}^{a-1} i(z,t) - \sum_{z=x}^{a-1} {}^{g}d_x(z,t) \qquad \text{B.7}$$

Reducing and reorganising B.7 leads to

$$ {}^{g}d_x(a,t) = q(a,t) \cdot \sum_{z=x}^{a-1} [g(z,t) - {}^{g}d_x(z,t)] \qquad \text{B.8}$$

with $a > x \geq 0$ and ${}^{g}d_x(x,t) = 0$

Eidesstattliche Versicherung

Ich erkläre hiermit, dass ich die vorliegende Arbeit ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.

Die Arbeit wurde bisher weder im Inland noch im Ausland in gleicher oder ähnlicher Form einer Prüfungsbehörde zur Erlangung eines akademischen Grades vorgelegt.

Wien, den 19.07.2013

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