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Feasibility and Performance of Relay-Aided Interference Alignment

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Contents

Chapter 1.

Introduction

1.1. Motivation

Wireless radio communication has experienced explosive growth in the last decades. Thanks to both theoretical and technological advances, the achievable data rate of practical wireless radio communication systems has dramatically increased from a few bits per second of wireless telegraphy to a peak rate of 1 Gbit/s as specified in the Long Term Evolution Advanced (LTE-A) standard [3GP16]. However, the demand for a higher data rate has never ceased. Moreover, the increment in the data rate enables many modern applications of wireless radio communications, e.g., wireless internet, wearable devices, internet of things, etc., which have changed our lifestyle. It is expected that, as new wireless radio communication techniques are developed and the achievable data rates further increase, more innovative applications will become reality in the future. For these reasons, pursuing higher data rates is always one of the main objectives of research in wireless radio communications.

In state-of-the-art wireless communication theory, the multiuser interference is considered as a major performance-limiting factor. Multiuser interferences are caused by the physical phenomenon in which radio waves superpose while propagating through the same medium. Therefore, if a wireless channel is shared by multiple users, the data transmissions of different users mutually interfere with each other. From a communication theory perspective, this results in a saturation of achievable data rates. More precisely, with increasing transmit powers, both the useful signal power and the interference power of each user increase and, consequently, its achievable data rate approaches a limit. In order to avoid multiuser interferences, conventional interference management approaches, such as frequency division multiple access (FDMA) and time division multiple access

(TDMA), separate the data transmissions of different users by orthogonal channel access. These conventional interference management approaches have been widely used in current wireless radio communications systems. However, when using these conventional approaches, the resources, e.g., the spectrum in case of FDMA and the time in case of TDMA, that can be assigned to each user is inversely proportional to the total number of users. This effect is interpreted as "cake-cutting" in [CJ08], and is also considered as the major drawback of these conventional interference management approaches. Therefore, better approaches which yield more efficient resource usage and, consequently, higher data rates are required for future wireless radio communication systems.

Recently, it has been discovered that every user in a multiuser interference network is able to achieve nearly half of the data rate that he could achieve without interference [GCJ11]. In other words, everyone could get "half the cake" [GCJ11]. This can be achieved by a novel interference management approach known as interference alignment (IA). IA aims at eliminating the multiuser interferences through cooperatively designed filters of different users. If IA can be achieved, the multiuser interferences can be completely nullified and, consequently, the achievable data rate of each user no longer saturates as the transmit powers increase. As compared to the aforementioned conventional interference management approaches, IA is able to achieve an impressive performance gain in large interference networks with lots of users.

In literature, a variety of IA schemes have been proposed and studied, such as IA using time extensions [CJ08], IA using multiple antennas which is referred to as multiple-input-multiple-output (MIMO) IA [GCJ11], and IA using relays which is referred to as relay-aided IA [GCJ11, NMK10]. As compared to other IA schemes, especially the most extensively studied MIMO IA, few results on relay-aided IA have been published. Nevertheless, relay-aided IA is still a valuable IA scheme and an interesting research topic mainly due to the following three advantages. First, relay-aided IA requires only few resource extensions which are required by the type of the relays. For instance, relay-aided IA using half-duplex relays, which cannot receive and transmit simultaneously, requires only two time slots. Second, relay-aided IA requires only few antennas at the source and destination nodes. Sometimes, even a single antenna at each source and destination node is sufficient. Third, many relay-aided IA problems can be solved linearly, which yields closed-form solutions.

Like many other IA schemes, relay-aided IA is usually based on the idealized assumption that the wireless channel is fully known. However, the acquisition of full channel knowledge is very challenging or even impossible in practice. Recently, some IA schemes which do not require full channel knowledge have been proposed, e.g., IA with outdated channel knowledge [MAT12], topological IA [Jaf14], and blind IA [GWJ11]. Unfortunately, little work has been done for relay-aided IA without full channel knowledge so far.

This thesis focuses on relay-aided IA. The research topics include the IA solutions, the feasibility conditions, relay-aided IA with partial channel knowledge, and the achievable performances. These research topics will be discussed in a few representative wireless relay interference networks.

1.2. State of the art

1.2.1. Degrees of freedom

The research on IA is closely related to the degrees of freedom (DoFs) of wireless channels. The DoF is a performance metric representing the characteristics of the achievable rate in the high signal-to-noise-ratio (SNR) regime [JS08]. It is also equivalently described by various researchers as the multiplexing gain or the pre-log factor [TV05]. Let $R_{\mathrm{sum}}(P_{\mathrm{tot}})$ denote an achievable sum rate in a wireless channel under the total transmit power constraint P_{tot} for all the transmitting nodes. Then, the corresponding sum DoF is defined as

$$\mathrm{DoF} = \lim_{P_{\mathrm{tot}} \to \infty} \frac{R_{\mathrm{sum}}(P_{\mathrm{tot}})}{\log\left(P_{\mathrm{tot}}\right)}. \tag{1.1}$$

Graphically, the sum DoF corresponds to the asymptotic slope of the sum rate curve, when it is plotted as a function of the logarithm of the SNR. Moreover, the maximum achievable sum DoF in a wireless channel is referred to as the sum DoF of the channel. The sum DoF of a channel is also the maximum number of data symbols that can be transmitted through the channel for a single channel use without interfering each other. Similar to the sum DoF, the achievable DoF region have been defined in [JS08, CJ08] as well.

In point-to-point scenarios, a single source node communicates with a single destination node. If both nodes are equipped with multiple antennas, the wireless channel between the two nodes can be modeled as a MIMO channel. Considering independently identically distributed (i.i.d.) white Gaussian noise at the receive antennas of the destination node, the capacity of this channel has been well studied [Fos96, Tel99]. According to (1.1), the DoF of this channel is equal to the rank of the channel matrix, and it can be achieved via water-filling power allocation. Besides, equally allocating the transmit power to all the transmit antennas is also able to achieve the DoF of the channel. In fact, it is well-known that both power allocations, i.e., the water-filling power allocation and the equal power allocation, asymptotically yield the same performance in the high-SNR regime. In other words, the DoF of a MIMO channel can be achieved both with and without the channel state information (CSI) at the source node.

Although the concept of DoF can be used to characterize the high-SNR performance in point-to-point scenarios, it is more often used in multiuser interference networks. For instance, in a network consisting of a single source node and multiple destination nodes, the wireless channel between them can be modeled as a broadcast channel (BC). If each source and destination node is equipped with a single antenna and i.i.d. white Gaussian noise is considered at the receive antennas of the destination nodes, the channel is referred to as a single-input-single-output (SISO) Gaussian BC or simply a Gaussian BC, which belongs to the class of degraded BCs [Cov72]. The capacity region of this channel is known and can be achieved via superposition coding [Cov72, GK11]. Substituting the sum capacity of this channel in (1.1) yields that the sum DoF of this channel is one. That is to say, for a single channel use, only one data symbol can be transmitted through the channel without interference. In the sense of achieving the sum DoF of a Gaussian BC, simple orthogonal channel access schemes such as time-sharing are optimal as well. Furthermore, the capacity regions of the MIMO Gaussian BC [YC04, VT03, VJG03] and the MIMO Gaussian multiple access channel (MAC) [CV93, VJG03] are also known, as well as the DoFs of these channels.

Consider a network consisting of multiple source-destination node pairs. The corresponding wireless channel is referred to as an interference channel (IC) [Ahl74, Car78]. In general, the capacity regions of ICs are yet unknown [GEK+11]. However, inner and outer bounds of the capacity regions of ICs have been extensively studied in the past decades [GK11]. It has been shown that some simple transmission schemes, e.g., time division, treating interference as noise, and simultaneous decoding, are able to achieve the capacity of IC in some special cases. For instance, an IC consisting of only two source-destination node pairs with a single antenna at each node and i.i.d. white Gaussian noise at the receive antennas of the destination nodes, which is also known as a two-user SISO Gaussian IC, has been investigated by many researchers. In such a channel, treating interference as noise is optimal for the sum-capacity if the interference is weak [SKC09, AV09, MK09], and simultaneous decoding is optimal if the interference is "very strong" [Car75] or "strong" [Sat78, HK81]. Besides, the Han-Kobayashi coding scheme proposed in [HK81] generalizes the aforementioned simple schemes. The corresponding Han-Kobayashi inner bound is the best known bound on the capacity region of the discrete memoryless IC, and it is tight for all the ICs with known capacity regions [GK11]. It has also been proven that the Han-Kobayashi inner bound differs by no more than half a bit per rate component from the capacity region in a two-user Gaussian IC [TT07, ETW08]. However, there are almost no results on the capacity regions of the ICs with more than two source-destination node pairs except for some special cases, e.g., the deterministic IC where the channel is noiseless [GJ11, BG11] and the many-to-one and one-to-many ICs [BPT10].

In recent years, high-SNR performances in ICs have attracted a lot of attention. Despite that the capacity regions of general ICs remain unknown, the DoFs of a number of ICs have been found. In [HMN05] and [JF07], the sum DoFs of the two-user SISO and MIMO ICs have been found, respectively. Besides, the authors of [HMN05] have also shown that the sum DoF of the K-user SISO IC is at most $K/2$, i.e., at most $1/2$ DoF per user is achievable. The achievability of this DoF bound has been shown in [CJ08] using IA. Following the work of [CJ08], the sum DoF and the DoF region of the K-user MIMO IC have been found in [GJ08] and [KKEKÑ16], respectively. Moreover, by using IA, the achievable DoFs in the K-user MIMO IC for a single channel use have been addressed in [BCT11] and in [YGJK10].

Besides the ICs, the DoFs of various other channels derived from multiuser interference networks have also been investigated. The X channel is a model for networks where every source node transmits an individual message to every destination node. The DoF of the two-user MIMO X channel has been studied in [JS08, MAMK08]. The MIMO X channel with more than two source and destination nodes is considered in [CJ09b], where an outer bound of the DoF region is described and an achievable sum DoF has been found.

Relay channels contain additional relays, besides the source and destination nodes. The DoFs of two kinds of relay channels, one of which includes a single source-destination node pair and the other includes multiple source-destination node pairs, have been investigated in [BNOP06] and in [MBN05], respectively. In [CJ09a], a relay channel where there is a message from every node to every other node, i.e., essentially an X channel with relays, is considered. The DoF of this channel has been found. Comparing the DoF of these relay channels with the DoF of the ICs and the X channels, it is concluded in [CJ09a] that the deployment of additional relays does not increase the DoFs of wireless channels.

1.2.2. Concept of interference alignment

Since IA is able to achieve the DoFs of many multiuser interference networks as introduced in Subsection 1.2.1, it is a promising technique for both theoretical research and future applications of wireless radio communications. The basic idea of IA can be shortly summarized as nulling interferences through cooperatively designed filters. Specifically, IA sacrifices half of the dimensions of the signal space at each destination node for all interferences, such that the interferences can be nullified using a zero-forcing (ZF) receive filter whereas a non-zero interference-free component of every useful signal can be recovered. Therefore, IA can also be considered as cooperative ZF.

The concept of IA can be illustrated using the following example, which has been introduced in [GCJ08]. Consider a network consisting of three source-destination node pairs with two antennas being equipped at each node, as shown in Figure 1.1. The corresponding wireless channel is a three-user MIMO IC. Note that the signals being transmitted via the two antennas of each source node span a two-dimensional transmit signal space at the source node. Similarly, the signals being received via the receive antennas span a two-dimensional receive signal space at each destination node. Suppose each source node wants to transmit a single data symbol intended for the corresponding destination node. Each source node pre-codes its data symbol using a linear transmit filter, which specifies a one-dimensional subspace of the transmit signal space at the source node. The pre-coded signals will then be manipulated, i.e., be scaled and rotated, by the channel and superpose at the destination nodes causing interferences. In [GCJ08], it has been shown that by using cooperatively designed transmit filters at the source nodes, the two interferences at each destination node can be aligned in a one-dimensional subspace of the two-dimensional receive signal space, which is referred to as the interference subspace. Furthermore, the useful signal for each destination node almost surely has a non-zero component in the orthogonal complement of the interference subspace. This interference-free component of the useful signal can be recovered by a ZF receive filter. Employing this IA scheme, in total three data symbols can be transmitted through the channel without interferences. Thus, a sum DoF of three, or 1/2 DoF per user per antenna, is achieved with a single channel use. This is also the DoF of the channel. In contrast to this IA scheme, if conventional interference management approaches such as TDMA

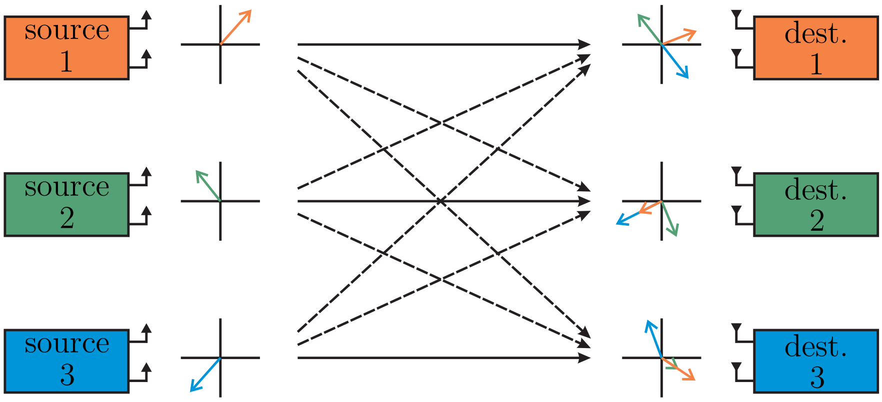


Figure 1.1.: IA in a three-user MIMO IC with two antennas at each source and destination node [GCJ08]. By using cooperatively designed linear transmit filters at the source nodes, the two interferences at every destination node can be aligned in a one-dimensional interference subspace of the two-dimensional receive signal space. Thus, a none-zero interference-free component of the useful signal for each destination node can be recovered from the orthogonal complement of the interference subspace by a ZF receive filter.

or FDMA are employed, only two data symbols can be transmitted through the channel for a single channel use, which corresponds to a DoF of only 1/3 per user per antenna.

1.2.3. Interference alignment with full channel knowledge

State-of-the-art IA schemes can be classified into two categories based on their requirements for the channel knowledge. IA schemes which require full channel knowledge will be briefly reviewed in this section, and the others, which do not require full channel knowledge, will be reviewed in Section 1.2.4. In this thesis, full channel knowledge has the following three implications. Firstly, the global CSI must be known, i.e., the CSI of every channel in the entire network has to be known. In order to design the filters for IA, the global CSI shall be available either at a central unit or at every node in the network. Secondly, the CSI must be perfect, i.e., the error in both the channel estimation and the feedback has to be zero. Thirdly, the instantaneous CSI must be known, i.e., the knowledge of the current channel has to be available before the transmission.

IA can be achieved using time extensions [JS08, CJ08, NGJV12]. The first explicit IA scheme was presented in [JS08] for the two-user MIMO X channel, which uses multiple time extensions and is able to achieve the DoF of the channel. Following the same idea, another IA scheme using time extensions was proposed in [CJ08] to show that the DoF of the K-user IC is achievable. The authors of [CJ08] first considered a time-varying K-user SISO IC, where the proposed IA scheme is employed to transmit $Kd + 1$ data symbols, i.e., each source node transmits d data symbols and one of them transmits one additional data symbol. In order to completely nullify the interferences, $2d + 1$ time slots are required. When the transmit filters at the source nodes are properly designed, the interferences will fall into an interference subspace of the $(2d + 1)$-dimensional receive signal space at every destination node, and the useful signals can be recovered by ZF receive filters. As the number of data symbols d grows, the achievable sum DoF of this IA scheme asymptotically approaches $K/2$, i.e., the sum DoF of the K-user SISO IC. In the same paper, this scheme was also extended to show the achievability of the sum DoF of the time-varying K-user MIMO IC with the same number of antennas being equipped at all the source and destination nodes. Other IA schemes using time extensions, e.g., ergodic IA [NGJV12] which requires that the channel coefficients vary in an ergodic fashion, also need infinite time extensions to achieve the sum DoF of the K-user SISO IC.

IA can also be achieved using multiple antennas instead of time extensions [GCJ08, GCJ11, TGR09]. These IA schemes are usually referred to as MIMO IA. Considering a constant K-user MIMO IC, the IA scheme using time extensions in [CJ08] is no longer applicable. In order to achieve IA in such a channel, the authors of [GCJ08] and [GCJ11] proposed an iterative algorithm, i.e., the interference leakage minimization (ILM) algorithm, which aims at minimizing the total residual interference power. For some special MIMO ICs where the number of node pairs, the numbers of antennas at the nodes, and the desired DoF to be achieved are carefully chosen, a closed-form solution of MIMO IA has been found in [TGR09]. For instance, the example with three node pairs, two antennas at each node, and each source node transmitting a single data symbol, which has been introduced in Section 1.2.2, is such a special case. Different from IA using time extensions, MIMO IA cannot achieve 1/2 DoF per user per antenna except for the three-user MIMO IC. The achievable DoF of MIMO IA in an IC strongly depends on the number of node pairs, the numbers of antennas at the nodes [YGJK10, BCT11, RLL12], and even the network topology [GG11]. The feasibility conditions for MIMO IA in fully connected ICs have been addressed in [YGJK10, BCT11, RLL12] exploiting results from algebraic geometry. For instance, in a constant K-user MIMO IC with N antennas at all the source and destination nodes, the achievable DoF per user is upper bounded by $2N/(K+1)$. In other words, in order to achieve a desired DoF, the number N of antennas at each node must proportionally increase with the network size K. In contrast to fully connected ICs, the number of required antennas for MIMO IA in partially connected ICs does not increase with the network size if each node is only connected to a limited number of other nodes [GG11]. However, there are few results on IA in partially connected ICs beside [GG11]. Following the same idea of MIMO IA in ICs, algorithms and feasibility conditions for MIMO IA in cellular networks have been investigated in many publications as well [ST08, SHT11, SNJC12, LY13, SY15].

In contrast to IA using time extensions and MIMO IA, relay-aided IA [GCJ08, NMK10, NLL10, ASW11, GASK+13] is able to achieve the DoFs of relay channels with only few time extensions, and the numbers of antennas at the source and destination nodes do not need to increase with the network size. For relay-aided IA, amplify-and-forward (AF) relays are considered. The relays are usually assumed to be working in the half-duplex mode. Consequently, at least two time slots are required to complete the transmission. The AF relays linearly process the signals received in the previous time slot and then forward them in the next time slot. By carefully choosing the relay processing filters, the virtual channels between the source and destination nodes can be manipulated in order to help aligning interferences at the destination nodes. It has been shown in [GCJ08, NMK10] that relay-aided IA achieves 1/2 DoF per user in a relay IC with only a single antenna at each source and destination node. Relay-aided IA for the special case where the relays are deployed close to the destination nodes has been considered in [NLL10]. The authors of [ASW11] showed that the relay-aided IA problems can be solved linearly and proposed a closed-form solution. Specially, the concept of relay-aided IA can be utilized for bi-directional communications employing a two-way relay [GASK+13]. Relay-aided IA for unidirectional communications will be further discussed in detail in this thesis.

For the IA schemes introduced above, i.e., IA using time extensions, MIMO IA, and relay-aided IA, the data symbols are assumed to be i.i.d. circularly symmetric complex Gaussian symbols. In fact, the concept of IA has also been extended for other distributions of data symbols, e.g., asymmetric complex symbols [CJW10] and coded symbols using multilevel or lattice codes [CJS09, EO09]. In [CJW10], the authors proposed an IA scheme exploiting the channel phase, which is known as phase alignment, for constant SISO ICs. This IA scheme is shown to achieve a sum DoF of at least 1.2, which outperforms the 1 DoF that can be achieved assuming circularly symmetric Gaussian symbols in these channels. In [CJS09] and [EO09], IA schemes based on multilevel codes and lattice codes have been considered, respectively. These IA schemes are also able to achieve more than 1 DoF for some special SISO ICs.

1.2.4. Interference alignment without full channel knowledge

The IA schemes introduced in the previous subsection aim at maximizing the achievable DoFs in the corresponding channels. To this end, all the interferences must be perfectly aligned and nullified at every destination node. Therefore, all of these IA schemes are based on an idealized assumption that the channel is fully known. However, despite channel estimation errors[1], the acquisition of full channel knowledge is still very challenging or even impossible in practice. For instance, in pilot-based systems, the CSI shall be estimated at the receivers with the help of known pilot signals and then fed back to either a central unit or every other node in the entire network. On the one hand, the length of pilot signals depends on the total number of transmit antennas. Therefore, in large networks consisting of many transmit antennas or in channels which change fast, a large portion of time, as well as energy, shall be used for channel estimation. In the extreme case where the required length of pilot signals exceeds the coherence time of the channel, meaningful channel estimation is even impossible. On the other hand, the quality of the feedback of channel estimates depends on the feedback links. In practice, the CSI feedback is often imperfect due to quantization errors, and delayed. For these reason, various IA schemes which do not require full channel knowledge have also been investigated.

Some of these IA schemes focus on dealing with imperfect or delayed CSI feedback. The influence of imperfect CSI feedbacks on the achievable DoFs has been well investigated in [Jin06, VV10, KV10] and the references therein. It has been found that if the quality of CSI feedbacks improves sufficiently fast with the logarithm of the SNR, the DoF that could be achieved with perfect CSI feedback is still achievable with imperfect CSI feedback. As compared to imperfect CSI feedback, delayed CSI feedback is more challenging in general. In the extreme case, the CSI feedback may be even completely outdated, i.e., be independent

[1] It has been shown in [HH03] that the mean square error (MSE) of the channel estimate of a block fading MIMO channel is asymptotically inversely proportional to the transmit power in the high-SNR regime. Consequently, if the transmit power is sufficiently large, the channel estimation error can be considered as negligibly small in the sense that the power of the residual interference due to channel estimation error is comparable with the noise power.

of the current channel state. In [MAT12], a transmission scheme based on the concept of IA has been proposed to show the surprising result that delayed CSI feedback, even if it is completely outdated, always improves the achievable DoF in a MISO BC. Extended from this work, an IA scheme for a two-user MIMO IC with outdated CSI has been proposed in [VV12]. Furthermore, IA schemes with outdated CSI for SISO and MIMO ICs consisting of more than two users have been studied in [MJS12, MC12, AGK13] and [TAV14, TAV16], respectively.

Other IA schemes without full channel knowledge focus on reducing the amount of CSI that needs to be estimated and/or fed back. Two representative examples of these IA schemes are topological IA [Jaf14] and blind IA [GWJ11]. Topological IA was first proposed for partially connected SISO ICs in [Jaf14]. Instead of the perfect knowledge of channel coefficients, only the topology of the channel is assumed to be known, i.e., only a "1-bit CSI" representing the presence of each link is available. The achievable DoF with this type of channel knowledge is studied. The author of [Jaf14] has shown that this problem can be translated to an index coding problem, which has been introduced in [BK98]. The concept of topological IA has also been extended to ICs with fast fading [NA15, GNA15], ICs with transmitter cooperation [YG15], and MIMO ICs [SJ14]. Unfortunately, the index coding problem itself is an open problem and there only exist valid solutions for some special cases. It has been shown in [SZL16] that evaluating the achievable DoFs of topological IA is non-deterministic polynomial time (NP) hard.

Blind IA does not require any knowledge of the channel coefficients at the transmitters. It can be simply achieved in a class of heterogeneous block fading channels [Jaf12], where certain users experience smaller coherence time than the others. Interestingly, it has been shown in [GWJ11] that blind IA can also be achieved exploiting reconfigurable antennas. A reconfigurable antenna is able to dynamically adjust its radiation patterns, which can be conceptually modeled as antenna selection, so that the receiver is capable of switching its receiving mode among several preset modes. In [GWJ11], blind IA is proposed for a MISO BC where each receiver is equipped with a single reconfigurable antenna. This IA scheme is then extended to MIMO BCs in [WGJ10, YJK17]. Blind IA using reconfigurable antennas in ICs has been studied in [LZL14, Wan14]. However, a supersymbol shall be constructed in order to achieve blind IA. Throughout the transmission of a supersymbol, the channel must remain constant, which requires that the coherence time of the channel has to be sufficiently large.

1.3. Objectives and outline of this thesis

This thesis focuses on the following four closely related aspects of relay-aided IA. These aspects are not only fundamental research topics but also of practical relevance, which make the problem of relay-aided IA challenging and interesting.

- **IA solutions:** The relay-aided IA schemes that will be proposed in this thesis are based on a two-hop transmission scheme employing half-duplex AF relays, which can be applied in a wide range of relay networks. In order to achieve relay-aided IA, the transmit filters, the receive filters, and the relay processing filters shall be cooperatively designed such that all the interferences can be perfectly nullified and that a non-zero component of each useful signal can be recovered. Such a set of properly designed filters is referred to as an IA solution. A linearization approach will be proposed in order to find all the relevant IA solutions, from an engineering perspective, in closed-form.

- **Feasibility conditions:** In the context of IA, feasibility conditions determine whether or not IA solutions exist for a given network. If at least one IA solution exists, IA is feasible. The feasibility conditions for the proposed relay-aided IA schemes will be addressed. Towards this end, the characteristics of the sets of IA solutions will be investigated. Furthermore, concepts and tools from graph theory will be employed.

- **Relay-aided IA with partial channel knowledge:** Relay-aided IA without full channel knowledge will also be studied in this thesis. Specifically, a relay-aided IA scheme with partial channel knowledge will be proposed for a class of partially connected relay networks as a case study. The goal is to reduce the required CSI for relay-aided IA while not affecting the achievable DoF.

- **Performance optimizations:** IA only aims at nullifying interferences and, therefore, is suboptimal in noise-limited cases. In this thesis, the achievable sum rates of the proposed relay-aided IA schemes will be investigated and optimized as well. In order to perform a fair comparison with other interference management approaches, sum power constraints, which are essentially energy constraints, will be considered.

The four aspects of relay-aided IA mentioned above will be discussed via a case study of the following three types of relay networks.

- **Fully connected ad-hoc networks:** Ad-hoc networks have various applications in practice, such as device-to-device communications, home networks, sensor networks, and distributed control networks [Gol05]. In this thesis, the considered ad-hoc networks consist of multiple source-destination node pairs and relays, where every source node communicates with the corresponding destination node, and the relays shall assist in the unidirectional communications employing relay-aided IA.

• **Fully connected cellular networks:** Mobile cellular networks are the most successful commercial wireless radio communication networks worldwide. They have been deployed over most of the inhabited area with 95% of the global population being covered [ITU16]. Future mobile cellular networks are expected to incorporate new architectures such as heterogeneous networks and small cells to improve the coverage and data rate. Due to the increased density of mobile devices, interferences, especially the inter-cell interferences, severely influence the performance. In this thesis, cellular networks with relays being deployed among the cells for relay-aided IA will be considered as well.

• **Partially connected ad-hoc networks:** In practical mobile radio communications networks, especially in large networks with many nodes, the channel conditions between different nodes may vary widely due to large-scale propagation effects such as path loss and shadowing [Gol05] or due to the radiation patterns of antennas. Therefore, the interferences from a few interferers, e.g., the nearby interferers or the interferers with a line-of-sight link, could be significantly stronger than the others and dominate the total interference power at a receiver. Furthermore, the interferences from some interferers, e.g., the faraway interferers, could be comparable to or even below the noise level at the chosen transmit power. For these reasons, some comparatively weak interferences can be ignored. In other words, the network can be considered as a partially connected network. In this thesis, relay-aided IA in a class of partially connected ad-hoc networks will be investigated too.

The topologies and configurations of these three types of relay networks will be introduced in Chapter 2 along with other assumptions. In this thesis, the fully connected ad-hoc networks are considered as the basic network. The fully connected cellular networks are considered as an extension of the fully connected ad-hoc networks in the sense that the source and destination node are replaced by base stations (BSs) and mobile stations (MSs). Partially connected ad-hoc networks are extension of fully connected ad-hoc networks by taking partial connectivity into consideration.

In Chapter 3, relay-aided IA in fully connected ad-hoc networks will be investigated. A relay-aided IA scheme will be proposed. Firstly, the conditions for achieving relay-aided IA will be formulated. Then, a linearization approach will be proposed, which can be used to find the IA solutions in closed-form. The proposed linearization approach applies not only to fully connected ad-hoc networks but also to the other two types of relay networks considered in this thesis. Thirdly, the feasibility conditions for relay-aided IA in fully connected ad-hoc networks will be studied. Finally, the achievable sum rate will be optimized either under a total sum power constraint or under individual sum power constraints. Part of the contents in this chapter has been published in the author's preliminary work [LASG+13a].

In Chapter 4, relay-aided IA in full connected cellular networks will be investigated. The problem of achieving relay-aided IA in the entire network will be decomposed into two subproblems, i.e., inter-cell interference nulling and intra-cell interference management. The former subproblem can be solved following the idea of relay-aided IA in fully connected ad-hoc networks. The latter subproblem can be solved employing beamforming. In this thesis, two widely used linear beamforming techniques, i.e., ZF and MMSE, will be considered for intra-cell interference management. Furthermore, the uplink-downlink duality of relay-aided IA in the considered fully connected cellular networks will be studied exploiting the reciprocity of radio channels. The duality of both inter-cell interference nulling and intra-cell interference management will be investigated. Part of the contents in this chapter has been published in the author's preliminary work [LASG+15].

Partially connected ad-hoc networks will be considered in Chapter 5, where the entire network consists of multiple subnetworks. Each subnetwork is a small fully connected ad-hoc network. However, different subnetworks are only partially connected to each other. The discussions in this chapter focus on the feasibility conditions. The case with full channel knowledge will be considered first as a benchmark. A relay-aided IA scheme with partial channel knowledge will be proposed afterwards. Using the proposed scheme, the IA solution for the entire network can be found by finding a proper solution for each individual subnetwork sequentially. A parallelization approach based on the topology of the network will be proposed in order to speed up this process. The performances achieved by relay-aided IA with both full channel knowledge and partial channel knowledge will be compared. Part of the contents in this chapter has been published in the author's preliminary work [LASG+13b, LASG+14, LPKW15].

Finally, Chapter 6 concludes this thesis.

Chapter 2.

System model

2.1. Basic network: Fully connected ad-hoc network

The basic networks being considered in this thesis are fully connected ad-hoc networks. The considered ad-hoc networks consist of K source-destination node pairs and Q AF relays, as depicted in Figure 2.1. Each source node intends to communicate with the corresponding destination node, and the relays shall assist in the unidirectional transmission employing relay-aided IA. The relays are assumed to operate in the half-duplex mode, i.e., they cannot receive and transmit simultaneously. The source nodes, the destination nodes, and the relays are generally assumed to be equipped with multiple antennas. This thesis focuses on the symmetric case where all the source and destination nodes have the same number of antennas. Let N antennas be equipped at each source node and at each destination node for transmitting and receiving, respectively. The q-th relay is assumed to have M_q antennas for both transmitting and receiving[1].

The discrete-time narrowband channel model [Gol05] will be considered. This model can be realized, for instance, by considering a single subcarrier in practical orthogonal frequency division multiplexing (OFDM) systems. The channels are assumed to be time invariant throughout the transmission. Let the channel from the j-th source node to the k-th destination node be denoted by the $N \times N$ matrix $\mathbf{H}_{\mathrm{DS}}^{(k,j)}$. Let the channel from the j-th source node to the q-th relay be denoted by the $M_q \times N$ matrix $\mathbf{H}_{\mathrm{RS}}^{(q,j)}$. Let the channel from the q-th relay to the k-th destination node be denoted by the $N \times M_q$ matrix $\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}$. The channel matrices are also shown in Figure 2.1. Considering multi-path propagations and rich scattering environments, the entries of all the channel matrices are assumed to

[1] Considering the differences in the radio-frequency chains being used for transmitting and receiving, M_q pairs of radio-frequency chains are actually available at the q-th relay.

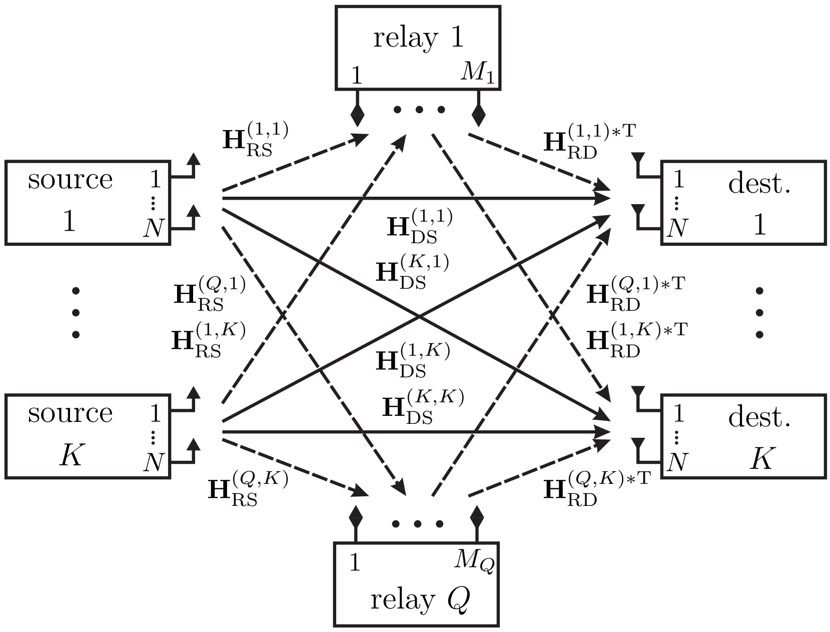


Figure 2.1.: The considered fully connected ad-hoc networks consist of K source-destination node pairs and Q AF relays which assist in the unidirectional data transmission from the source nodes to the corresponding destination nodes.

be independently drawn from a continuous distribution over the complex field $\mathbb{C}$. This implies that the considered ad-hoc networks are fully connected since all the channel matrices are almost surely non-zero. Moreover, full channel knowledge is assumed, i.e., the global, perfect, and instantaneous CSI is assumed to be known by a central unit or by every node in the network. Furthermore, additive noise is considered both at the destination nodes and at the relays. For the performance analysis and numerical simulations in this thesis, Rayleigh channels and i.i.d. white Gaussian noise will be considered. However, it is worthwhile to emphasize that the relay-aided IA schemes to be discussed in this thesis are not restricted to Rayleigh channels and Gaussian noise.

Suppose each source node intends to transmit D data symbols through the network to the corresponding destination node. A synchronized two-hop transmission scheme is considered, where the synchronization can be easily realized in OFDM systems because of the long symbol durations and the guard intervals [GMRW03]. The two-hop transmission scheme uses two time slots. In the first time slot, each source node linearly pre-codes its data symbols and transmits the pre-coded signals towards all the relays and destination nodes. The signals being received by each relay in the first time slot, including the relay noise, will be linearly processed and, then, forwarded towards all the destination nodes in a second time slot. Meanwhile, each source node pre-codes the same data symbols and transmits again towards all the destination nodes in the second time slot. However, the data symbols can be pre-coded differently as compared to the first time slot. The signals being received by the destination nodes in the two time slots are combined using linear receive filters.

Let the data symbols to be transmitted by the j-th source node be denoted by the vector $\mathbf{d}^{(j)} \in \mathbb{C}^D$. Let the $2N \times D$ matrix $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$ be referred to as the transmit filter of the j-th source node, where $\mathbf{V}_1^{(j)}$ and $\mathbf{V}_2^{(j)}$ are the $N \times D$ pre-coding matrices being used by the j-th source node in the first and in the second time slot, respectively. The signals being received by the k-th destination node in the first time slot can then be denoted by the $N \times 1$ vector

$$\mathbf{e}_1^{(k)} = \sum_{j=1}^{K} \mathbf{H}_{\mathrm{DS}}^{(k,j)} \mathbf{V}_1^{(j)} \mathbf{d}^{(j)} + \mathbf{n}_{\mathrm{D},1}^{(k)}, \tag{2.1}$$

where $\mathbf{n}_{\mathrm{D},1}^{(k)} \in \mathbb{C}^N$ is the noise received by the k-th destination node in the first time slot. Similarly, the signals being received by the q-th relay in the first time slot can be denoted by the $M_q \times 1$ vector

$$\mathbf{e}_{\mathrm{R}}^{(q)} = \sum_{j=1}^{K} \mathbf{H}_{\mathrm{RS}}^{(q,j)} \mathbf{V}_1^{(j)} \mathbf{d}^{(j)} + \mathbf{n}_{\mathrm{R}}^{(q)}, \tag{2.2}$$

where $\mathbf{n}_{\mathrm{R}}^{(q)} \in \mathbb{C}^{M_q}$ is the noise received by the q-th relay. Let the linear processing filter of the q-th relay be represented by the $M_q \times M_q$ matrix $\mathbf{G}^{(q)}$. Thus, $\mathbf{G}^{(q)}\mathbf{e}_{\mathrm{R}}^{(q)}$ represents the signals to be transmitted by the q-th relay in the second time slot. The signals being received by the k-th destination node in the second time slot can therefore be denoted by the $N \times 1$ vector

$$\mathbf{e}_2^{(k)} = \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{e}_{\mathrm{R}}^{(q)} + \sum_{j=1}^{K} \mathbf{H}_{\mathrm{DS}}^{(k,j)} \mathbf{V}_2^{(j)} \mathbf{d}^{(j)} + \mathbf{n}_{\mathrm{D},2}^{(k)}, \tag{2.3}$$

where $\mathbf{n}_{\mathrm{D},2}^{(k)} \in \mathbb{C}^N$ is the noise at the k-th destination node in the second time slot. Let the receive filter, which is used to combine the signals being received by the k-th destination node in both time slots, be denoted by the $2N \times D$ matrix

$\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$, where $\mathbf{U}_1^{(k)}$ and $\mathbf{U}_2^{(k)}$ are the $N \times D$ combining matrices. Then the output data symbols of the k-th destination node are given by

$$\begin{aligned}\tilde{\mathbf{d}}^{(k)} &= \begin{bmatrix}\mathbf{U}_1^{(k)*\mathrm{T}} & \mathbf{U}_2^{(k)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{e}_1^{(k)}\\ \mathbf{e}_2^{(k)}\end{bmatrix}\\ &= \sum_{j=1}^{K}\begin{bmatrix}\mathbf{U}_1^{(k)*\mathrm{T}} & \mathbf{U}_2^{(k)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{DS}}^{(k,j)} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)} & \mathbf{H}_{\mathrm{DS}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}\mathbf{d}^{(j)} + \mathbf{n}_{\mathrm{eff}}^{(k)}\\ &= \mathbf{H}_{\mathrm{eff}}^{(k,k)}\mathbf{d}^{(k)} + \sum_{j\neq k}\mathbf{H}_{\mathrm{eff}}^{(k,j)}\mathbf{d}^{(j)} + \mathbf{n}_{\mathrm{eff}}^{(k)},\end{aligned} \tag{2.4}$$

where the $D \times D$ channel matrix $\mathbf{H}_{\mathrm{eff}}^{(k,j)}$ is given by

$$\mathbf{H}_{\mathrm{eff}}^{(k,j)} = \begin{bmatrix}\mathbf{U}_1^{(k)*\mathrm{T}} & \mathbf{U}_2^{(k)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{DS}}^{(k,j)} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)} & \mathbf{H}_{\mathrm{DS}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}, \tag{2.5}$$

and the noise being given by

$$\begin{aligned}\mathbf{n}_{\mathrm{eff}}^{(k)} &= \begin{bmatrix}\mathbf{U}_1^{(k)*\mathrm{T}} & \mathbf{U}_2^{(k)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{n}_{\mathrm{D},1}^{(k)}\\ \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{n}_{\mathrm{R}}^{(q)} + \mathbf{n}_{\mathrm{D},2}^{(k)}\end{bmatrix}\\ &= \mathbf{U}_1^{(k)*\mathrm{T}}\mathbf{n}_{\mathrm{D},1}^{(k)} + \mathbf{U}_2^{(k)*\mathrm{T}}\mathbf{n}_{\mathrm{D},2}^{(k)} + \mathbf{U}_2^{(k)*\mathrm{T}}\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{n}_{\mathrm{R}}^{(q)}\end{aligned} \tag{2.6}$$

includes the noise at the k-th destination node in both time slots and the noise being forwarded by the relays. In (2.4), the term $\mathbf{H}_{\mathrm{eff}}^{(k,k)}\mathbf{d}^{(k)}$ represents the useful signals for the k-th destination node, and the term $\sum_{j\neq k}\mathbf{H}_{\mathrm{eff}}^{(k,j)}\mathbf{d}^{(j)}$ represents the interferences. Thus, by applying the two-hop transmission scheme, the considered ad-hoc network is converted into an effective MIMO IC with the effective channel matrices $\mathbf{H}_{\mathrm{eff}}^{(k,j)}$ and the effective noises $\mathbf{n}_{\mathrm{eff}}^{(k)}$. Moreover, the effective channel matrices $\mathbf{H}_{\mathrm{eff}}^{(k,j)}$ as well as the effective noises $\mathbf{n}_{\mathrm{eff}}^{(k)}$ can be manipulated by adapting the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$, the receive filters $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$, and the relay processing filters $\mathbf{G}^{(q)}$.

Although the capacity of the effective MIMO IC as given in (2.4) is unknown, the achievable sum rate can still be computed for any specific transmit filters, receive filters, and relay processing filters by treating interferences as noise. Assume that the transmit data symbols $\mathbf{d}^{(j)}$ of each source node are i.i.d. circularly symmetric Gaussian symbols with zero mean and unit variance, i.e.,

$$\mathrm{E}\left\{\mathbf{d}^{(j)}\mathbf{d}^{(j)*\mathrm{T}}\right\} = \mathbf{I}_D, \quad \forall j \in \{1,\ldots,K\}, \tag{2.7}$$

holds, with $\mathbf{I}_D$ denoting the $D \times D$ identity matrix. Thus, $\mathrm{tr}\left(\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}}\right)$ and $\mathrm{tr}\left(\mathbf{V}_2^{(j)}\mathbf{V}_2^{(j)*\mathrm{T}}\right)$ are the average transmit powers of the j-th source node in the first and in the second time slot, respectively. Since the average transmit powers of a source node could be different in the two time slots, which is likely to be the case in order to achieve relay-aided IA as will be seen in the remaining part of this thesis, the sum transmit power

$$P_{\mathrm{S}}^{(j)} = \mathrm{tr}\left(\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}} + \mathbf{V}_2^{(j)}\mathbf{V}_2^{(j)*\mathrm{T}}\right) \tag{2.8}$$

will be considered for each source node. The sum transmit power $P_{\mathrm{S}}^{(j)}$ can be interpreted as the transmit energy consumed by the j-th source node throughout the transmission divided by the duration of a single time slot. The average transmit power of the q-th relay can be denoted by

$$P_{\mathrm{R}}^{(q)} = \mathrm{tr}\left(\mathrm{E}\left\{\mathbf{G}^{(q)}\mathbf{e}_{\mathrm{R}}^{(q)}\mathbf{e}_{\mathrm{R}}^{(q)*\mathrm{T}}\mathbf{G}^{(q)*\mathrm{T}}\right\}\right), \tag{2.9}$$

where $\mathbf{e}_{\mathrm{R}}^{(q)}$ is given by (2.2). Furthermore, define the total sum transmit power as

$$P_{\mathrm{tot}} = \sum_{j=1}^{K} P_{\mathrm{S}}^{(j)} + \sum_{q=1}^{Q} P_{\mathrm{R}}^{(q)}, \tag{2.10}$$

which can be interpreted as the total transmit energy consumed by all the source nodes and relays throughout the transmission divided by the duration of a single time slot. The total sum transmit power P_{tot} will be convenient for performance comparison between the proposed relay-aided IA schemes and other interference management approaches. Moreover, the noises $\mathbf{n}_{\mathrm{D},1}^{(k)}$, $\mathbf{n}_{\mathrm{D},2}^{(k)}$, and $\mathbf{n}_{\mathrm{R}}^{(k)}$ are assumed to be independent circularly symmetric white Gaussian noise with zero mean and covariance matrices $\sigma_{\mathrm{D}}^2\mathbf{I}_N$, $\sigma_{\mathrm{D}}^2\mathbf{I}_N$, and $\sigma_{\mathrm{R}}^2\mathbf{I}_{M_q}$, respectively. Let $\mathbf{S}_{\mathrm{I+N}}^{(k)}$ denote the covariance matrix of interference plus noise in the output data symbols $\widetilde{\mathbf{d}}^{(k)}$ of the k-th destination node. Then, the achievable sum rate R_{sum} can be computed as

$$R_{\mathrm{sum}} = \frac{1}{2}\sum_{k=1}^{K} \log_2\left(\det\left(\mathbf{I}_N + \mathbf{H}_{\mathrm{eff}}^{(k,k)*\mathrm{T}}\mathbf{S}_{\mathrm{I+N}}^{(k)-1}\mathbf{H}_{\mathrm{eff}}^{(k,k)}\right)\right) \tag{2.11}$$

and is measured in bits per channel use. In (2.11), the factor 1/2 results from the use of two time slots.

Finally, two more assumptions will be made for the considered fully connected ad-hoc networks. First, it is always assumed that $D = N$ holds. This can be argued as follows. The fully connected ad-hoc networks can be considered as a K-user MIMO IC with some additional relays. On the one hand, it has been introduced in Section 1.2.1 that the sum DoF of a K-user MIMO IC with N antennas at each source and destination node is $KN/2$, if the channel matrices are of full rank. On the other hand, the deployment of additional relays cannot increase the achievable DoF [CJ09a]. Therefore, the sum DoF of the considered fully connected ad-hoc networks is almost surely $KN/2$, if the channel coefficients are independently drawn from a continuous distribution. However, if the two-hop transmission scheme is employed, it can be seen from (2.11) that the maximum achievable sum DoF is $KD/2$, providing that the D data symbols intended for every destination node can be successfully transmitted without interferences. Hence, the number D of data symbols must be smaller than or equal to the number N of antennas at a source or destination node, otherwise interference-free transmission is impossible. If $D = N$ holds, the sum DoF of the considered ad-hoc network, i.e., a sum DoF of $KN/2$ in the almost sure sense, can be achieved employing the two-hop transmission scheme. Second, it is always assumed that the network has $K \geq 3$ source-destination node pairs. If the network has only two source-destination node pairs, the sum DoF of the network can be easily achieved using conventional interference management approaches, e.g., TDMA, without relays. This case is excluded from this thesis.

2.2. Extended network: Fully connected cellular network

In this thesis, fully connected cellular networks with relays being deployed among the cells for relay-aided IA will be considered as well. The considered cellular networks consist of K cells, where a single BS being equipped with N antennas communicates with N single-antenna MSs in each cell. Q half-duplex AF relays are deployed, where q-th relay is assumed to have M_q antennas. In this thesis, both the uplink and the downlink transmissions will be considered. Such a cellular network can be considered as an extension of the ad-hoc networks introduced in Section 2.1 in the sense that the source and destination nodes are replaced by BSs and MSs. For instance in the uplink, the BSs play the role of destination nodes, and the MSs of each cell play the role of a source node. However, the MSs cannot perform joint signal processing. Resulting from this, both inter- and intra-cell interferences have to be taken into consideration. In the following, the system model for the uplink transmission will be introduced first. The downlink transmission will be briefly introduced afterwards.

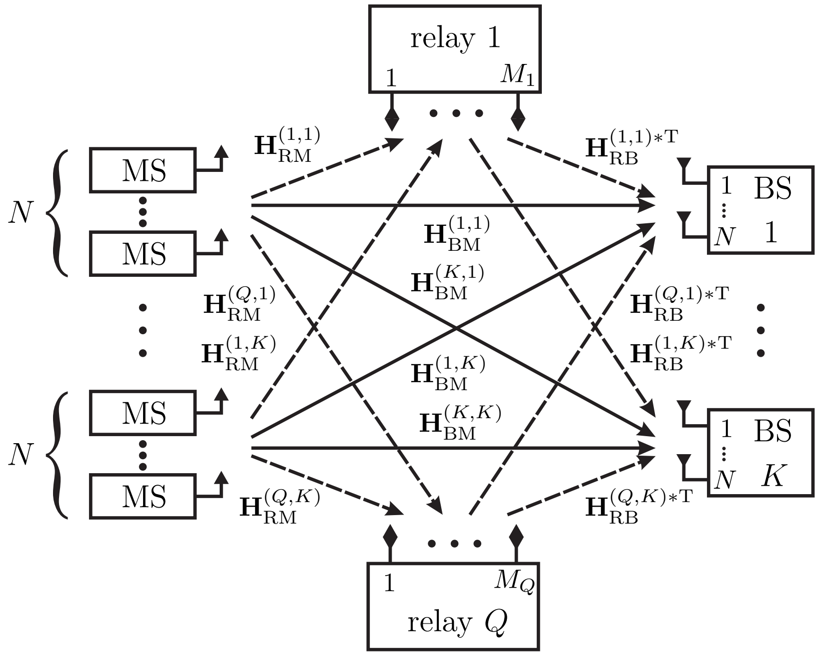


Figure 2.2.: In the uplink of the considered cellular network, the single-antenna MSs transmit to the corresponding BSs using the two-hop transmission scheme with the help of Q half-duplex AF relays.

Similar to the ad-hoc networks, a discrete-time narrowband time-invariant channel model will be considered for the cellular networks. In the uplink, the MSs transmit data symbols towards the BSs through the channel, as depicted in Figure 2.2. The channel matrices for the uplink transmission are also shown in Figure 2.2 and they are defined as follows. Let the channel from the MSs of the j-th cell to the k-th BS be denoted by the $N \times N$ matrix $\mathbf{H}_{\text{BM}}^{(k,j)}$. Let the channel from the MSs of the j-th cell to the q-th relay be denoted by the $M_q \times N$ matrix $\mathbf{H}_{\text{RM}}^{(q,j)}$. Let the channel from the q-th relay to the k-th BS be denoted by the $N \times M_q$ matrix $\mathbf{H}_{\text{RB}}^{(q,k)*\text{T}}$. The entries of these channel matrices are assumed to be independently drawn from a continuous distribution over $\mathbb{C}$, which implies that the cellular network is fully connected. Full channel knowledge is assumed, i.e., the global, perfect, and instantaneous CSI is assumed to be known by a central unit or by every node in the network. Furthermore, additive noise is considered at both the BSs and the relays. For the performance analysis and numerical simulations in this thesis, Rayleigh channels and i.i.d. white Gaussian noise will be considered.

The synchronized two-hop transmission scheme is again employed in the cellular networks. In the uplink, each MS intends to transmit a single data symbol to the BS of the corresponding cell. Let the data symbol to be transmitted by the n-th MS of the j-th cell be denoted by $d_{\mathrm{UL}}^{(j,n)} \in \mathbb{C}$. To keep conformity between the equations in this thesis, the N data symbols to be transmitted by the MSs of the j-th cell are stacked in the $N \times 1$ vector $\mathbf{d}_{\mathrm{UL}}^{(j)}$. Let the transmit filters being used by the MSs of the j-th cell be stacked in the $2N \times N$ matrix $\begin{bmatrix}\mathbf{V}_{\mathrm{UL},1}^{(j)} \\ \mathbf{V}_{\mathrm{UL},2}^{(j)}\end{bmatrix}$. Specially, since the MSs cannot perform joint transmission, both pre-coding matrices $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ and $\mathbf{V}_{\mathrm{UL},2}^{(j)}$ are $N \times N$ diagonal matrices, whose diagonal entries are the pre-coding coefficients of the N MSs of the j-cell in the first and the second time slot, respectively. Let the linear processing filter of the q-th relay be represented by the $M_q \times M_q$ matrix $\mathbf{G}_{\mathrm{UL}}^{(q)}$. Let the receive filter[2], which is used to combine the signals received by the k-th BS, be denoted by the $2N \times N$ matrix $\begin{bmatrix}\mathbf{U}_{\mathrm{UL},1}^{(k)} \\ \mathbf{U}_{\mathrm{UL},2}^{(k)}\end{bmatrix}$. Using the aforementioned notations, the effective channel matrix $\mathbf{H}_{\mathrm{UL,eff}}^{(k,j)}$ from the MSs of the j-th cell to the k-th BS in the uplink reads

$$\mathbf{H}_{\mathrm{UL,eff}}^{(k,j)} = \begin{bmatrix}\mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} & \mathbf{U}_{\mathrm{UL},2}^{(k)*\mathrm{T}}\end{bmatrix} \begin{bmatrix} \mathbf{H}_{\mathrm{BM}}^{(k,j)} & \mathbf{0} \\ \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{H}_{\mathrm{RM}}^{(q,j)} & \mathbf{H}_{\mathrm{BM}}^{(k,j)} \end{bmatrix} \begin{bmatrix}\mathbf{V}_{\mathrm{UL},1}^{(j)} \\ \mathbf{V}_{\mathrm{UL},2}^{(j)}\end{bmatrix}. \tag{2.12}$$

The effective noise at the k-th BS reads

$$\mathbf{n}_{\mathrm{UL,eff}}^{(k)} = \mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \mathbf{n}_{\mathrm{BS},1}^{(k)} + \mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \mathbf{n}_{\mathrm{BS},2}^{(k)} + \mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{n}_{\mathrm{UL,R}}^{(q)}, \tag{2.13}$$

where $\mathbf{n}_{\mathrm{BS},1}^{(k)} \in \mathbb{C}^N$ and $\mathbf{n}_{\mathrm{BS},2}^{(k)} \in \mathbb{C}^N$ are the noises at the k-th BS in the first and in the second time slot, respectively, and $\mathbf{n}_{\mathrm{UL,R}}^{(q)} \in \mathbb{C}^{M_q}$ is the noise at the q-th relay. Then the output data symbols of the k-th BS can be given by

$$\widetilde{\mathbf{d}}_{\mathrm{UL}}^{(k)} = \mathbf{H}_{\mathrm{UL,eff}}^{(k,k)} \mathbf{d}_{\mathrm{UL}}^{(k)} + \sum_{j \neq k} \mathbf{H}_{\mathrm{UL,eff}}^{(k,j)} \mathbf{d}_{\mathrm{UL}}^{(j)} + \mathbf{n}_{\mathrm{UL,eff}}^{(k)}, \tag{2.14}$$

where the first term on the right hand side represents the useful signals for the k-th BS superposed with the intra-cell interferences, and the second term represents the inter-cell interferences. Suppose the transmit data symbols $d_{\mathrm{UL}}^{(j,n)}$ of

[2] In this thesis, the filters of the BSs are assumed to be linear due to their simplicity and optimality in terms of the achievable DoF, as will be shown later in this section. Nevertheless, the proposed relay-aided IA scheme can be combined with non-linear signal processing techniques, e.g., successive interference cancellation (SIC) and superposition coding, to further improve the achievable sum rate, as will be briefly mentioned in Chapter 4.

different MSs are i.i.d. circularly symmetric Gaussian symbols with zero mean and unit variance. The noises $\mathbf{n}_{\mathrm{BS},1}^{(k)}$, $\mathbf{n}_{\mathrm{BS},2}^{(k)}$, and $\mathbf{n}_{\mathrm{R}}^{(q)}$ are assumed to be independent circularly symmetric white Gaussian noises with zero mean and covariance matrices $\sigma_{\mathrm{BS}}^2\mathbf{I}_N$, $\sigma_{\mathrm{BS}}^2\mathbf{I}_N$, and $\sigma_{\mathrm{R}}^2\mathbf{I}_{M_q}$, respectively. Define $\gamma_{\mathrm{UL}}^{(k,n)}$ to be the signal-to-interference-plus-noise ratio (SINR) of the n-th data symbol in the output data symbols $\widetilde{\mathbf{d}}_{\mathrm{UL}}^{(k)}$ of the k-th BS. Note that for $\gamma_{\mathrm{UL}}^{(k,n)}$, both the inter- and intra-cell interferences shall be taken into account. Thus the achieved sum rate in the uplink reads

$$R_{\mathrm{sum,UL}} = \frac{1}{2}\sum_{k=1}^{K}\sum_{n=1}^{N}\log_2\left(1+\gamma_{\mathrm{UL}}^{(k,n)}\right). \tag{2.15}$$

In the downlink, the transmission direction is reversed as compared to the uplink while applying the two-hop transmission scheme, as depicted in Figure 2.3. In the considered cellular networks, reciprocal downlink channels are assumed. This is the case, for instance, in a low mobility scenario considering time-division duplexing (TDD) systems. That is to say, the channel matrices in the downlink are assumed to be the conjugate transposed versions of those in the uplink[3]. In particular, the channel from the k-th BS to the MSs of the j-th cell is assumed to be $\mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}}$. The channel from the k-th BS to the q-th relay is assumed to be $\mathbf{H}_{\mathrm{RB}}^{(q,k)}$. The channel from the q-th relay to the MSs of the j-th cell is assumed to be $\mathbf{H}_{\mathrm{RM}}^{(q,j)*\mathrm{T}}$. The downlink channel matrices are also shown in Figure 2.3. Similarly, one can define the downlink versions of the other notations that have been defined in the uplink. These include the data symbols $\mathbf{d}_{\mathrm{DL}}^{(k)} \in \mathbb{C}^N$ to be transmitted by the k-th BS, the transmit filter $\begin{bmatrix}\mathbf{V}_{\mathrm{DL},1}^{(k)}\\ \mathbf{V}_{\mathrm{DL},2}^{(k)}\end{bmatrix}$ of the k-th BS, the processing filter $\mathbf{G}_{\mathrm{DL}}^{(q)}$ of the q-th relay, the receive filters $\begin{bmatrix}\mathbf{U}_{\mathrm{DL},1}^{(j)}\\ \mathbf{U}_{\mathrm{DL},2}^{(j)}\end{bmatrix}$ of the MSs of the j-th cell, the effective channel matrix $\mathbf{H}_{\mathrm{DL,eff}}^{(j,k)}$ from the k-th BS to the MSs of the j-th cell, the effective noise $\mathbf{n}_{\mathrm{DL,eff}}^{(j)}$ at the MSs of the j-th cell, and the output data symbols $\widetilde{\mathbf{d}}_{\mathrm{DL}}^{(j)}$ of the MSs of the j-th cell. Note that in the downlink, the MSs cannot jointly process the received signals. Therefore, both combining matrices $\mathbf{U}_{\mathrm{DL},1}^{(j)}$ and $\mathbf{U}_{\mathrm{DL},2}^{(j)}$ are diagonal matrices. Moreover, suppose the data symbols $d_{\mathrm{DL}}^{(k,n)}$ intended for the different MSs are i.i.d. circularly symmetric Gaussian symbols with zero mean and unit variance. The noises at the MSs of the j-th cell in the first and second

[3] In fact, the reciprocal downlink channel matrices should be the transposed uplink channel matrices, since a radio channel provides the same attenuation and phase shift for both transmission directions. The complex conjugation is added for notational conformity.

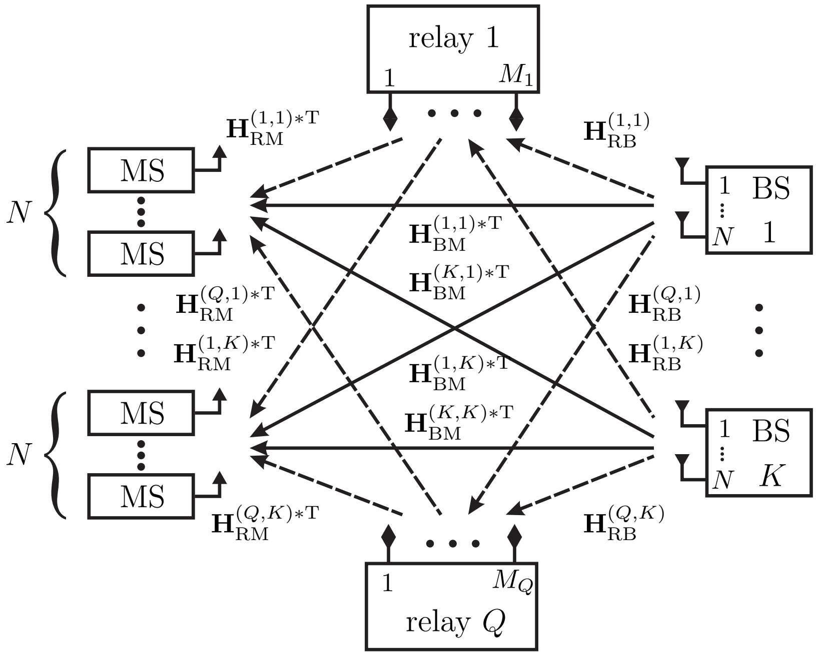


Figure 2.3.: In the downlink of the considered cellular network, the transmission direction is reversed as compared to the uplink and reciprocal channels are assumed.

time slots and the noise at the q-th relay are denoted by $\mathbf{n}_{\mathrm{MS},1}^{(j)} \in \mathbb{C}^N$, $\mathbf{n}_{\mathrm{MS},2}^{(j)} \in \mathbb{C}^N$, and $\mathbf{n}_{\mathrm{R}}^{(q)} \in \mathbb{C}^{M_q}$, respectively. They are assumed to be independent circularly symmetric white Gaussian noises with zero mean and covariance matrices $\sigma_{\mathrm{MS}}^2\mathbf{I}_N$, $\sigma_{\mathrm{MS}}^2\mathbf{I}_N$, and $\sigma_{\mathrm{R}}^2\mathbf{I}_{M_q}$, respectively. Define $\gamma_{\mathrm{DL}}^{(j,n)}$ to be SINR of the output data symbol of the n-th MS of the j-th cell. Thus, the achievable sum rate in the downlink reads

$$R_{\mathrm{sum,DL}} = \frac{1}{2}\sum_{j=1}^{K}\sum_{n=1}^{N}\log_2\left(1+\gamma_{\mathrm{DL}}^{(j,n)}\right). \tag{2.16}$$

In the considered cellular networks, the achievable sum DoF per cell is N if the channel coefficients are independently drawn from a continuous distribution. Therefore, the sum DoF of the considered cellular networks is almost surely $KN/2$. Furthermore, it can be seen from (2.15) and (2.16) that the sum DoF of $KN/2$ can be achieved in both the uplink and the downlink by using the two-hop transmission scheme, providing that N data symbols can be successfully transmitted in each cell without interferences. Moreover, even if a cell has more than N MSs, only N MSs can be simultaneously served by the BS in order to achieve interference-free transmissions. Finally, it is also assumed that the considered cellular networks always have $K \geq 3$ cells to exclude the cases where the sum DoF of the network can be easily achieved using conventional interference management approaches, e.g., TDMA, without relays.

2.3. Extended network: Partially connected ad-hoc network

In this thesis, relay-aided IA in a class of partially connected ad-hoc networks will be investigated, too. The considered partially connected ad-hoc networks result from ignoring certain weak interferences in the fully connected ad-hoc networks as introduced in Section 2.1. Correspondingly, these links are considered to be absent in the system model, i.e., the channel coefficients of these links are set to zero.

The topology of the considered partially connected ad-hoc networks is described as follows. Consider an ad-hoc network consisting of Q groups of source-destination node pairs and half-duplex AF relays. Each group will be referred to as a subnetwork in this thesis. The Q subnetworks are disjoint, meaning that different subnetworks do not have common nodes or relays. The channels of the interior of a subnetwork will be referred to as the intra-subnetwork links. In every subnetwork, the intra-subnetwork links are assumed to be sufficiently strong such that all of them are considered to be present. This is usually the case, for instance, if the source-destination node pairs and the relays in the same subnetwork are deployed close to each other. In other words, each subnetwork can be considered as a fully connected ad-hoc network as introduced in Section 2.1. Furthermore, the channel between a source and a destination node in different subnetworks will be referred to as an inter-subnetwork direct link, and the channel between a relay in one subnetwork and a source or destination node in another subnetwork will be referred to as an inter-subnetwork relay link. In contrast to the intra-subnetwork links, the inter-subnetwork links are assumed to be much weaker, e.g., due to great distances or due to the radiation patterns of antennas. Specifically, all the inter-subnetwork relay links are assumed to be absent[4]. Nevertheless, a few inter-subnetwork direct links are assumed to be present, e.g., the inter-subnetwork direct links between nodes that are close to the common

[4] The case where some inter-subnetwork relay links are present is considered in the author's preliminary work [LPKW16], but will not be included in this thesis.

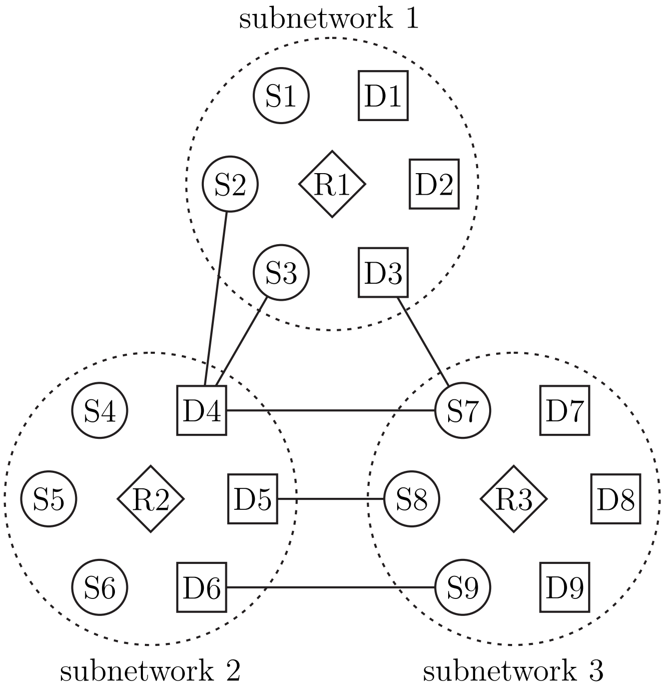


Figure 2.4.: A partially connected ad-hoc network consisting of three subnetworks. Each subnetwork is of size three, i.e., has three source-destination node pairs along with a single relay. The three subnetworks are partially connected to each other by multiple present inter-subnetwork direct links. The intra-subnetwork links are not depicted for simplicity.

boundary of neighboring subnetworks. In other words, the subnetworks are partially connected to each other by present inter-subnetwork direct links. In order to focus on the main results, it is furthermore assumed in this thesis that each source and destination node has only a single antenna and that each subnetwork has only a single relay for simplicity. Moreover, let K_q denote the number of source-destination node pairs in the q-th subnetwork and be referred to as the size of the subnetwork. Thus, the entire network has $\sum_{q=1}^{Q} K_q = K$ single-antenna source-destination node pairs and Q multi-antenna relays. An example of such partially connected ad-hoc networks consisting of three subnetworks is depicted in Figure 2.4.

The channel model that has been considered for fully connected ad-hoc networks can also be considered for the partially connected ad-hoc networks, i.e., a discrete-time narrowband channel which is time invariant throughout the transmission is considered and all the channel coefficients are assumed to be independently drawn from a continuous distribution over the complex field $\mathbb{C}$. In order to incorporate the partial connectivity, the channel coefficients of the absent links are set to zero, due to their negligibly small channel gains. Furthermore, the two-hop transmission scheme, which has been introduced in Section 2.1, is employed. Thus, the output data symbol $\widetilde{d}^{(k)}$ of the k-th destination node can also be given by (2.4). However, in order to clarify the specialty of the considered partially connected ad-hoc networks, let $\widetilde{d}^{(k)}$ be reformulated as follows taking the partial connectivity into account. Let $\Omega_{\{q\}} \subseteq \{1, \ldots, K\}$ denote the index set of the node pairs belonging to the q-th subnetwork. Suppose the k-th destination node belongs to the q-th subnetwork, i.e., $k \in \Omega_{\{q\}}$. Then, $\widetilde{d}^{(k)}$ can be reformulated as

$$\widetilde{d}^{(k)} = h_{\text{eff}}^{(k,k)} d^{(k)} + \sum_{\substack{j \in \Omega_{\{q\}} \\ j \neq k}} h_{\text{eff}}^{(k,j)} d^{(j)} + \sum_{i \notin \Omega_{\{q\}}} h_{\text{eff}}^{(k,i)} d^{(i)} + n_{\text{eff}}^{(k)}, \tag{2.17}$$

where $h_{\text{eff}}^{(k,j)}$ and $n_{\text{eff}}^{(k)}$ are the effective channel coefficient and the effective noise, respectively. In (2.17), the first term on the right hand side represents the useful signal for the k-th destination node. The second term represents the interferences from the other source nodes in the same subnetwork as the k-th destination node, which will be referred to as intra-subnetwork interferences. The third term represents the interferences from the source nodes in other subnetworks, which will be referred to as inter-subnetwork interferences. Furthermore, depending on the location of the j-th source node, the effective channel coefficient $h_{\text{eff}}^{(k,j)}$ can be given by one of the following three forms.

- If the j-th source node belongs to the same subnetwork as the k-th destination node, the transmitted signal propagates through both the intra-subnetwork direct link $h_{\text{DS}}^{(k,j)}$ and the q-th relay. Then, $h_{\text{eff}}^{(k,j)}$ is given by

$$h_{\text{eff}}^{(k,j)} = \begin{bmatrix} u_1^{(k)*\text{T}} & u_2^{(k)*\text{T}} \end{bmatrix} \begin{bmatrix} h_{\text{DS}}^{(k,j)} & 0 \\ \mathbf{h}_{\text{RD}}^{(q,k)*\text{T}} \mathbf{G}^{(q)} \mathbf{h}_{\text{RS}}^{(q,j)} & h_{\text{DS}}^{(k,j)} \end{bmatrix} \begin{bmatrix} v_1^{(j)} \\ v_2^{(j)} \end{bmatrix}. \tag{2.18}$$

- If the j-th source node does not belong to the same subnetwork as the k-th destination node, but they are connected by a present inter-subnetwork direct link, the transmitted signal propagates only through the present inter-subnetwork direct link $h_{\text{DS}}^{(k,j)}$. Then, $h_{\text{eff}}^{(k,j)}$ is given by

$$h_{\text{eff}}^{(k,j)} = \begin{bmatrix} u_1^{(k)*\text{T}} & u_2^{(k)*\text{T}} \end{bmatrix} \begin{bmatrix} h_{\text{DS}}^{(k,j)} & 0 \\ 0 & h_{\text{DS}}^{(k,j)} \end{bmatrix} \begin{bmatrix} v_1^{(j)} \\ v_2^{(j)} \end{bmatrix}. \tag{2.19}$$

- If the j-th source node does not belong to the same subnetwork as the k-th destination node, and they are not connected by any present inter-subnetwork direct link either,

$$h_{\text{eff}}^{(k,j)} = 0 \tag{2.20}$$

follows.

In general, the sum DoFs of the considered partially connected ad-hoc networks depend on the network topology, i.e., the subnetwork sizes and the presence of inter-subnetwork direct links. For instance, in the trivial case where each subnetwork has only a single source-destination node pair and no inter-subnetwork direct links are present, the sum DoF of the network is K. In order to exclude such special cases, it is always assumed that each subnetwork is at least of size three. Under this assumption, the sum DoF of the considered partially connected ad-hoc networks is almost surely $K/2$, if the channel coefficients are independently drawn from a continuous distribution. Furthermore, the maximum achievable sum DoF of the two-hop transmission scheme is $K/2$ in the considered partially connected ad-hoc networks, providing that the data symbol intended for every destination node can be successfully transmitted without interferences.

In this thesis, a relay-aided IA scheme without full channel knowledge will be proposed for the considered partially connected ad-hoc networks. More precisely, only the following three types of CSI are assumed to be known by every subnetwork, i.e., either by a central unit in the subnetwork or by every node and relay in the subnetwork.

- **Intra-subnetwork CSI:** Every subnetwork knows perfect and instantaneous CSI of the intra-subnetwork links in itself.
- **Network topology:** Every subnetwork knows the sizes of all subnetworks and the presence of inter-subnetwork direct links.
- **Side information:** Every subnetwork obtains a few complex-valued numbers from other subnetworks, which are referred to as side information in this thesis. The side information can be considered as some kind of compressed CSI, and it enables cooperation between different subnetworks. More details on side information will be explained in Chapter 5.

The three types of CSI mentioned above will be referred to as partial channel knowledge in this thesis.

Chapter 3.

Relay-aided interference alignment in fully connected ad-hoc networks

3.1. Overview

In this chapter, relay-aided IA in fully connected ad-hoc networks will be investigated. As introduced in Section 2.1, all the channel coefficients in such networks are assumed to be independently drawn from a continuous distribution, which means all the channel coefficients are almost surely non-zero. Furthermore, full channel knowledge is assumed, i.e., the global, perfect, and instantaneous CSI is assumed to be known by a central unit or by every node in the network. The basic idea of the proposed relay-aided IA scheme and the general results on the feasibility and performances will be presented in this chapter.

The conditions for achieving relay-aided IA will be discussed first. In this thesis, these conditions are classified into two categories, i.e., the interference-nulling (IN) conditions and the validity conditions. The IN conditions are some non-linear equations which, as the name suggests, ensure that all the interferences at every destination node are perfectly aligned and then nullified. The validity conditions are some non-linear inequalities which ensure that a non-zero interference-free component of the useful signals of each destination node can be recovered. The problem of achieving relay-aided IA in fully connected ad-hoc networks is basically to find the IA solutions, i.e., to find the transmit filters, the receive filters, and the relay processing filters which satisfy all the IN conditions while not violating any of the validity conditions.

Due to the non-linearity of the IN conditions and the validity conditions, little is known about how to efficiently find an IA solution or the characteristics of the set of IA solutions. In this thesis, a linearization approach is proposed such that the set of IA solutions can be described analytically. Firstly, some new variables will be introduced, which provide alternative descriptions of the vector spaces specified by the transmit filters and the receive filters. With the help of these new variables, the solutions of the non-linear IN conditions can then be mapped to a linear space called the IN solution space. Furthermore, the solutions which violate the validity conditions can be mapped to some subsets of the IN solution space, which are referred to as the invalid IN solution subsets. If each source and destination node is equipped with a single antenna, these invalid IN solution subsets are linear subspaces of the IN solution space. If each source and destination node has more than one antenna, these invalid IN solution subsets are non-linear algebraic sets. Finally, besides these invalid IN solutions, the other IN solutions in the IN solution space, i.e., the valid IN solutions, correspond to the IA solutions.

In the context of IA, feasibility conditions determine whether or not IA solutions exist. If at least one IA solution exists, IA is feasible. For MIMO IA, this problem is relatively simple since all the non-trivial IN solutions are almost surely valid if the channel coefficients are independently drawn from a continuous distribution [GCJ08]. However, for relay-aided IA, a non-trivial IN solution may be invalid for all channel realizations. It may even occur that every IN solution is an invalid one, resulting that no valid IN solution exists. In this chapter, the feasibility conditions for relay-aided IA in fully connected ad-hoc networks will be addressed. For the single-antenna case, it will be investigated under which condition the invalid IN solutions only form strict linear subspaces, i.e., hyperplanes, of the IN solution space. For the multi-antenna case, it will be investigated under which condition the invalid solutions do not fill up the entire IN solution space and, meanwhile, form a negligibly small subset of the IN solution space. Furthermore, for both the single- and multi-antenna cases, a randomly picked IN solution is almost surely valid if relay-aided IA is feasible.

Given any valid IN solution, the corresponding transmit and receive filters are not unique. The question of how to design these filters aiming at maximizing the achievable sum rate will be answered in this chapter. For the proposed relay-aided IA scheme, each source node transmits in the first and second time slot likely with different transmit powers, and the relays only transmit in the second time slot. Therefore, sum power constraints, which are essentially energy constraints, will be considered. Specifically, the sum rate maximization will be carried out either under a total sum power constraint, which is essentially a total transmit energy constraint, or under individual sum power constraints, which are essentially individual transmit energy constraints for the different source nodes and relays. In the former case, the sum rate maximization problem is a convex problem and can be solved in closed-form. In the latter case, the sum rate maximization problem is non-convex. However, a suboptimal solution is proposed, which can be readily obtained using standard convex optimization tools.

3.2. Interference-nulling and validity conditions

In the considered fully-connected ad-hoc networks, an IA solution is just a set of relay processing filters $\mathbf{G}^{(q)}$, transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$ of the source nodes, and receive filters $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$ of the destination nodes which achieves the DoF of the network, i.e., a sum DoF of $KN/2$ in the almost sure sense. In this section, the conditions that an IA solution must satisfy will be introduced. As discussed in Section 2.1, each source node shall transmit N data symbols to the corresponding destination node using the two-hop transmission scheme in order to achieve a sum DoF of $KN/2$. Thus both the transmit filters and the receive filters shall be $2N \times N$ matrices. To facilitate the following discussions, recall that the $N \times N$ matrix

$$\mathbf{H}_{\text{eff}}^{(k,j)} = \begin{bmatrix}\mathbf{U}_1^{(k)*\text{T}} & \mathbf{U}_2^{(k)*\text{T}}\end{bmatrix} \begin{bmatrix}\mathbf{H}_{\text{DS}}^{(k,j)} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\text{RD}}^{(q,k)*\text{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,j)} & \mathbf{H}_{\text{DS}}^{(k,j)}\end{bmatrix} \begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix} \tag{3.1}$$

has been defined as the effective channel matrix between the j-th source node and the k-th destination node in (2.5). The components of an effective channel matrix are illustrated in Figure 3.1, where each of the depicted "vectors" represents a subspace in order to graphically show the relations among them. Firstly, the transmit filter $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$ of the source node specifies a subspace of dimension $\text{rank}\left(\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}\right) \leq N$ of the $2N$-dimensional transmit signal space at the j-th source node. The signals being transmitted by the j-th source node can be represented by vectors in this subspace. Secondly, the two-hop transmission scheme exploiting relays creates a virtual $2N \times 2N$ MIMO channel

$$\begin{bmatrix}\mathbf{H}_{\text{DS}}^{(k,j)} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\text{RD}}^{(q,k)*\text{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,j)} & \mathbf{H}_{\text{DS}}^{(k,j)}\end{bmatrix} \tag{3.2}$$

between the j-th source node and k-th destination node, which rotates and scales the transmitted signals. Note that the virtual MIMO channel matrix (3.2) has

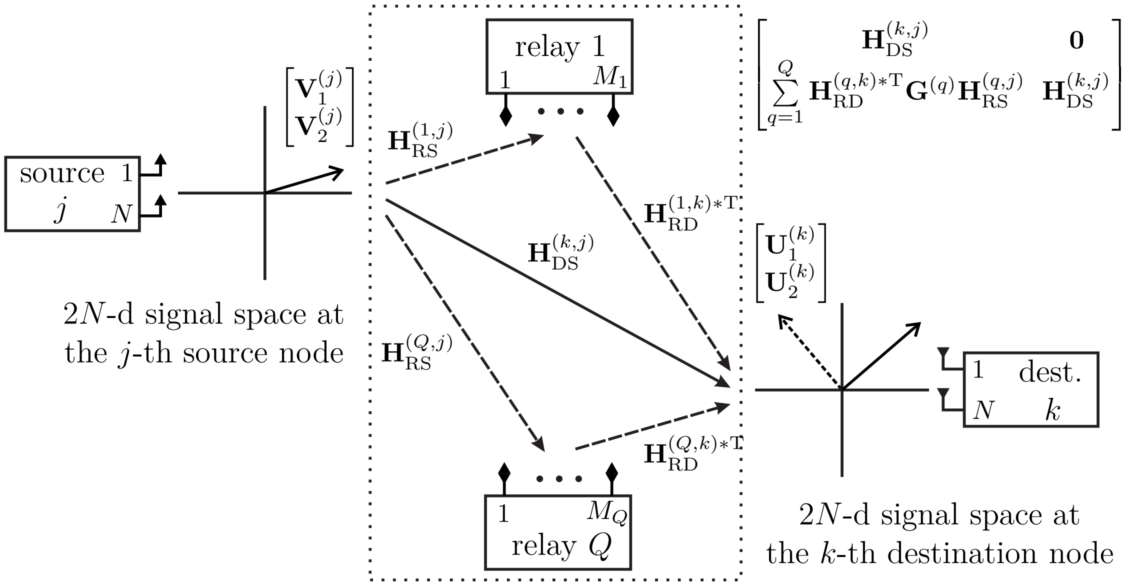


Figure 3.1.: Components of an effective channel matrix between a source node and a destination node in an ad-hoc network, i.e., the transmit filter, a virtual $2N \times 2N$ MIMO channel, and the receive filter. Each depicted "vector" represents a subspace.

a block triangular structure with identical diagonal blocks $\mathbf{H}_{\mathrm{DS}}^{(k,j)}$. Therefore, the relays play an essential role in rotating the transmitted signals. Especially if each source and destination node only has a single antenna, the rotation will be solely due to the relays[1]. Finally, only a component of the signals received from the j-th source node is contained in the output data symbols of the k-th destination node. This component is basically a projection of the received signals on the subspace specified by the receive filter $\begin{bmatrix} \mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)} \end{bmatrix}$.

In order to achieve the DoF of the network, all the interferences at every destination node have to be perfectly nullified first. To this purpose, the subspace specified by the receive filter $\begin{bmatrix} \mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)} \end{bmatrix}$ must be the orthogonal complement of the subspace spanned by the all the interferences at the k-th destination node, which will be referred to as the interference subspace. In other words, the transmit fil-

[1] In the real plane, a 2×2 matrix with such a structure represents a (scaled) vertical shear mapping [Wei99]. The off-diagonal entry is called the shear factor, which determines the angle by which a horizontal vector tilts.

ters, the receive filters, and the relay processing filters in the entire network must be cooperatively designed such that the following $K(K-1)N^2$ equalities given in the matrix form

$$\mathbf{H}_{\text{eff}}^{(k,j)} = \begin{bmatrix}\mathbf{U}_1^{(k)*\text{T}} & \mathbf{U}_2^{(k)*\text{T}}\end{bmatrix} \begin{bmatrix} \mathbf{H}_{\text{DS}}^{(k,j)} & \mathbf{0} \\ \sum_{q=1}^{Q} \mathbf{H}_{\text{RD}}^{(q,k)*\text{T}} \mathbf{G}^{(q)} \mathbf{H}_{\text{RS}}^{(q,j)} & \mathbf{H}_{\text{DS}}^{(k,j)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)} \end{bmatrix} = \mathbf{0}_N, \quad \forall j \neq k, \tag{3.3}$$

are satisfied. In this thesis, equalities such as (3.3) will be referred to as IN conditions.

However, it has to be emphasized that not every solution of the IN conditions (3.3) is able to achieve the DoF of the network. This can be demonstrated by the following examples.

Example 3.1. The trivial solution of the IN conditions (3.3), i.e., the solution with all the transmit filters, the receive filters, and the relay processing filters being zero matrices, achieves zero DoF, because no data symbol can be transmitted through the network.

Example 3.2. If any transmit or receive filter is rank deficient, i.e., has a rank less than N, the N data symbols to be transmitted between the corresponding source-destination node pair are not separable. Therefore, the achievable sum DoF is less than $KN/2$.

Example 3.3. Firstly, let the transmit filter of the j-th source node be chosen as $\begin{bmatrix} \mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^{(j)} \\ \alpha \mathbf{Z}^{(j)} \end{bmatrix}$, where α is a common scaling factor used by all the source nodes and $\mathbf{Z}^{(j)}$ is an arbitrarily chosen pre-coding matrix. Secondly, choose all the relay processing filters as zero matrices, i.e., let the relays be muted. By doing so, every virtual MIMO channel matrix (3.2) becomes a block diagonal matrix with identical diagonal blocks $\mathbf{H}_{\text{DS}}^{(k,j)}$. Therefore, the signals being received by a destination node in the second time slot are only a scaled version of the signals being received in the first time slot. In other words, the interference subspace of every destination node is the N-dimensional subspace spanned by $\begin{bmatrix} \mathbf{I}_N \\ \alpha \mathbf{I}_N \end{bmatrix}$. Finally, simply combining the received signals of the two time slots with the receive filter $\begin{bmatrix} \mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)} \end{bmatrix} = \begin{bmatrix} -\alpha \mathbf{I}_N \\ \mathbf{I}_N \end{bmatrix}$ nullifies all the interferences at each destination node. In spite that the above choice of the transmit filters, the receive filters, and the relay processing filters forms a non-trivial solution of the IN conditions (3.3), it results in that the useful signals also fall into the interference subspaces and will be nullified by the receive filters together with the interferences, and hence that zero DoF is achieved.

Besides these three examples, there could exist other solutions of the IN conditions (3.3) which cannot achieve the DoF of the network and hence are not IA solutions. In general, a solution of the IN conditions (3.3) which also achieves the DoF of the network must ensure that the N data symbols intended for each destination node are separable, i.e., the effective channel matrix $\mathbf{H}_{\text{eff}}^{(k,k)}$ between each source-destination node pair is of full rank. This can be translated to the following K inequality conditions

$$\det\left(\mathbf{H}_{\text{eff}}^{(k,k)}\right) = \det\left(\begin{bmatrix}\mathbf{U}_1^{(k)*\mathrm{T}} & \mathbf{U}_2^{(k)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\text{DS}}^{(k,k)} & \mathbf{0} \\ \sum_{q=1}^{Q}\mathbf{H}_{\text{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,k)} & \mathbf{H}_{\text{DS}}^{(k,k)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{(k)} \\ \mathbf{V}_2^{(k)}\end{bmatrix}\right) \neq 0, \quad \forall k. \tag{3.4}$$

In this thesis, inequalities such as (3.4) will be referred to as validity conditions. Clearly, only the solutions of the IN conditions (3.3) which do not violate any of the K validity conditions (3.4) are able to achieve the DoF of the network and are hence the IA solutions.

3.3. Linearization approach

Note that if the relay processing filters $\mathbf{G}^{(q)}$, the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)}\end{bmatrix}$, and the receive filters $\begin{bmatrix}\mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)}\end{bmatrix}$ are chosen as variables, the IN conditions (3.3) and the validity conditions (3.4) are trilinear equalities and inequalities, respectively, and are difficult to analyze. Furthermore, as compared to numerically approaching a single IA solution, it is of greater interest to analytically find the set of all of them or, at least, to find a set of the relevant ones. For these reasons, a linearization approach is proposed in this thesis. The proposed linearization approach is based on the following assumption. The consequences of this assumption will be discussed later in this section.

Assumption 3.1. In the considered fully connected ad-hoc networks, it is always assumed that the pre-coding matrix $\mathbf{V}_1^{(j)}$ of each source node in the first time slot and the combining matrix $\mathbf{U}_2^{(k)}$ of each destination node in the second time slot are invertible matrices, i.e.,

$$\operatorname{rank}\left(\mathbf{V}_1^{(j)}\right) = \operatorname{rank}\left(\mathbf{U}_2^{(k)}\right) = N, \quad \forall j,k \tag{3.5}$$

holds.

Under Assumption 3.1, the transmit and the receive filters can be factorized as

$$\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix} = \begin{bmatrix}\mathbf{I}_N\\ \mathbf{V}^{(j)}\end{bmatrix}\mathbf{V}_1^{(j)} \quad \text{and} \quad \begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix} = \begin{bmatrix}\mathbf{U}^{(k)}\\ \mathbf{I}_N\end{bmatrix}\mathbf{U}_2^{(k)}, \quad \forall j,k, \tag{3.6}$$

where the $N \times N$ matrices $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$ are given by

$$\mathbf{V}^{(j)} = \mathbf{V}_2^{(j)}\mathbf{V}_1^{(j)-1} \quad \text{and} \quad \mathbf{U}^{(k)} = \mathbf{U}_1^{(k)}\mathbf{U}_2^{(k)-1}, \quad \forall j,k, \tag{3.7}$$

respectively. Note that as long as $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$ are invertible, $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$ are able to uniquely represent the subspaces specified by $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$ and $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$, respectively[2]. Exploiting the matrices $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$, the effective channel matrix $\mathbf{H}_{\text{eff}}^{(k,j)}$ can be reformulated as

$$\begin{aligned}\mathbf{H}_{\text{eff}}^{(k,j)} &= \mathbf{U}_2^{(k)*\mathrm{T}}\begin{bmatrix}\mathbf{U}^{(k)*\mathrm{T}} & \mathbf{I}_N\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\text{DS}}^{(k,j)} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\text{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,j)} & \mathbf{H}_{\text{DS}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{V}^{(j)}\end{bmatrix}\mathbf{V}_1^{(j)}\\ &= \mathbf{U}_2^{(k)*\mathrm{T}}\left(\sum_{q=1}^{Q}\mathbf{H}_{\text{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,j)} + \mathbf{H}_{\text{DS}}^{(k,j)}\mathbf{V}^{(j)} + \mathbf{U}^{(k)*\mathrm{T}}\mathbf{H}_{\text{DS}}^{(k,j)}\right)\mathbf{V}_1^{(j)}.\end{aligned} \tag{3.8}$$

Therefore, under Assumption 3.1, the IN conditions (3.3) and the validity conditions (3.4) can be equivalently reformulated as

$$\sum_{q=1}^{Q}\mathbf{H}_{\text{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,j)} + \mathbf{H}_{\text{DS}}^{(k,j)}\mathbf{V}^{(j)} + \mathbf{U}^{(k)*\mathrm{T}}\mathbf{H}_{\text{DS}}^{(k,j)} = \mathbf{0}, \quad \forall k \neq j, \tag{3.9}$$

[2] Geometrically, the entries of $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$ can be interpreted as trigonometric functions of some complex(-valued) angles, following the definition on this subject in [Sch01]. For instance, if each source and destination node has only a single antenna, $v^{(j)} = v_2^{(j)}/v_1^{(j)}$ is the cotangent of the complex angle defined by the transmit filter $(v_1^{(j)}, v_2^{(j)})^{\mathrm{T}}$ and the axis $(1,0)^{\mathrm{T}}$, and $u^{(k)} = u_1^{(k)}/u_2^{(k)}$ is the tangent of the complex angle defined by the receive filter $(u_1^{(k)}, u_2^{(k)})^{\mathrm{T}}$ and the axis $(1,0)^{\mathrm{T}}$.

and

$$\det\left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,k)}+\mathbf{H}_{\mathrm{DS}}^{(k,k)}\mathbf{V}^{(k)}+\mathbf{U}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)\neq 0,\quad \forall k, \tag{3.10}$$

respectively, where the relay processing filters $\mathbf{G}^{(q)}$ and the matrices $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$, instead of the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$ and the receive filters $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$, are chosen as variables. Since the reformulated IN conditions (3.9) are linear in the new variables $\mathbf{G}^{(q)}$, $\mathbf{V}^{(j)}$, and $\mathbf{U}^{(k)}$, they will be referred to as the linearized IN conditions. A solution and the solution space of the linearized IN conditions (3.9) will be referred to as an IN solution and the IN solution space $\mathbb{S}_{\mathrm{IN}}$, respectively. Furthermore, an IN solution in $\mathbb{S}_{\mathrm{IN}}$ is called an invalid IN solution with respect to the k-th node pair if it violates the k-th reformulated validity condition (3.10). All the invalid IN solutions with respect to the k-th node pair form a subset $\mathbb{S}_{\mathrm{inv}}^{(k)}$ of the IN solution space $\mathbb{S}_{\mathrm{IN}}$. In general, $\mathbb{S}_{\mathrm{inv}}^{(k)}$ is not a linear subspace of $\mathbb{S}_{\mathrm{IN}}$, except for the case where each source and destination node only has a single antenna so that the reformulated validity conditions (3.10) reduce to linear inequalities. Moreover, a valid IN solution is not invalid with respect to any of the K node pairs. Thus the set of valid IN solutions can be denoted by $\mathbb{S}_{\mathrm{IN}}\setminus\bigcup\mathbb{S}_{\mathrm{inv}}^{(k)}$.

The linearization approach described above can be considered as a surjective mapping from a subset of the solution set of the non-linear IN conditions (3.3), which fulfills Assumption 3.1, to the linear IN solution space $\mathbb{S}_{\mathrm{IN}}$. Inversely, every solution of the non-linear IN conditions (3.3) fulfilling Assumption 3.1 can be constructed using an IN solution in $\mathbb{S}_{\mathrm{IN}}$ along with a set of properly designed full-rank matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$. However, the solutions of the non-linear IN conditions (3.3) which do not fulfil Assumption 3.1, i.e., the ones with at least one of $\mathbf{V}_1^{(j)}$ or $\mathbf{U}_2^{(k)}$ being rank deficient, cannot be found using the proposed linearization approach. In the following, it will be argued that these solutions are of little relevance from an engineering perspective and, therefore, it is viable to consider Assumption 3.1.

If each source and destination node is equipped with a single antenna, the matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$ reduce to scalars, and they can be denoted by $v_1^{(j)}$ and $u_2^{(k)}$, respectively. In this case, a solution of the non-linear IN conditions (3.3) which does not fulfill Assumption 3.1 has at least one of the $v_1^{(j)}$ or $u_2^{(k)}$ being zero. However, in a fully-connected ad-hoc network with $K\geq 3$ node pairs, such a solution results in that no useful signal can be received, and it is therefore not an IA solution. This can be shown as follows. Suppose $v_1^{(1)}$ is zero, i.e., the first source node is muted in the first time slot. Then it must transmit in the second time slot. In order to align the caused interferences with the first source node, the other source nodes must also be muted in the first time slot and only transmit in the second time slot. However, all the destination nodes must be shut down in the second time slot, i.e., all $u_2^{(k)}$ must be zero, otherwise the interferences cannot be nullified. Consequently, no useful signal can be received. The same consequence follows if at least one of the $u_2^{(k)}$ is zero.

If each source and destination node is equipped with more than one antenna, a solution of the non-linear IN conditions (3.3) with at least one $\mathbf{V}_1^{(j)}$ or $\mathbf{U}_2^{(k)}$ being rank deficient could still be an IA solution. However, from an engineering point of view, rank-deficient matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$ cannot fully exploit the multiple antennas being equipped at the source and destination nodes. Hence, these solutions are obviously less favourable if the interference is not the only concern.

Remark 3.1. If each source and destination node has a single antenna, the proposed linearization approach is able to find all the IA solutions. If each source and destination node has more than one antenna, the proposed linearization approach is able to find almost all the IA solutions, except for some irrelevant ones.

Based on the proposed linearization approach, the relay-aided IA problem in fully connected ad-hoc networks can be solved as follows. Firstly, one can determine the IN solution space $\mathbb{S}_{\text{IN}}$ by solving the linearized IN conditions (3.9). Secondly, for any valid IN solution in $\mathbb{S}_{\text{IN}} \setminus \bigcup \mathbb{S}_{\text{inv}}^{(k)}$, one can choose a set of full-rank matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$, and use (3.6) to obtain an IA solution. In fact, the choice of $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$ does not influence the achievable DoF. Hence, they can be designed for sum rate maximization.

3.4. Feasibility conditions

3.4.1. Single-antenna source and destination nodes

In this section, the feasibility conditions for relay-aided IA in the considered fully-connected ad-hoc networks will be addressed. Specifically, it will be investigated when the valid IN solution set $\mathbb{S}_{\text{IN}} \setminus \bigcup \mathbb{S}_{\text{inv}}^{(k)}$ is a non-empty set, or equivalently, when the union $\bigcup \mathbb{S}_{\text{inv}}^{(k)}$ of the invalid solution subsets does not fill up the entire IN solution space $\mathbb{S}_{\text{IN}}$. The two cases where each each source and destination node has a single antenna and has more than one antenna will be discussed separately. In both cases, each relay may have multiple antennas. The single-antenna case will be discussed first in this subsection.

In order to clarify the differences to the multi-antenna case and to simplify the notations, the channel matrices and the filters will be replaced by scalars or vectors, correspondingly, in this subsection. Specifically, the channel between a source node and a destination node will be denoted by $h_{\mathrm{DS}}^{(k,j)}$. The channel between a source node and a relay will be denoted by the $M_q \times 1$ vector $\mathbf{h}_{\mathrm{RS}}^{(q,j)}$. The channel between a relay and a destination node will be denoted by the $1 \times M_q$ row vector $\mathbf{h}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}$. Furthermore, the transmit filter of a source node and the receive filter of a destination node can be denoted by the 2×1 vectors $(v_1^{(j)}, v_2^{(j)})^{\mathrm{T}}$ and $(u_1^{(k)}, u_2^{(k)})^{\mathrm{T}}$, respectively. The processing filter of a relay will still be denoted by the $M_q \times M_q$ matrix $\mathbf{G}^{(q)}$. Using these notations, the linearized IN conditions (3.9) and validity conditions (3.10) can be rewritten as

$$\sum_{q=1}^{Q} \mathbf{h}_{\mathrm{RD}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RS}}^{(q,j)} + h_{\mathrm{DS}}^{(k,j)} v^{(j)} + h_{\mathrm{DS}}^{(k,j)} u^{(k)*} = 0, \quad \forall j \neq k, \tag{3.11}$$

and

$$\sum_{q=1}^{Q} \mathbf{h}_{\mathrm{RD}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RS}}^{(q,k)} + h_{\mathrm{DS}}^{(k,k)} v^{(k)} + h_{\mathrm{DS}}^{(k,k)} u^{(k)*} \neq 0, \quad \forall k, \tag{3.12}$$

respectively, where $v^{(k)} = v_2^{(j)}/v_1^{(j)}$ and $u^{(k)} = u_1^{(k)}/u_2^{(k)}$ are the corresponding scalar forms of the matrices $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$ defined in (3.7). Note that in the single-antenna case, not only the IN conditions (3.11) but also the validity conditions (3.12) are linear in the variables $\mathbf{G}^{(q)}$, $v^{(j)}$, and $u^{(k)}$.

Let the variables of the linearized IN conditions (3.9) and the reformulated validity conditions (3.10) be stacked in a $(\sum_{q=1}^{Q} M_q^2 + 2K) \times 1$ vector as

$$\mathbf{x} = \left[\left[\mathrm{vec}\left(\left[\mathbf{G}^{(1)} \ \cdots \ \mathbf{G}^{(Q)} \right] \right) \right]^{\mathrm{T}} \ v^{(1)} \ \ldots \ v^{(K)} \ u^{(1)*} \ \ldots \ u^{(K)*} \right]^{\mathrm{T}}, \tag{3.13}$$

where $\mathrm{vec}(\cdot)$ denotes the vectorization of a matrix, see Appendix A. Thus, the system of linear equations consisting of all the linearized IN conditions (3.11) can be written in the matrix form as

$$\mathbf{A}_{\mathrm{IN}} \mathbf{x} = \left[\mathbf{A}_{\mathrm{RL}} \ \ \mathbf{A}_{\mathrm{DL}} \right] \mathbf{x} = \mathbf{0}, \tag{3.14}$$

where $\mathbf{A}_{\mathrm{IN}}$ is interpreted as a partitioned matrix consisting of the $K(K-1) \times$

$\sum_{q=1}^{Q} M_q^2$ matrix

$$\mathbf{A}_{\mathrm{RL}} = \begin{bmatrix} \mathbf{h}_{\mathrm{RS}}^{(1,1)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,2)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,1)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,2)*\mathrm{T}} \\ \mathbf{h}_{\mathrm{RS}}^{(1,1)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,3)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,1)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,3)*\mathrm{T}} \\ \vdots & & \vdots \\ \mathbf{h}_{\mathrm{RS}}^{(1,1)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,K)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,1)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,K)*\mathrm{T}} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \mathbf{h}_{\mathrm{RS}}^{(1,K)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,1)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,K)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,1)*\mathrm{T}} \\ \mathbf{h}_{\mathrm{RS}}^{(1,K)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,2)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,K)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,2)*\mathrm{T}} \\ \vdots & & \vdots \\ \mathbf{h}_{\mathrm{RS}}^{(1,K)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,K-1)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,K)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,K-1)*\mathrm{T}} \end{bmatrix}, \tag{3.15}$$

where $\otimes$ denotes the Kronecker product, and the $K(K-1) \times 2K$ matrix

$$\mathbf{A}_{\mathrm{DL}} = \begin{bmatrix} h_{\mathrm{DS}}^{(2,1)} & 0 & \cdots & 0 & 0 & h_{\mathrm{DS}}^{(2,1)} & 0 & \cdots & 0 \\ h_{\mathrm{DS}}^{(3,1)} & 0 & \cdots & 0 & 0 & 0 & h_{\mathrm{DS}}^{(3,1)} & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \ddots & \vdots \\ h_{\mathrm{DS}}^{(K,1)} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & h_{\mathrm{DS}}^{(K,1)} \\ \vdots & & & & & & & & \vdots \\ \vdots & & & & & & & & \vdots \\ 0 & \cdots & 0 & h_{\mathrm{DS}}^{(1,K)} & h_{\mathrm{DS}}^{(1,K)} & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & h_{\mathrm{DS}}^{(2,K)} & 0 & h_{\mathrm{DS}}^{(2,K)} & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & h_{\mathrm{DS}}^{(K-1,K)} & 0 & \cdots & 0 & h_{\mathrm{DS}}^{(K-1,K)} & 0 \end{bmatrix}. \tag{3.16}$$

Then the IN solution space $\mathbb{S}_{\mathrm{IN}}$ can be denoted by

$$\mathbb{S}_{\mathrm{IN}} = \mathrm{null}\left(\mathbf{A}_{\mathrm{IN}}\right), \tag{3.17}$$

where null $(\cdot)$ denotes the null space of a matrix. The dimension of the IN solution space $\mathbb{S}_{\mathrm{IN}}$ is

$$\dim \mathbb{S}_{\mathrm{IN}} = \sum_{q=1}^{Q} M_q^2 + 2K - \mathrm{rank}\left(\mathbf{A}_{\mathrm{IN}}\right). \tag{3.18}$$

Furthermore, the left hand side of the k-th validity condition (3.12) can be written as $\left[\mathbf{b}^{(k)\mathrm{T}} \;\; \mathbf{c}^{(k)\mathrm{T}}\right] \mathbf{x}$, where $\mathbf{b}^{(k)\mathrm{T}}$ is a $1 \times \sum_{q=1}^{Q} M_q^2$ row vector given by

$$\mathbf{b}^{(k)\mathrm{T}} = \left[\mathbf{h}_{\mathrm{RS}}^{(1,k)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,k)*\mathrm{T}} \;\; \cdots \;\; \mathbf{h}_{\mathrm{RS}}^{(Q,k)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,k)*\mathrm{T}}\right], \tag{3.19}$$

and $\mathbf{c}^{(k)\mathrm{T}}$ is a $1 \times 2K$ row vector defined as

$$\mathbf{c}^{(k)\mathrm{T}} = h_{\mathrm{DS}}^{(k,k)} \left[\mathbf{i}_k^{\mathrm{T}} \;\; \mathbf{i}_k^{\mathrm{T}} \right] \tag{3.20}$$

with $\mathbf{i}_i$ denoting a $K \times 1$ index vector with the k-th entry being one and the others zero. Hence, the invalid solution subset $\mathbb{S}_{\mathrm{inv}}^{(k)}$ with respect to the k-th node pair is the solution space of the system of linear equations

$$\mathbf{A}_{\mathrm{inv}}^{(k)} \mathbf{x} = \begin{bmatrix} \mathbf{A}_{\mathrm{RL}} & \mathbf{A}_{\mathrm{DL}} \\ \mathbf{b}^{(k)\mathrm{T}} & \mathbf{c}^{(k)\mathrm{T}} \end{bmatrix} \mathbf{x} = \mathbf{0}. \tag{3.21}$$

Clearly, $\mathbb{S}_{\mathrm{inv}}^{(k)}$ is a linear subspace of the IN solution space $\mathbb{S}_{\mathrm{IN}}$ in the single-antenna case, which can be denoted by

$$\mathbb{S}_{\mathrm{inv}}^{(k)} = \mathrm{null}\left(\mathbf{A}_{\mathrm{inv}}^{(k)} \right). \tag{3.22}$$

The dimension of $\mathbb{S}_{\mathrm{inv}}^{(k)}$ is

$$\dim \mathbb{S}_{\mathrm{inv}}^{(k)} = \sum_{q=1}^{Q} M_q^2 + 2K - \mathrm{rank}\left(\mathbf{A}_{\mathrm{inv}}^{(k)} \right). \tag{3.23}$$

In the single-antenna case, if every invalid IN solution subset $\mathbb{S}_{\mathrm{inv}}^{(k)}$ is a strict subspace, i.e., a hyperplane, of the IN solution space $\mathbb{S}_{\mathrm{IN}}$, then $\bigcup \mathbb{S}_{\mathrm{inv}}^{(k)}$ is the union of a finite number K of such hyperplanes. Since these hyperplanes cannot fill up the entire IN solution space $\mathbb{S}_{\mathrm{IN}}$, infinitely many valid IN solutions still exist in $\mathbb{S}_{\mathrm{IN}}$. This is graphically illustrated in Figure 3.2. Based on this idea, the following proposition gives a necessary and sufficient feasibility condition for relay-aided IA in fully connected ad-hoc networks with single-antenna source and destination nodes.

Proposition 3.1. In a fully connected ad-hoc network with single-antenna source and destination nodes, relay-aided IA is feasible if and only if each invalid IN solution subset $\mathbb{S}_{\mathrm{inv}}^{(k)}$ is a strict subspace of the IN solution space $\mathbb{S}_{\mathrm{IN}}$, i.e.,

$$\dim \mathbb{S}_{\mathrm{inv}}^{(k)} = \dim \mathbb{S}_{\mathrm{IN}} - 1 \tag{3.24}$$

holds for all $k = 1, \ldots, K$.

Proof. See Appendix D. □

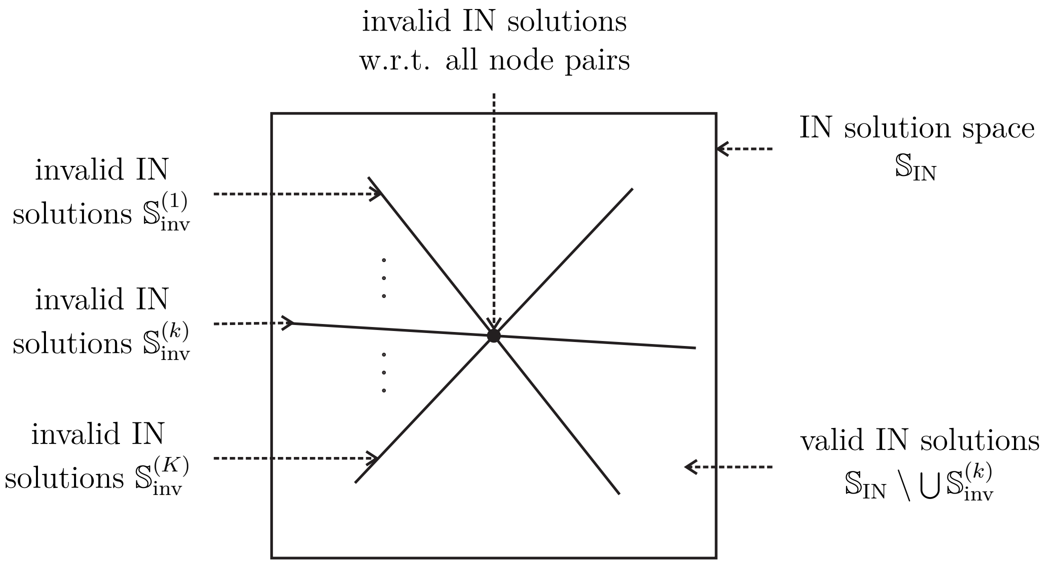


Figure 3.2.: Illustration of the IN solution space $\mathbb{S}_{\mathrm{IN}}$ and the invalid IN solution subsets $\mathbb{S}_{\mathrm{inv}}^{(k)}$ in ad-hoc networks with single-antenna source and destination nodes. In this case, the invalid IN solution subsets are linear subspaces of the IN solution space. Relay-aided IA is feasible if and only if every invalid IN solution subset is a strict subspace, i.e., a hyperplane, of the IN solution space.

In order to use Proposition 3.1 to determine whether or not relay-aided IA is feasible in a fully connected ad-hoc network with some specific configuration, i.e., with a certain number K of node pairs, a number Q of relays, and the numbers M_q of relay antennas, the key is to determine the ranks of the matrices $\mathbf{A}_{\mathrm{IN}}$ and $\mathbf{A}_{\mathrm{inv}}^{(k)}$. In principle, the ranks of these matrices depend on channel realizations. However, if the channel coefficients are assumed to be independently drawn from a continuous distribution, the ranks of these matrices can be determined in the almost sure sense by exploiting the special structures of them.

Firstly, note that the submatrix $\mathbf{A}_{\mathrm{RL}}$ of $\mathbf{A}_{\mathrm{IN}}$ and the submatrix $\begin{bmatrix}\mathbf{A}_{\mathrm{RL}}\\ \mathbf{b}^{(k)\mathrm{T}}\end{bmatrix}$ of $\mathbf{A}_{\mathrm{inv}}^{(k)}$, where $\mathbf{A}_{\mathrm{RL}}$ and $\mathbf{b}^{(k)\mathrm{T}}$ are defined in (3.15) and (3.19), respectively, are related to the Khatri-Rao product. More information on the Kronecker product and the Khatri-Rao product can be found in Appendix A. Exploiting this structure, it can be shown that both matrices $\mathbf{A}_{\mathrm{RL}}$ and $\begin{bmatrix}\mathbf{A}_{\mathrm{RL}}\\ \mathbf{b}^{(k)\mathrm{T}}\end{bmatrix}$ are almost surely of full rank, which is stated by the following proposition.

Proposition 3.2. In a fully connected ad-hoc network with single-antenna source and destination nodes, if the channel coefficients are independently drawn from a continuous distribution, then

$$\operatorname{rank}\left(\mathbf{A}_{\mathrm{RL}}\right) = \min\left\{K(K-1), \sum_{q=1}^{Q} M_q^2\right\} \tag{3.25}$$

and

$$\operatorname{rank}\left(\begin{bmatrix}\mathbf{A}_{\mathrm{RL}}\\ \mathbf{b}^{(k)\mathrm{T}}\end{bmatrix}\right) = \min\left\{K(K-1)+1, \sum_{q=1}^{Q} M_q^2\right\} \tag{3.26}$$

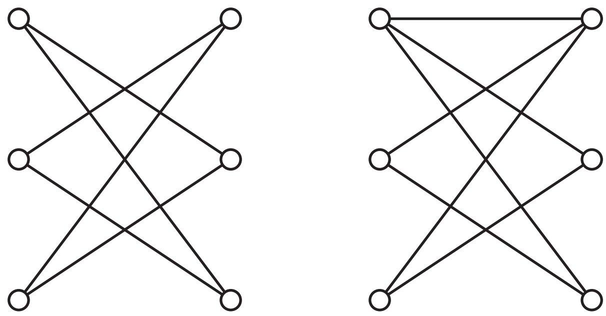
hold with probability one, where $\mathbf{A}_{\mathrm{RL}}$ and $\mathbf{b}^{(k)\mathrm{T}}$ are defined in (3.15) and (3.19), respectively.

Proof. See Appendix D. □

Secondly, the ranks of the submatrix $\mathbf{A}_{\mathrm{DL}}$ of $\mathbf{A}_{\mathrm{IN}}$ and the submatrix $\begin{bmatrix}\mathbf{A}_{\mathrm{DL}}\\ \mathbf{c}^{(k)\mathrm{T}}\end{bmatrix}$ of $\mathbf{A}_{\mathrm{inv}}^{(k)}$, where $\mathbf{A}_{\mathrm{DL}}$ and $\mathbf{c}^{(k)\mathrm{T}}$ are defined in (3.16) and (3.20), respectively, can be determined employing graph theory. A brief introduction on the useful concepts and results from graph theory can be found in Appendix B. Consider a bipartite graph $\mathcal{G} = (\mathcal{V}_{\mathrm{S}}, \mathcal{V}_{\mathrm{D}}, \mathcal{E})$ for a given fully connected ad-hoc network. The two subsets $\mathcal{V}_{\mathrm{S}}$ and $\mathcal{V}_{\mathrm{D}}$ of vertices of $\mathcal{G}$ correspond to the source and destination nodes in the considered network, respectively. For all $k \neq j$, the j-th vertex in $\mathcal{V}_{\mathrm{S}}$ and the k-th vertex in $\mathcal{V}_{\mathrm{D}}$, i.e., the j-th source node and the k-th destination node in the considered network, are adjacent via an edge in $\mathcal{E}$. However, a source node and its corresponding destination node are nonadjacent via an edge in $\mathcal{E}$. For example, the diagram of $\mathcal{G}$ for a fully-connected ad-hoc network with three source-destination node pairs is shown in Figure 3.3a, and the incidence matrix $\mathbf{\Psi}_{\mathcal{G}}$ of $\mathcal{G}$ is given by

$$\mathbf{\Psi}_{\mathcal{G}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}. \tag{3.27}$$

For a fully-connected ad-hoc network with $K \geq 3$ node pairs, the graph $\mathcal{G}$ must be a connected graph on $2K$ vertices. Therefore, a spanning tree of $\mathcal{G}$, whose edges correspond to a basis of the row space of the incidence matrix $\mathbf{\Psi}_{\mathcal{G}}$, must



(a) $\mathcal{G}$ (b) $\mathcal{G}_1$

Figure 3.3.: Bipartite graphs (a) $\mathcal{G}$ and (b) $\mathcal{G}_1$ for a fully-connected ad-hoc network with $K = 3$ source-destination node pairs, which are both connected graphs on $2K = 6$ vertices. Consequently, the incidence matrices of $\mathcal{G}$ and $\mathcal{G}_1$ are both of rank $2K - 1 = 5$.

have $2K-1$ edges. Hence, the rank of $\mathbf{\Psi}_{\mathcal{G}}$ is $2K-1$. Furthermore, note that the matrix $\mathbf{A}_{\text{DL}}$ is obtained by scaling each row of $\mathbf{\Psi}_{\mathcal{G}}$ by the corresponding channel coefficient $h_{\text{DS}}^{(k,j)}$, i.e.,

$$\mathbf{A}_{\text{DL}} = \text{diag}\left(\left[h_{\text{DS}}^{(2,1)} \; \cdots \; h_{\text{DS}}^{(K,1)} \; \cdots \; \cdots \; h_{\text{DS}}^{(1,K)} \; \cdots \; h_{\text{DS}}^{(K-1,K)}\right]\right) \mathbf{\Psi}_{\mathcal{G}} \tag{3.28}$$

holds, where $\text{diag}(\cdot)$ denotes a diagonal matrix. Since all the channel coefficients $h_{\text{DS}}^{(k,j)}$ are almost surely non zero in a fully connected network, the rank of $\mathbf{A}_{\text{DL}}$ is almost surely equal to the rank of $\mathbf{\Psi}_{\mathcal{G}}$, i.e., is almost surely $2K-1$ too. Moreover, let the graph $\mathcal{G}_i$, $i = 1, \ldots, K$, be obtained by adding to the graph $\mathcal{G}$ a single edge between the k-th source-destination node pair. For example, the graph $\mathcal{G}_1$ in a fully connected ad-hoc network with three source-destination node pairs is shown in Figure 3.3b. Note that the incidence matrix $\mathbf{\Psi}_{\mathcal{G}_i}$ of the graph $\mathcal{G}_i$ has the same structure as the matrix $\begin{bmatrix}\mathbf{A}_{\text{DL}} \\ \mathbf{c}^{(k)\text{T}}\end{bmatrix}$. For the same reason as discussed before, the matrix $\begin{bmatrix}\mathbf{A}_{\text{DL}} \\ \mathbf{c}^{(k)\text{T}}\end{bmatrix}$ is almost surely of rank $2K-1$ as well.

Proposition 3.3. In a fully connected ad-hoc network with $K \geq 3$ single-antenna source and destination nodes, if the channel coefficients are independently drawn from a continuous distribution, then

$$\text{rank}\left(\mathbf{A}_{\text{DL}}\right) = \text{rank}\begin{bmatrix}\mathbf{A}_{\text{DL}} \\ \mathbf{c}^{(k)\text{T}}\end{bmatrix} = 2K-1 \tag{3.29}$$

holds with probability one, where $\mathbf{A}_{\text{DL}}$ and $\mathbf{c}^{(k)\text{T}}$ are defined in (3.16) and (3.20), respectively.

Proof. See above discussion. □

Finally, the column space of $\mathbf{A}_{\rm IN}$ is spanned by the column spaces of $\mathbf{A}_{\rm RL}$ and $\mathbf{A}_{\rm DL}$, both of which have $K(K-1)$ rows. Importantly, the entries of $\mathbf{A}_{\rm RL}$ and $\mathbf{A}_{\rm DL}$ are independent. Hence, it can be concluded that the column spaces of $\mathbf{A}_{\rm RL}$ and $\mathbf{A}_{\rm DL}$ are almost surely disjoint, i.e., they only intersect at the origin, as long as the sum of their dimensions does not exceed $K(K-1)$. A similar conclusion holds for the column spaces of $\begin{bmatrix}\mathbf{A}_{\rm RL}\\ \mathbf{b}^{(k)\rm T}\end{bmatrix}$ and $\begin{bmatrix}\mathbf{A}_{\rm DL}\\ \mathbf{c}^{(k)\rm T}\end{bmatrix}$. Based on these, the ranks of $\mathbf{A}_{\rm IN}$ and $\mathbf{A}_{\rm inv}^{(k)}$ can be determined exploiting the results on the ranks of the matrices $\mathbf{A}_{\rm RL}$, $\mathbf{A}_{\rm DL}$, $\begin{bmatrix}\mathbf{A}_{\rm RL}\\ \mathbf{b}^{(k)\rm T}\end{bmatrix}$, and $\begin{bmatrix}\mathbf{A}_{\rm DL}\\ \mathbf{c}^{(k)\rm T}\end{bmatrix}$ in Proposition 3.2 and Proposition 3.3.

Proposition 3.4. In a fully connected ad-hoc network with $K \geq 3$ single-antenna source and destination nodes, if the channel coefficients are independently drawn from a continuous distribution, then

$$\operatorname{rank}\left(\mathbf{A}_{\rm IN}\right) = \min\left\{K(K-1), \sum_{q=1}^{Q} M_q^2 + 2K - 1\right\}, \tag{3.30}$$

and

$$\operatorname{rank}\left(\mathbf{A}_{\rm inv}^{(k)}\right) = \min\left\{K(K-1)+1, \sum_{q=1}^{Q} M_q^2 + 2K - 1\right\}, \quad \forall k = 1, \ldots, K, \tag{3.31}$$

hold with probability one, where the matrices $\mathbf{A}_{\rm IN}$ and $\mathbf{A}_{\rm inv}^{(k)}$ are defined in (3.15) and (3.21), respectively.

Proof. See Appendix D. □

Then, a proper relation between the number Q of relays, the number M_q of relay antennas, and the number K of node pairs such that relay-aided IA is almost surely feasible can be given as follows.

Proposition 3.5. In a fully-connected ad-hoc network with $K \geq 3$ single-antenna source and destination nodes, if the channel coefficients are independently drawn from a continuous distribution, then relay-aided IA is feasible with probability one if and only if

$$\dim \mathbb{S}_{\rm IN} = \max\left\{\sum_{q=1}^{Q} M_q^2 - K(K-3), 1\right\} \geq 2 \tag{3.32}$$

or equivalently

$$\sum_{q=1}^{Q} M_q^2 - K(K-3) - 2 \geq 0 \tag{3.33}$$

holds.

Proof. This can be directly deduced from Proposition 3.1 and Proposition 3.4. □

Regarding the IN solution space $\mathbb{S}_{\mathrm{IN}}$ and the feasibility conditions for relay-aided IA in fully connected ad-hoc networks with single-antenna source and destination nodes, a few remarks need to be further discussed.

Remark 3.2. By Proposition 3.4, $\mathrm{rank}\,(\mathbf{A}_{\mathrm{IN}}) \leq \sum_{q=1}^{Q} M_q^2 + 2K - 1$ holds. Therefore, $\dim \mathbb{S}_{\mathrm{IN}} \geq 1$ follows. That is to say, for relay-aided IA, some non-trivial IN solutions always exist in the IN solution space $\mathbb{S}_{\mathrm{IN}}$. These non-trivial IN solutions are the scaled versions of the IN solution with $\mathbf{G}^{(q)} = \mathbf{0}_{M_q}$, $\forall q$, and $v^{(k)} = -u^{(j)*} = 1$, $\forall j, k$, which correspond to the invalid IN solutions being discussed in Example 3.3 in Section 3.2.

Remark 3.3. If relay-aided IA is feasible, the IN solution space $\mathbb{S}_{\mathrm{IN}}$ must have at least two dimensions. Moreover, the invalid IN solution subsets $\mathbb{S}_{\mathrm{inv}}^{(k)}$ are almost surely hyperplanes of the IN solution space $\mathbb{S}_{\mathrm{IN}}$. That is to say, for the special case where the IN solution space $\mathbb{S}_{\mathrm{IN}}$ is exactly two-dimensional, all the invalid IN solution subspaces are almost surely the one-dimensional subspace spanned by the invalid IN solution with $\mathbf{G}^{(q)} = \mathbf{0}_{M_q}$, $\forall q$, and $v^{(k)} = -u^{(j)*} = 1$, $\forall j, k$, and all the other IN solutions in $\mathbb{S}_{\mathrm{IN}}$ are almost surely valid.

Remark 3.4. If relay-aided IA is feasible, an IN solution which is randomly picked from the IN solution space $\mathbb{S}_{\mathrm{IN}}$ is almost surely a valid one following from Proposition 3.1.

Remark 3.5. In fully connected ad-hoc networks, the number Q of relays and the number M_q of relay antennas required for relay-aided IA grow rapidly with the number K of node pairs. In large networks with lots of node pairs, effective methods to reduce the required numbers of relays and relay antennas are beneficial. Such a method will be proposed in Chapter 5 exploiting partial connectivity.

3.4.2. Multi-antenna source and destination nodes

In this section, the case where each source and destination node has $N \geq 2$ antennas is considered. As compared to the single-antenna case, having more than one antenna at the source and destination nodes causes the following three major differences. Firstly, in the multi-antenna case, some IA solutions, in spite of being less relevant, cannot be mapped to a valid IN solution in $\mathbb{S}_{\text{IN}} \setminus \bigcup \mathbb{S}_{\text{inv}}^{(k)}$ using the proposed linearization approach. Therefore, strictly speaking, the feasibility condition to be derived in this subsection is only a sufficient condition for the multi-antenna case. Secondly, although the IN solution space $\mathbb{S}_{\text{IN}}$ can still be represented as the null space of a matrix $\mathbf{A}_{\text{IN}}$ in the multi-antenna case, the structure of $\mathbf{A}_{\text{IN}}$ is more complicated as compared to the single-antenna case, which also influences the rank of $\mathbf{A}_{\text{IN}}$ as well as the dimension of $\mathbb{S}_{\text{IN}}$. Thirdly, the reformulated validity conditions (3.10) are non-linear in the multi-antenna case, which results in that the invalid IN solution subsets $\mathbb{S}_{\text{inv}}^{(k)}$ are no longer linear subspaces of the IN solution space $\mathbb{S}_{\text{IN}}$. Due to these reasons, the discussion in this subsection follows a slightly different line as compared to Subsection 3.4.1.

In the multi-antenna case, let the variables of the linearized IN conditions (3.9), i.e., the relay processing filters $\mathbf{G}^{(q)}$ and the matrices $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$, be stacked in a $(\sum_{q=1}^{Q} M_q^2 + 2KN^2) \times 1$ vector as

$$\mathbf{x} = \text{vec}\left(\begin{bmatrix}\mathbf{G}^{(1)} & \cdots & \mathbf{G}^{(Q)} & \mathbf{V}^{(1)} & \cdots & \mathbf{V}^{(K)} & \mathbf{U}^{(1)*\text{T}} & \cdots & \mathbf{U}^{(K)*\text{T}}\end{bmatrix}\right). \tag{3.34}$$

Thus the matrix form of the system of linear equations consisting of all the linearized IN conditions (3.9) can be written as

$$\mathbf{A}_{\text{IN}}\mathbf{x} = \begin{bmatrix}\mathbf{A}_{\text{RL}} & \mathbf{A}_{\text{DL},1} & \mathbf{A}_{\text{DL},2}\end{bmatrix}\mathbf{x} = \mathbf{0}, \tag{3.35}$$

where $\mathbf{A}_{\text{IN}}$ is interpreted as a partitioned matrix consisting of the $K(K-1)N^2 \times$

$\sum_{q=1}^{Q} M_q^2$ matrix

$$\mathbf{A}_{\mathrm{RL}} = \begin{bmatrix} \mathbf{H}_{\mathrm{RS}}^{(1,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(1,2)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RS}}^{(Q,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(Q,2)*\mathrm{T}} \\ \mathbf{H}_{\mathrm{RS}}^{(1,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(1,3)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RS}}^{(Q,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(Q,3)*\mathrm{T}} \\ \vdots & & \vdots \\ \mathbf{H}_{\mathrm{RS}}^{(1,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(1,K)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RS}}^{(Q,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(Q,K)*\mathrm{T}} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \mathbf{H}_{\mathrm{RS}}^{(1,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(1,1)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RS}}^{(Q,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(Q,1)*\mathrm{T}} \\ \mathbf{H}_{\mathrm{RS}}^{(1,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(1,2)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RS}}^{(Q,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(Q,2)*\mathrm{T}} \\ \vdots & & \vdots \\ \mathbf{H}_{\mathrm{RS}}^{(1,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(1,K-1)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RS}}^{(Q,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RD}}^{(Q,K-1)*\mathrm{T}} \end{bmatrix}, \tag{3.36}$$

the $K(K-1)N^2 \times KN^2$ matrix

$$\mathbf{A}_{\mathrm{DL},1} = \begin{bmatrix} \mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(2,1)} & 0 & \cdots & 0 \\ \mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(3,1)} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(K,1)} & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & 0 & \mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(1,K)} \\ 0 & \cdots & 0 & \mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(2,K)} \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & \mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(K-1,K)} \end{bmatrix}, \tag{3.37}$$

and the $K(K-1)N^2 \times KN^2$ matrix

$$\mathbf{A}_{\mathrm{DL},2} = \begin{bmatrix} 0 & \mathbf{H}_{\mathrm{DS}}^{(2,1)\mathrm{T}} \otimes \mathbf{I}_N & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{H}_{\mathrm{DS}}^{(3,1)\mathrm{T}} \otimes \mathbf{I}_N & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \mathbf{H}_{\mathrm{DS}}^{(K,1)\mathrm{T}} \otimes \mathbf{I}_N \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \mathbf{H}_{\mathrm{DS}}^{(1,K)\mathrm{T}} \otimes \mathbf{I}_N & 0 & \cdots & 0 & 0 \\ 0 & \mathbf{H}_{\mathrm{DS}}^{(2,K)\mathrm{T}} \otimes \mathbf{I}_N & \cdots & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathbf{H}_{\mathrm{DS}}^{(K-1,K)\mathrm{T}} \otimes \mathbf{I}_N & 0 \end{bmatrix}. \tag{3.38}$$

Then the IN solution space can be given by

$$\mathbb{S}_{\mathrm{IN}} = \mathrm{null}\left(\mathbf{A}_{\mathrm{IN}}\right), \tag{3.39}$$

and its dimension reads

$$\dim \mathbb{S}_{\mathrm{IN}} = \sum_{q=1}^{Q} M_q^2 + 2KN^2 - \mathrm{rank}\left(\mathbf{A}_{\mathrm{IN}}\right). \tag{3.40}$$

In the multi-antenna case, the matrix $\mathbf{A}_{\mathrm{RL}}$ also has a structure that is related to the Khatri-Rao product. Exploiting this, it can be shown that $\mathbf{A}_{\mathrm{RL}}$ is almost surely of full rank if the channel coefficients are independently drawn from a continuous distribution. The matrix $\left[\mathbf{A}_{\mathrm{DL},1}\ \ \mathbf{A}_{\mathrm{DL},2}\right]$ can be considered as an expanded version of the matrix $\mathbf{A}_{\mathrm{DL}}$ in the single-antenna case in the sense that the non-zero entries $h_{\mathrm{DS}}^{(k,j)}$ of $\mathbf{A}_{\mathrm{DL}}$ are replaced by the blocks $\mathbf{I}_N \otimes \mathbf{H}_{\mathrm{DS}}^{(k,j)}$ in $\mathbf{A}_{\mathrm{DL},1}$ and $\mathbf{H}_{\mathrm{DS}}^{(k,j)\mathrm{T}} \otimes \mathbf{I}_N$ in $\mathbf{A}_{\mathrm{DL},2}$, respectively. In spite of this similarity, the rank of $\left[\mathbf{A}_{\mathrm{DL},1}\ \ \mathbf{A}_{\mathrm{DL},2}\right]$ cannot be determined in the same way as the single-antenna case employing graph theory, because $\left[\mathbf{A}_{\mathrm{DL},1}\ \ \mathbf{A}_{\mathrm{DL},2}\right]$ has more than two non-zero entries in each row. Instead, properties of commuting matrices[3] will be exploited to find the rank of $\left[\mathbf{A}_{\mathrm{DL},1}\ \ \mathbf{A}_{\mathrm{DL},2}\right]$. Taking all these similarities and differences into consideration, the rank of $\mathbf{A}_{\mathrm{IN}}$ in the multi-antenna case can be given by the following proposition, which generalizes the result of Proposition 3.4.

[3] Two square matrices $\mathbf{A}$ and $\mathbf{B}$ are said to commute if $\mathbf{AB} = \mathbf{BA}$ holds [HJ90].

Proposition 3.6. In a fully connected ad-hoc network with $K \geq 3$ source and destination node pairs, where each source and destination node is equipped with N antennas, if the channel coefficients are independently drawn from a continuous distribution, then

$$\text{rank}\left(\mathbf{A}_{\text{IN}}\right) = \begin{cases} \min\left\{6N^2, \sum_{q=1}^{Q} M_q^2 + 6N^2 - N\right\} & \text{for } K = 3 \\ \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2 + 2KN^2 - 1\right\} & \text{for } K > 3 \end{cases} \tag{3.41}$$

holds with probability one, where the matrix $\mathbf{A}_{\text{IN}}$ is defined in (3.35).

Proof. See Appendix D. □

In the multi-antenna case, the reformulated validity conditions (3.10) are nonlinear in the variables $\mathbf{G}^{(q)}$, $\mathbf{V}^{(j)}$, and $\mathbf{U}^{(k)}$. Thus the invalid IN solution subsets $\mathbb{S}_{\text{inv}}^{(k)}$ are no longer linear subspaces of the IN solution space $\mathbb{S}_{\text{IN}}$. In fact, each $\mathbb{S}_{\text{inv}}^{(k)}$ is the intersection of the IN solution space $\mathbb{S}_{\text{IN}}$ and the solution set of the polynomial equation

$$p_k = \det\left(\sum_{q=1}^{Q} \mathbf{H}_{\text{RD}}^{(q,k)*\text{T}} \mathbf{G}^{(q)} \mathbf{H}_{\text{RS}}^{(q,k)} + \mathbf{H}_{\text{DS}}^{(k,k)} \mathbf{V}^{(k)} + \mathbf{U}^{(k)*\text{T}} \mathbf{H}_{\text{DS}}^{(k,k)}\right) = 0. \tag{3.42}$$

In other words, $\mathbb{S}_{\text{inv}}^{(k)}$ is an algebraic set following the definition on this subject in [Har77]. More importantly, each $\mathbb{S}_{\text{inv}}^{(k)}$ could be neither a hypersurface of $\mathbb{S}_{\text{IN}}$ nor equal to the entire $\mathbb{S}_{\text{IN}}$[4]. This complicates the problem of directly characterizing the properties, e.g., the dimension, of $\mathbb{S}_{\text{inv}}^{(k)}$, and then using these properties to derive the feasibility conditions. In the following, an alternative approach will be considered. For a given IN solution $\mathbf{x} \in \mathbb{S}_{\text{IN}}$, the polynomial p_k as given in (3.42) can be considered as a polynomial $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ of the channel matrix $\mathbf{H}_{\text{DS}}^{(k,k)}$. If an IN solution $\mathbf{x}$ yields that $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ is a trivial polynomial, this IN solution must be an invalid IN solution with respect to the k-th node pair, i.e., $\mathbf{x} \in \mathbb{S}_{\text{inv}}^{(k)}$, regardless of the channel realization of $\mathbf{H}_{\text{DS}}^{(k,k)}$. Otherwise, if an IN solution $\mathbf{x}$ yields that $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ is a non-trivial polynomial in $\mathbf{H}_{\text{DS}}^{(k,k)}$, this IN solution is almost surely not an invalid IN solution with respect to the k-th node pair, since $\mathbf{H}_{\text{DS}}^{(k,k)}$

[4] Although it can be proved that the solution set of the polynomial equation $p_k = 0$ is an irreducible algebraic set, i.e., a single hypersurface rather than the union of multiple hypersurfaces, the intersection of two irreducible algebraic sets, i.e., the intersection of the solution set of $p_k = 0$ and the IN solution space $\mathbb{S}_{\text{IN}}$ in the case considered here, is not necessarily an irreducible algebraic set [Har77].

is random and independent of the IN solution $\mathbf{x}$. In fact, whether or not there exists an IN solution $\mathbf{x}$ such that $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$ a non-trivial polynomial strongly depends on the dimension of the IN solution space $\mathbb{S}_{\mathrm{IN}}$. These results are stated in the following proposition.

Proposition 3.7. In a fully-connected ad-hoc network with $K \geq 3$ source and destination node pairs, where each source and destination node is equipped with N antennas, if the channel coefficients are independently drawn from a continuous distribution, then the following two statements hold true.

(1) If the IN solution space $\mathbb{S}_{\mathrm{IN}}$ is one-dimensional, every IN solution nullifies the polynomial p_k.
(2) If the IN solution space $\mathbb{S}_{\mathrm{IN}}$ has at least two dimensions, a randomly picked IN solution nullifies the polynomial p_k with probability zero.

Proof. See Appendix D. □

Following from Proposition 3.6 and Proposition 3.7, the feasibility conditions for relay-aided IA in the multi-antenna case based on the proposed linearization approach can be given.

Proposition 3.8. In a fully connected ad-hoc network with $K \geq 3$ source and destination node pairs, where each source and destination node is equipped with $N \geq 2$ antennas, if the channel coefficients are independently drawn from a continuous distribution, then relay-aided IA based on the proposed linearization approach is almost surely feasible if and only if

$$\dim \mathbb{S}_{\mathrm{IN}} \geq 2 \tag{3.43}$$

holds, or equivalently,

(a) $K = 3$, or
(b) $\sum_{q=1}^{Q} M_q^2 - K(K-3)N^2 - 2 \geq 0$ for $K > 3$

holds.

Proof. This can be directly deduced from Proposition 3.6 and Proposition 3.7. □

If the source and the destination nodes have more than one antenna, the IN solution space $\mathbb{S}_{\mathrm{IN}}$ and the feasibility conditions for relay-aided IA in fully-connected ad-hoc networks have a few major differences as compared to those in the single-antenna case, especially for the networks with exactly $K = 3$ source-destination node pairs.

Remark 3.6. If the network has exactly $K = 3$ source-destination node pairs, the IN solution space $\mathbb{S}_{\mathrm{IN}}$ has at least N dimensions. The reason is that the null space of the matrix $\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix}$ is N dimensional due to its special structure. For $K > 3$, the null space of the matrix $\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix}$ is always one dimensional, regardless of the number N of antennas at the source and destination nodes. In both cases, the one-dimensional subspace spanned by the non-trivial invalid IN solution with $\mathbf{G}^{(q)} = \mathbf{0}_{M_q}$, $\forall q$, and $\mathbf{V}^{(j)} = -\mathbf{U}^{(k)*\mathrm{T}} = \mathbf{I}_N$, $\forall j, k$, which corresponds to the invalid IN solutions being discussed in Example 3.3 in Section 3.2, always exists in the IN solution space $\mathbb{S}_{\mathrm{IN}}$.

Remark 3.7. If the network has exactly $K = 3$ source-destination node pairs and each source and destination node has $N \geq 2$ antennas, relay-aided IA is almost surely feasible regardless of the number Q of relays and the number M_q of relay antennas. In the extreme case without the deployment of relays, the considered two-hop transmission scheme is equivalent to transmitting twice over a constant MIMO IC, namely, the effective channel matrix becomes

$$\begin{aligned}\mathbf{H}_{\mathrm{eff}}^{(k,j)} &= \begin{bmatrix}\mathbf{U}_1^{(k)*\mathrm{T}} & \mathbf{U}_2^{(k)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{DS}}^{(k,j)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{\mathrm{DS}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)}\end{bmatrix} \\ &= \mathbf{U}_2^{(k)*\mathrm{T}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,j)}\mathbf{V}^{(j)} + \mathbf{U}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(k,j)}\right)\mathbf{V}_1^{(j)}.\end{aligned} \tag{3.44}$$

Therefore, only the matrices $\mathbf{V}^{(j)}$ and $\mathbf{U}^{(k)}$ shall be considered as the variables of the linearized IN conditions. Correspondingly, the IN solution space shall be given by

$$\mathbb{S}_{\mathrm{IN}} = \mathrm{null}\left(\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix}\right). \tag{3.45}$$

Since the IN solution space $\mathbb{S}_{\mathrm{IN}}$ is N-dimensional, a randomly picked IN solution is almost surely a valid one according to the discussions in this subsection. That is to say, the proposed IA scheme provides an alternative closed-form solution to achieve the DoF of the three-user constant MIMO IC with $N \geq 2$ antennas at each node.

3.5. Performance optimization

3.5.1. Performance measure

From any valid IN solution in the IN solution space $\mathbb{S}_{\mathrm{IN}}$, the transmit and receive filters can be constructed with a set of full-rank matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$, which can be designed aiming at performance optimization without influencing the achievable DoF. In this section, the optimizations of the matrices $\mathbf{V}_1^{(j)}$ and $\mathbf{U}_2^{(k)}$ will be carried out based on a given valid IN solution. The problem of how to find the "best" valid IN solution is out of the scope of this thesis.

The achievable sum rate in the entire network will be considered as the performance measure. Employing the two-hop transmission scheme, the achievable sum rate R_{sum} has been given in (2.11) in Section 2.1. For a valid IN solution, if the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)}\end{bmatrix}$ and the receive filters $\begin{bmatrix}\mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)}\end{bmatrix}$ are constructed using (3.6), the interferences can be perfectly nullified. That is to say, the entire network can be converted into K non-interfering MIMO channels. Then, R_{sum} can be simplified as

$$R_{\mathrm{sum}} = \frac{1}{2}\sum_{k=1}^{K}\log_2\left(\frac{\det\left(\mathbf{S}_{\mathrm{noise}}^{(k)} + \mathbf{H}_{\mathrm{eff}}^{(k,k)}\mathbf{H}_{\mathrm{eff}}^{(k,k)*\mathrm{T}}\right)}{\det\left(\mathbf{S}_{\mathrm{noise}}^{(k)}\right)}\right), \tag{3.46}$$

where

$$\mathbf{H}_{\mathrm{eff}}^{(k,k)} = \mathbf{U}_2^{(k)*\mathrm{T}}\left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,k)} + \mathbf{H}_{\mathrm{DS}}^{(k,k)}\mathbf{V}^{(k)} + \mathbf{U}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)\mathbf{V}_1^{(k)} \tag{3.47}$$

is the effective channel matrix between the k-th source-destination node pair, and

$$\mathbf{S}_{\mathrm{noise}}^{(k)} = \mathbf{U}_2^{(k)*\mathrm{T}}\left(\sigma_{\mathrm{D}}^2\mathbf{I}_N + \sigma_{\mathrm{D}}^2\mathbf{U}^{(k)*\mathrm{T}}\mathbf{U}^{(k)} + \sigma_{\mathrm{R}}^2\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{G}^{(q)*\mathrm{T}}\mathbf{H}_{\mathrm{RD}}^{(q,k)}\right)\mathbf{U}_2^{(k)} \tag{3.48}$$

is the covariance matrix of the effective noise at the k-th destination node.

In a given valid IN solution, the matrix $\mathbf{V}^{(j)}$ fully determines the ratio of the transmit powers of the j-th source node in the first and second time slot, see (3.6). That is to say, each source node usually has to transmit with different powers in the two time slots in order to achieve relay-aided IA. Furthermore, the relays only transmit in the second time slot. Therefore, it is more reasonable to consider energy constraints rather than power constraints for a fair comparison with other interference management approaches. Recall that the data symbols $\mathbf{d}^{(j)}$ are assumed to be i.i.d. Gaussian symbols with unit variance, see Section 2.1. Then the sum transmit power $P_{\mathrm{S}}^{(j)}$ of the j-th source node, which can be interpreted as the transmit energy consumed by a source node in two time slots divided by the duration of a single time slot as defined in (2.8), can be rewritten using the IN solution as

$$\begin{aligned} P_{\mathrm{S}}^{(j)} &= \mathrm{tr}\left(\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}} + \mathbf{V}_2^{(j)}\mathbf{V}_2^{(j)*\mathrm{T}}\right) \\ &= \mathrm{tr}\left(\mathbf{V}_1^{(j)*\mathrm{T}}\left(\mathbf{I}_N + \mathbf{V}^{(j)*\mathrm{T}}\mathbf{V}^{(j)}\right)\mathbf{V}_1^{(j)}\right). \end{aligned} \tag{3.49}$$

The transmit power $P_{\mathrm{R}}^{(q)}$ of the q-th relay in the second time slot, which has been defined in (2.9), can be written as

$$\begin{aligned} P_{\mathrm{R}}^{(q)} &= \mathrm{tr}\left(\mathrm{E}\left\{\mathbf{G}^{(q)}\mathbf{e}_{\mathrm{R}}^{(q)}\mathbf{e}_{\mathrm{R}}^{(q)*\mathrm{T}}\mathbf{G}^{(q)*\mathrm{T}}\right\}\right) \\ &= \mathrm{tr}\left(\mathbf{G}^{(q)}\left(\sum_{j=1}^{K}\mathbf{H}_{\mathrm{RS}}^{(q,j)}\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{RS}}^{(q,j)*\mathrm{T}} + \sigma_{\mathrm{R}}^2\mathbf{I}_{M_q}\right)\mathbf{G}^{(q)*\mathrm{T}}\right) \\ &= \sum_{j=1}^{K}\mathrm{tr}\left(\mathbf{V}_1^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{RS}}^{(q,j)*\mathrm{T}}\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)}\mathbf{V}_1^{(j)}\right) + \sigma_{\mathrm{R}}^2\mathrm{tr}\left(\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\right), \end{aligned} \tag{3.50}$$

where $\mathbf{e}_{\mathrm{R}}^{(q)}$ denotes the signals being received by the q-th relay in the first time slot and $\sigma_{\mathrm{R}}^2\mathbf{I}_{M_q}$ is the covariance matrix of the noise at the q-th relay, see Section 2.1. Thus,

$$P_{\mathrm{tot}} = \sum_{j=1}^{K} P_{\mathrm{S}}^{(j)} + \sum_{q=1}^{Q} P_{\mathrm{R}}^{(q)} \tag{3.51}$$

is the total sum transmit power in the entire network, which can be interpreted as the total transmit energy consumed by all the source nodes and the relays in the entire network divided by the duration of a single time slot.

In the following, the matrices $\mathbf{V}_1^{(j)}$ will be optimized based on a given valid IN solution aiming at maximizing the achievable sum rate R_{sum}. The matrices $\mathbf{U}_2^{(k)}$ do not influence the achievable sum rate R_{sum} and, hence, do not need to be considered as the optimization variables. Furthermore, two different types of sum power constraints, which are essentially energy constraints, will be considered in Subsection 3.5.2 and in Subsection 3.5.3, respectively.

3.5.2. Sum rate maximization under a total sum power constraint

In this subsection, a maximum total sum transmit power $P_{\text{tot,max}}$ is considered for all the source nodes and the relays in the entire network. This type of sum power constraint, which is essentially a total transmit energy constraint, does not suggest that some transmit energy is allowed to be transferred between different source nodes and relays. Instead, it provides a benchmark for the achievable sum rate when the overall network energy consumption is of interest. Thus, the corresponding sum rate maximization problem can be formulated as

$$\underset{\mathbf{V}_1^{(1)},\ldots,\mathbf{V}_1^{(K)}}{\text{maximize}} \quad R_{\text{sum}}, \tag{3.52}$$

$$\text{subject to} \quad \sum_{j=1}^{K} P_{\text{S}}^{(j)} + \sum_{q=1}^{Q} P_{\text{R}}^{(q)} \leq P_{\text{tot,max}}, \tag{3.53}$$

where R_{sum}, $P_{\text{S}}^{(j)}$, and $P_{\text{R}}^{(q)}$ are given in (3.46), (3.49), and (3.50), respectively. It will be shown in the following that in order to solve the above optimization problem, it suffices to optimize the eigenvectors and the eigenvalues of $\mathbf{V}_1^{(j)}$ separately, and a closed-form solution will be given.

Introduce $\widetilde{\mathbf{V}}_1^{(j)}$ as

$$\widetilde{\mathbf{V}}_1^{(j)} = \left(\mathbf{I}_N + \mathbf{V}^{(j)*\text{T}}\mathbf{V}^{(j)} + \sum_{q=1}^{Q} \mathbf{H}_{\text{RS}}^{(q,j)*\text{T}}\mathbf{G}^{(q)*\text{T}}\mathbf{G}^{(q)}\mathbf{H}_{\text{RS}}^{(q,j)}\right)^{\frac{1}{2}} \mathbf{V}_1^{(j)}, \quad \forall j = 1, \ldots, K, \tag{3.54}$$

so that the total sum power constraint (3.53) can be rewritten as

$$\sum_{j=1}^{K} \text{tr}\left(\widetilde{\mathbf{V}}_1^{(j)*\text{T}}\widetilde{\mathbf{V}}_1^{(j)}\right) \leq P_{\text{tot,max}} - \sigma_{\text{R}}^2 \sum_{q=1}^{Q} \text{tr}\left(\mathbf{G}^{(q)*\text{T}}\mathbf{G}^{(q)}\right). \tag{3.55}$$

Furthermore, let the receive filter $\mathbf{U}_2^{(k)}$ be chosen as the pre-whitening filter

$$\mathbf{U}_2^{(k)} = \left(\sigma_{\text{D}}^2\mathbf{I}_N + \sigma_{\text{D}}^2\mathbf{U}^{(k)*\text{T}}\mathbf{U}^{(k)} + \sigma_{\text{R}}^2 \sum_{q=1}^{Q} \mathbf{H}_{\text{RD}}^{(q,k)*\text{T}}\mathbf{G}^{(q)}\mathbf{G}^{(q)*\text{T}}\mathbf{H}_{\text{RD}}^{(q,k)}\right)^{-\frac{1}{2}}, \tag{3.56}$$

so that the covariance matrix $\mathbf{S}_{\text{noise}}^{(k)}$ of the effective noise at each destination node is an identity matrix. Thus, the achievable sum rate R_{sum} can be reformulated using $\widetilde{\mathbf{V}}_1^{(j)}$ and the pre-whitening receive filters in (3.56) as

$$R_{\text{sum}} = \frac{1}{2}\sum_{j=1}^{K} \log_2\left(\det\left(\mathbf{I}_N + \mathbf{A}^{(j)}\widetilde{\mathbf{V}}_1^{(j)}\widetilde{\mathbf{V}}_1^{(j)*\mathrm{T}}\mathbf{A}^{(j)*\mathrm{T}}\right)\right), \tag{3.57}$$

where the matrix $\mathbf{A}^{(j)}$ is given by

$$\begin{aligned}\mathbf{A}^{(j)} = &\left(\sigma_{\mathrm{D}}^2\mathbf{I}_N + \sigma_{\mathrm{D}}^2\mathbf{U}^{(k)*\mathrm{T}}\mathbf{U}^{(k)} + \sigma_{\mathrm{R}}^2\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{G}^{(q)*\mathrm{T}}\mathbf{H}_{\mathrm{RD}}^{(q,k)}\right)^{-\frac{1}{2}} \\ &\cdot\left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,j)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)} + \mathbf{H}_{\mathrm{DS}}^{(j,j)}\mathbf{V}^{(j)} + \mathbf{U}^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(j,j)}\right) \\ &\cdot\left(\mathbf{I}_N + \mathbf{V}^{(j)*\mathrm{T}}\mathbf{V}^{(j)} + \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RS}}^{(q,j)*\mathrm{T}}\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)}\right)^{-\frac{1}{2}}.\end{aligned} \tag{3.58}$$

Clearly, the maximizing $\widetilde{\mathbf{V}}_1^{(j)}$ can be found via optimal power allocation on the eigenmodes of $\mathbf{A}^{(j)}$ using the well-known "water-filling" algorithm [Gol05]. Let $\lambda_n^{(j)}$, $\forall n = 1, \ldots, N$, be the eigenvalues of the Hermitian matrix $\mathbf{A}^{(j)*\mathrm{T}}\mathbf{A}^{(j)}$ and the columns of the unitary matrix $\mathbf{W}^{(j)}$ be the corresponding eigenvectors. Thus, the optimum $\widetilde{\mathbf{V}}_1^{(j)}$ reads

$$\widetilde{\mathbf{V}}_{1,\text{opt}}^{(j)} = \mathbf{W}^{(j)}\begin{bmatrix}\sqrt{s_1^{(j)}} & & \\ & \ddots & \\ & & \sqrt{s_N^{(j)}}\end{bmatrix}, \quad \forall j = 1, \ldots, K, \tag{3.59}$$

where

$$s_n^{(j)} = \max\left\{0, S_{\mathrm{W}} - \lambda_n^{(j)-1}\right\}, \quad \forall n = 1, \ldots, N,\ \forall j = 1, \ldots, K, \tag{3.60}$$

and S_{W} is chosen such that $\sum_{j=1}^{K}\sum_{n=1}^{N} s_n^{(j)} = P_{\text{tot,max}} - \sigma_{\mathrm{R}}^2\sum_{q=1}^{Q}\operatorname{tr}\left(\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\right)$ holds. Finally, the optimum $\mathbf{V}_1^{(j)}$ is given by

$$\mathbf{V}_{1,\text{opt}}^{(j)} = \left(\mathbf{I}_N + \mathbf{V}^{(j)*\mathrm{T}}\mathbf{V}^{(j)} + \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RS}}^{(q,j)*\mathrm{T}}\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)}\right)^{-\frac{1}{2}}\widetilde{\mathbf{V}}_{1,\text{opt}}^{(j)}, \tag{3.61}$$

and the optimum R_{sum} reads

$$R_{\text{sum,opt}} = \frac{1}{2}\sum_{j=1}^{K}\sum_{n=1}^{N}\max\left\{0, \log_2\left(S_{\mathrm{W}}\lambda_n^{(j)}\right)\right\}. \tag{3.62}$$

Remark 3.8. It shall be noted that $\operatorname{tr}\left(\widetilde{\mathbf{V}}_1^{(j)*\mathrm{T}}\widetilde{\mathbf{V}}_1^{(j)}\right)$ is non-negative. Consequently, $P_{\mathrm{tot,max}} \geq \sigma_{\mathrm{R}}^2 \sum_{q=1}^{Q} \operatorname{tr}\left(\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\right)$ must hold, otherwise the feasible region of the considered sum rate maximization problem is an empty set. This can be explained as follows. The AF relays have to consume some energy to forward the received noise, as long as they are operating. The amount of energy required for this depends on the given IN solution, i.e., depends on $\sigma_{\mathrm{R}}^2 \sum_{q=1}^{Q} \operatorname{tr}\left(\mathbf{G}^{(q)*\mathrm{T}}\mathbf{G}^{(q)}\right)$. If the available transmit energy is too small, the given IN solution is not applicable. Nevertheless, one can always choose another valid IN solution, e.g., scale down the given IN solution, such that less energy is required to forward the relay noise.

Remark 3.9. If the total sum transmit power constraint $P_{\mathrm{tot,max}}$ is not sufficiently large, the "water-filling" algorithm may yield that some eigenmodes of the channel are not used, i.e., some $s_n^{(j)}$ may be zero. That is to say, the corresponding transmit filters are rank deficient. However, this will not occur if $P_{\mathrm{tot,max}}$ goes to infinity, and hence does not contradict with the results on the achievable DoF of relay-aided IA.

3.5.3. Sum rate maximization under individual sum power constraints

In this subsection, a maximum sum transmit power $P_{\mathrm{S,max}}^{(j)}$ is considered for the transmissions of each source node, and a maximum transmit power $P_{\mathrm{R,max}}^{(q)}$ is considered for the transmission of each relay in the second time slot. These sum power constraints, which are essentially individual transmit energy constraints, are motivated by the fact that the different source nodes and relays usually have individual energy supplies in wireless scenarios. However, the maximization of the sum rate R_{sum} over $\mathbf{V}_1^{(j)}$ subject to these individual sum power constraints is a non-convex problem, since the eigenvalues and the eigenvectors of $\mathbf{V}_1^{(j)}$ have to be jointly optimized. To the best knowledge of the author of this thesis, an effective method to solve this problem is yet unknown[5]. In the following, a simpler problem will be considered instead, where the eigenvectors of $\mathbf{V}_1^{(j)}$ are chosen according to the given IN solution and only the eigenvalues of $\mathbf{V}_1^{(j)}$ need to be optimized. The simplified problem is a convex problem which can be solved numerically

[5] This problem is closely related to the problem on the capacity of MIMO channels under total and per-antenna power constraints, which is still an open problem except for a few special cases, see [Loy17] and the references therein.

using convex optimization tools. However, due to the simplification, the obtained solution is a suboptimal solution of the original sum rate maximization problem under individual sum power constraints.

Again, by using the pre-whitening receive filter in (3.56), the achievable sum rate R_{sum} can be reformulated as

$$R_{\text{sum}} = \frac{1}{2}\sum_{j=1}^{K} \log_2\left(\det\left(\mathbf{I}_N + \mathbf{B}^{(j)}\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}}\mathbf{B}^{(j)*\mathrm{T}}\right)\right), \tag{3.63}$$

where the matrix $\mathbf{B}^{(j)}$ is given by

$$\mathbf{B}^{(j)} = \left(\sigma_{\mathrm{D}}^2\mathbf{I}_N + \sigma_{\mathrm{D}}^2\mathbf{U}^{(k)*\mathrm{T}}\mathbf{U}^{(k)} + \sigma_{\mathrm{R}}^2\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{G}^{(q)*\mathrm{T}}\mathbf{H}_{\mathrm{RD}}^{(q,k)}\right)^{-\frac{1}{2}} \cdot \left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RD}}^{(q,j)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)} + \mathbf{H}_{\mathrm{DS}}^{(j,j)}\mathbf{V}^{(j)} + \mathbf{U}^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(j,j)}\right). \tag{3.64}$$

Let $\lambda_n^{(j)}$, $\forall n = 1, \ldots, N$, be the eigenvalues of the Hermitian matrix $\mathbf{B}^{(j)*\mathrm{T}}\mathbf{B}^{(j)}$ and the columns of the unitary matrix $\mathbf{W}^{(j)}$ be the corresponding eigenvectors. Furthermore, restrict $\mathbf{V}_1^{(j)}$ to a normal matrix so that it can be decomposed as

$$\mathbf{V}_1^{(j)} = \mathbf{W}^{(j)}\begin{bmatrix}\sqrt{s_1^{(j)}} & & \\ & \ddots & \\ & & \sqrt{s_N^{(j)}}\end{bmatrix}, \quad \forall j = 1, \ldots, K. \tag{3.65}$$

With this restriction on $\mathbf{V}_1^{(j)}$, the following sum rate maximization problem can be considered:

$$\underset{s_n^{(j)}}{\text{maximize}} \quad \frac{1}{2}\sum_{j=1}^{K}\sum_{n=1}^{N}\log_2\left(1 + \lambda_n^{(j)}s_n^{(j)}\right), \tag{3.66}$$

$$\text{subject to} \quad P_{\mathrm{S}}^{(j)} \leq P_{\mathrm{S,max}}^{(j)}, \quad \forall j = 1, \ldots, K, \tag{3.67}$$

$$P_{\mathrm{R}}^{(q)} \leq P_{\mathrm{R,max}}^{(q)}, \quad \forall q = 1, \ldots, Q, \tag{3.68}$$

$$\mathbf{V}_1^{(j)} = \mathbf{W}^{(j)}\begin{bmatrix}\sqrt{s_1^{(j)}} & & \\ & \ddots & \\ & & \sqrt{s_N^{(j)}}\end{bmatrix}, \quad \forall j = 1, \ldots, K, \tag{3.69}$$

$$s_n^{(j)} \geq 0, \quad \forall n = 1, \ldots, N, \ j = 1, \ldots, K, \tag{3.70}$$

where $P_{\mathrm{S}}^{(j)}$ and $P_{\mathrm{R}}^{(q)}$ are given in (3.49) and (3.50), respectively. Clearly, the above optimization problem is a convex problem. The optimal $s_n^{(j)}$ can be obtained by solving the corresponding Karush-Kuhn-Tucker conditions. More details on this subject can be found in [BV04] and will not be further discussed in this thesis. For the numerical simulations in this thesis, this optimization problem is solved using CVX, a package for specifying and solving convex programs [CVX12, GB08].

3.6. Numerical simulations and results

In this section, the sum rate achieved by relay-aided IA in the considered fully connected ad-hoc networks will be investigated using numerical simulations and compared with a few other interference management approaches. Such a fully connected ad-hoc network has been shown in Figure 2.1. For all the scenarios considered in the following, the entries of the channel matrices $\mathbf{H}_{\mathrm{DS}}^{(k,j)}$, $\mathbf{H}_{\mathrm{RS}}^{(q,j)}$, and $\mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}$ are assumed to be independently drawn from the circularly symmetric complex Gaussian distribution with unit variance, i.e., i.i.d. Rayleigh channels with unit average channel gain are considered. The noises at the relays and at the destination nodes in both time slots are assumed to be additive i.i.d. circularly symmetric complex Gaussian noise with a common variance $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{D}}^2 = \sigma^2$. The performances achieved by relay-aided IA and the reference schemes will be measured by the achievable sum rate R_{sum} in bits per channel use. For a fair comparison among the difference schemes, the pseudo signal-to-noise ratio (PSNR), which is defined as

$$\gamma_{\mathrm{PSNR}} = \frac{P_{\mathrm{tot}}}{K\sigma^2}, \tag{3.71}$$

will be considered instead of SNR. The PSNR can be interpreted as an approximation[6] of the average SNR per receive antenna under unit average channel gain. For the comparison, the following interference management approaches will be considered as references.

- **TDMA without relays:** A total number of K time slots will be used. In each time slot, one of the K source nodes directly transmits to the corresponding destination node without the help of relays under an individual transmit power constraint $P_{\mathrm{tot}}/2K$. The achievable sum rate of this scheme is simply the average capacity of the K point-to-point MIMO channels between the K source-destination node pairs. In spite of being a simple transmission scheme, it is shown

[6] This is because P_{tot} also includes the power used by the relays to forward the relay noise.

to be optimal or nearly optimal in the half-duplex non-orthogonal AF relay channel [RTLN14].

• **Sum MSE minimization:** The two-hop transmission scheme is employed. The transmit filters, the receive filters, and the relay processing filters are alternately adapted aiming at minimizing the sum MSE across the destination nodes under a total sum transmit power constraint P_{tot}, see Appendix C.1.

• **Sum rate maximization:** The two-hop transmission scheme is employed. The transmit filters, the receive filters, and the relay processing filters are alternately adapted aiming at maximizing the sum rate under a total sum transmit power constraint P_{tot}, see Appendix C.2.

• **MIMO IA without relays:** A single time slot will be used. The source nodes directly transmit to the destination nodes without the help of relays under an individual transmit power constraint $P_{\text{tot}}/2K$. The transmit and receive filters are alternately adapted to minimize the total interference leakage using the ILM algorithm [GCJ08, GCJ11].

First consider a simple scenario with $K = 3$ single-antenna source-destination node pairs and $Q = 2$ single-antenna relays. The sum DoF of this network is 3/2. When using TDMA without relays, each source-destination node pair is only able to achieve a DoF of 1/3, which corresponds to a sum DoF of one, as shown by the dotted curve in Figure 3.4. Both sum MSE minimization and sum rate maximization are able to achieve satisfactory performances in the low-PSNR regime. However, the average sum DoFs achieved by these two approaches are similar to TDMA without relays, as shown by the dashed curved in Figure 3.4. This is because both the sum MSE minimization and sum rate maximization problems are non-convex, and the performances of these two approaches strongly depend on the initialization. When the algorithms are initialized randomly, which is the case considered in this thesis, it is difficult for them to converge to a solution which perfectly nullifies all interferences. In contrast to this, relay-aided IA is always able to perfectly nullify all interferences, and therefore, is able to achieve a sum DoF of 3/2, as shown by the solid curves in Figure 3.4. The curves marked by circles and squares indicate the achievable sum rates of a randomly picked valid IN solution, where the matrices $\mathbf{V}_1^{(j)}$ are designed for sum rate maximization under a total sum power constraint as proposed in Subsection 3.5.2 and under individual sum power constraints as proposed in Subsection 3.5.3, respectively. Furthermore, $P_{\text{S}}^{(j)} = P_{\text{tot}}/2K$ and $P_{\text{R}}^{(q)} = P_{\text{tot}}/2Q$ are assumed for the later case. Both of these two cases benefit from the higher sum DoF and outperform the

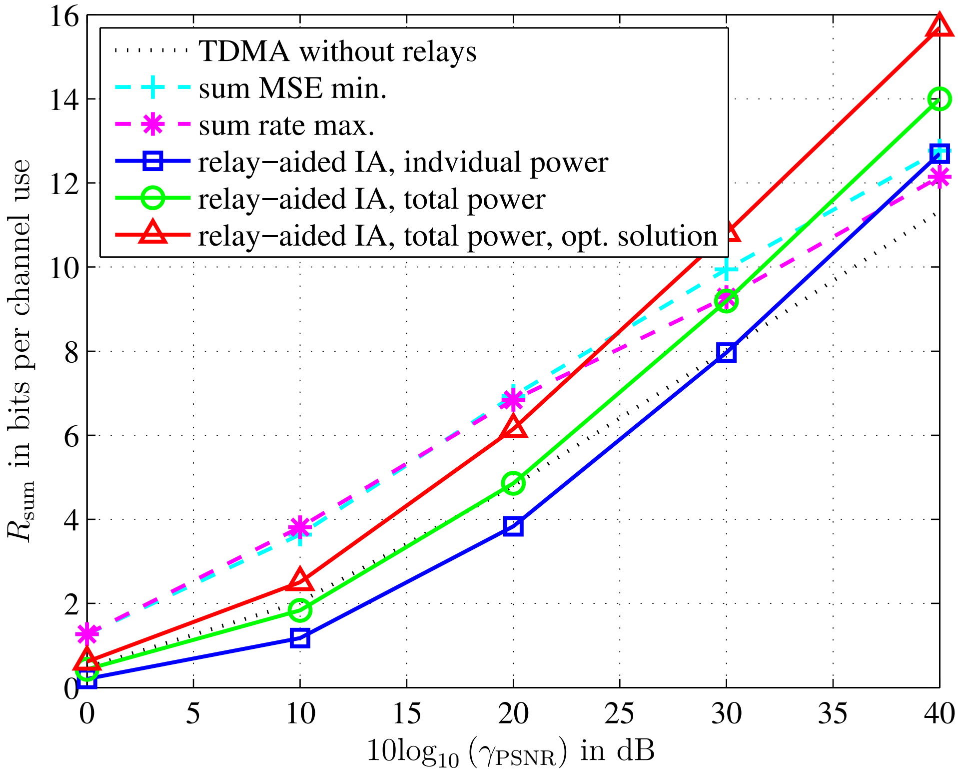


Figure 3.4.: The average achievable sum rates R_{sum} in bits per channel use as a function of the PSNR γ_{PSNR} in dB in a fully connected ad-hoc network with $K = 3$ single-antenna source-destination node pairs and $Q = 2$ single-antenna relays

aforementioned three reference approaches in the high-PSNR regime. However, relatively high PSNRs are required to see the sum rate gain in this scenario. This is partly because the IN solutions are randomly picked. Although how to find the "best" IN solution is not discussed in this thesis, the potential of optimizing the IN solution is revealed by the curve marked by triangles. In this case, the IN solution is chosen to be the one which achieves the highest sum rate under a total sum power constraint out of ten randomly picked IN solutions. Such a simple optimization already greatly improves the achievable sum rate, and outperforms the three reference approaches at lower PSNRs.

Next consider the multi-antenna case. $K = 4$ source-destination node pairs are considered so that the deployment of relays are necessary to achieve the DoF of the network. Each source and destination node is assumed to have $N = 2$ antennas. $Q = 2$ relays with $M_q = 3$ antennas each are considered to satisfy the feasibility condition. The sum DoF of this network is 4. In this network, TDMA without relays achieves a sum DoF of 2 since each source and destination node now has 2 antennas, as shown by the dotted curve in Figure 3.5. Again, both sum MSE minimization and sum rate maximization achieve satisfactory performances in the low-PSNR regime, but fail to converge to a solution which perfectly nullifies all interferences, as shown by the dashed curves in Figure 3.5. As compared to these reference approaches, relay-aided IA achieves the sum DoF of the network, as shown by the solid curves in Figure 3.5, which yields outstanding performances in the high-PSNR regime. In both Figure 3.4 and Figure 3.5, there is a gap between the sum rates achieved by relay-aided IA under a total sum power constraint and under individual sum power constraints. This is mainly due to the fact that the individual sum power constraints for all the source nodes and the relays cannot be simultaneously satisfied with equality, and some energy is hence wasted. For instance in Figure 3.5, relay-aided IA under the considered individual sum power constraints consumes ca. 56% of the total available energy at high PSNRs on average, which corresponds to a difference of ca. 2.5 dB in PSNR. In order to reduce the amount of wasted energy, a "better" IN solution shall be selected, instead of a randomly picked one. However, this is out of the scope of this thesis and will not be further discussed.

Finally, the performance achieved by relay-aided IA will be compared with MIMO IA, which achieves IA using multiple antennas of the source and destination node instead of the help of relays. Due to the difference in the feasibility conditions for relay-aided IA and for MIMO IA, two carefully chosen scenarios are considered in order to perform a fair comparison. In the first scenario, $K = 3$ source-destination node pairs and no relays are considered. Each source and destination node has $N = 2$ antennas. In this scenario, MIMO IA is able to achieve the DoF of the network, i.e., a sum DoF of 3. That is to say, each source node is able to transmit a single data symbol to the corresponding destination node without interference. By using the two-hop transmission scheme, relay-aided IA, although without the deployment of relays in this case, is also able to achieve the DoF of the network, as discussed in Remark 3.7. Namely, each source node can transmit two data symbols to the corresponding destination node using two time slots. The simulation results also show that both MIMO IA and relay-aided IA achieve the same sum DoF as depicted in Figure 3.6. In the second scenario, $K = 5$ source-destination node pairs, where each source and destination node has $N = 3$ antennas are considered. In this scenario, the feasibility condition for MIMO IA can be satisfied with equality [YGJK10]. Namely, each source node can

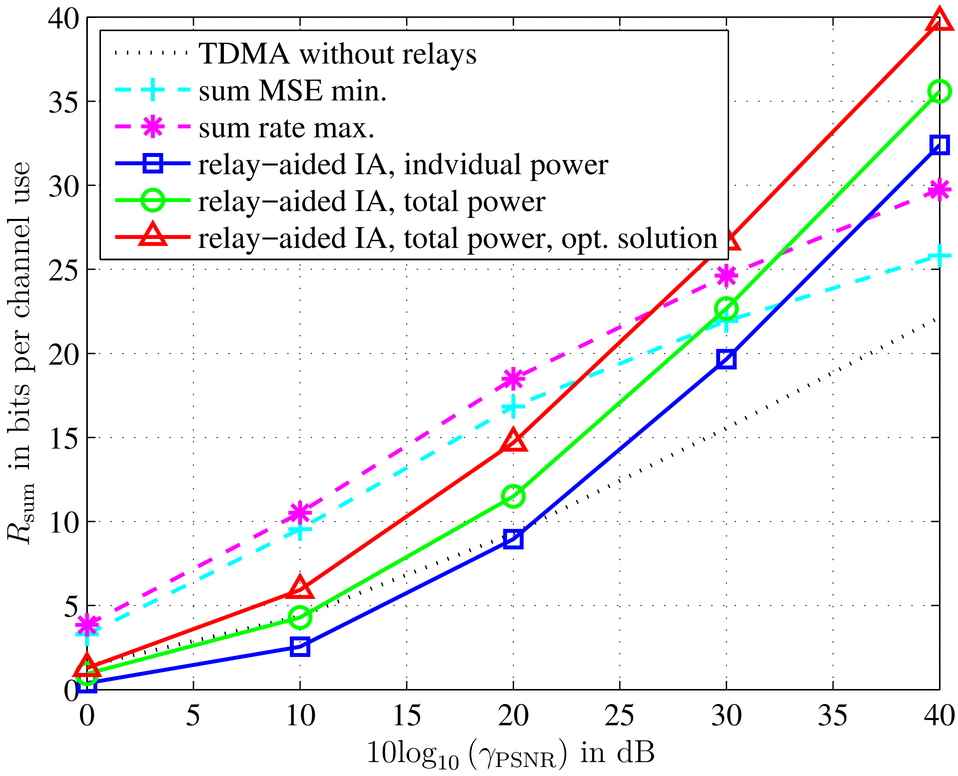


Figure 3.5.: The average achievable sum rates R_{sum} in bits per channel use as a function of the PSNR γ_{PSNR} in dB in a fully connected ad-hoc network with $K = 4$ source-destination node pairs and $Q = 2$ relays, where each source and destination node has $N = 2$ antennas and each relay has $M_q = 3$ antennas

transmit a single data symbol to the corresponding destination node and a sum DoF of 5 is achievable. In order to satisfy the feasibility condition for relay-aided IA, $Q = 6$ relays with $M_q = 4$ antennas each are considered. With the help of relays, relay-aided IA achieves a sum DoF of 7.5, which is the sum DoF of the network. The difference in the achievable sum DoFs of MIMO IA and relay-aided IA in this scenario can be seen from Figure 3.6. Due to the fact that the achievable sum DoF of MIMO IA approaches a limit of $2N$ as K increases [YGJK10], relay-aided IA is expected to further outperform MIMO IA in large networks with lots of source-destination node pairs.

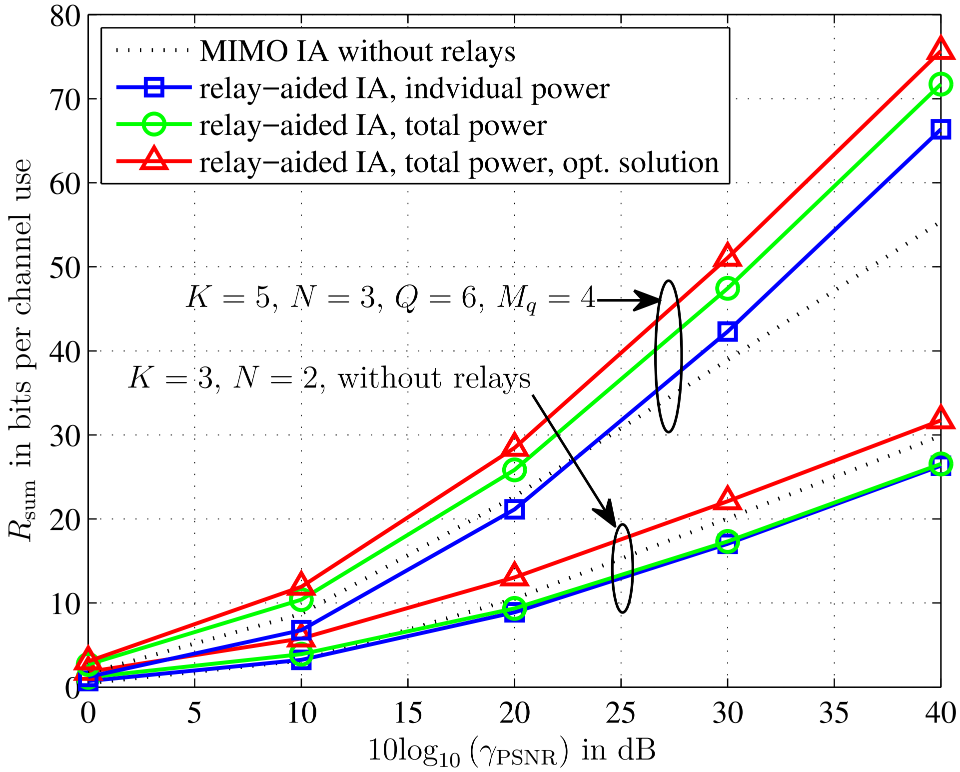


Figure 3.6.: The average achievable sum rates R_{sum} in bits per channel use as a function of the PSNR γ_{PSNR} in dB in 1) a fully connected ad-hoc network with $K = 3$ source-destination node pairs and no relays, where each source and destination node has $N = 2$ antennas, and 2) a fully connected ad-hoc network with $K = 5$ source-destination node pairs and $Q = 6$ relays, where each source and destination node has $N = 3$ antennas and each relay has $M_q = 4$ antennas

Chapter 4.

Relay-aided interference alignment in fully connected cellular networks

4.1. Overview

In this chapter, relay-aided IA in fully connected cellular networks will be investigated. In the considered fully connected cellular networks, all the channel coefficients are assumed to be independently drawn from a continuous distribution, see Section 2.2, which means all the channel coefficients are almost surely non-zero. Furthermore, full channel knowledge is assumed, i.e., the global, perfect, and instantaneous CSI is assumed to be known by a central unit or by every node in the network.

Comparing the considered fully connected cellular networks with fully connected ad-hoc networks, the following similarities and differences can be observed. The essential similarity is that the source and destination nodes in the ad-hoc networks are replaced by BSs and MSs in the cellular networks. For instance in the uplink, a BS with N antennas plays the role of a destination node, and the N single-antenna MSs of each cell play the role of a source node. Resulting from this, the sum DoFs of both networks are almost surely $KN/2$. However, the major difference is that the MSs cannot perform joint signal processing in cellular networks. Consequently, besides the inter-cell interferences, the intra-cell interferences have to be considered. Because of the aforementioned differences and similarities, the fully connected cellular networks are considered as an extension of the fully connected ad-hoc networks. The relay-aided IA scheme that has been proposed in the previous chapter shall be modified in order to achieve the DoF of the cellular networks.

In this chapter, the problem of achieving relay-aided IA in cellular networks will be decomposed into two subproblems, i.e., the inter-cell IN and the intra-cell interference management. This is realized by factorizing the transmit and receive filters of the MSs and BSs in a way similar to that in the ad-hoc networks. Since each MS only has a single antenna, the factorization of the filters of the MSs is similar to that in the single-antenna case of ad-hoc networks. In contrast, the factorization of the filters of the BSs is similar to that in the multi-antenna case of ad-hoc networks. Then, the inter-cell IN can be performed with the help of relays, following the idea of IN in the ad-hoc networks. The goal of inter-cell IN is to find the valid inter-cell IN solutions, i.e., the solutions which nullify all the inter-cell interferences and keep the resulting intra-cell channels full rank. Moreover, the inter-cell IN solution spaces and the feasibility conditions will be addressed. It has to be emphasized that in the cellular networks, a valid inter-cell IN solution does not guarantee the achievement of the DoF of the network unless the intra-cell interferences can be nullified as well. For intra-cell interference management, two widely used linear beamforming techniques, i.e., ZF and MMSE, will be considered at the BSs. Specially, ZF is always able to perfectly nullify the intra-cell interferences, whereas MMSE can asymptotically nullify the intra-cell interferences. For both ZF and MMSE, the corresponding achievable sum rates under a total sum transmit power constraint will be investigated.

In literature, the duality between the MAC and BC under a total power constraint has been well studied [VJG03, VT03, SSB07, HJU09, GJ10]. However, the duality of relay-aided IA in interfering MACs and BCs has not yet been considered. In this chapter, the duality between the uplink and the downlink transmissions in the considered fully connected cellular networks will be investigated exploiting channel reciprocity. The considered uplink-downlink duality has the following two implications. First, the inter-cell IN solutions in the uplink and the downlink are dual. That is to say, every valid inter-cell IN solution in the uplink uniquely corresponds to a valid inter-cell IN solution in the downlink, and vice versa. Second, based on the duality of inter-cell IN solutions, the beamforming matrices designed for intra-cell interference management as well as the achievable performances in the uplink and the downlink are also dual. More precisely, given a pair of dual valid inter-cell IN solutions, the achievable rate regions in the uplink and the downlink are dual under a total sum transmit power constraint.

4.2. Uplink transmission

4.2.1. Factorization of transmit filters

In this section, the uplink transmission will be considered first. The considered fully connected cellular networks, as modeled in Section 2.2, consist of K cells and Q half-duplex AF relays, where N single-antenna MSs transmit to a single BS with N antennas in each cell in the uplink and the q-th relay has M_q antennas. In order to achieve the DoF of this network in the uplink, i.e., a sum DoF of $KN/2$ in the almost sure sense, not only the inter-cell interferences but also the intra-cell interferences have to be perfectly nullified at every BS. Fortunately, the DoF of the network implies that at most N single-antenna MSs can be simultaneously served in each cell, as being discussed in Section 2.2. Therefore, the intra-cell interference can be handled by the BS alone using beamforming, e.g., using ZF. Hence, the relays only need to focus on nullifying the inter-cell interferences following the idea of IN in fully connected ad-hoc networks as discussed in the previous chapter.

Recall that the transmit filters used by the MSs of the j-th cell in the uplink can be stacked in the $2N \times N$ matrix $\begin{bmatrix} \mathbf{V}_{\mathrm{UL},1}^{(j)} \\ \mathbf{V}_{\mathrm{UL},2}^{(j)} \end{bmatrix}$, where the pre-coding matrices $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ and $\mathbf{V}_{\mathrm{UL},2}^{(j)}$ are $N \times N$ diagonal matrices since the MSs cannot perform joint transmission. Assuming that the pre-coding matrix $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ is of full rank, the factorization

$$\begin{bmatrix} \mathbf{V}_{\mathrm{UL},1}^{(j)} \\ \mathbf{V}_{\mathrm{UL},2}^{(j)} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{V}_{\mathrm{UL}}^{(j)} \end{bmatrix} \overline{\mathbf{Q}}_{\mathrm{UL}}^{(j)}, \quad \forall j, \tag{4.1}$$

can be considered, where $\overline{\mathbf{Q}}_{\mathrm{UL}}^{(j)}$ is an $N \times N$ full-rank diagonal matrix and $\mathbf{V}_{\mathrm{UL}}^{(j)}$ is an $N \times N$ diagonal matrix. Suppose

$$\overline{\mathbf{Q}}_{\mathrm{UL}}^{(j)} \overline{\mathbf{Q}}_{\mathrm{UL}}^{(j)*\mathrm{T}} = \mathbf{Q}_{\mathrm{UL}}^{(j)} \tag{4.2}$$

holds, where the n-th diagonal entry $q_{\mathrm{UL}}^{(j,n)}$ of $\mathbf{Q}_{\mathrm{UL}}^{(j)}$ does nothing but scales the transmit powers of the n-th MS of the j-th cell in both time slots. Therefore, it is harmless to choose

$$\overline{\mathbf{Q}}_{\mathrm{UL}}^{(j)} = \mathbf{Q}_{\mathrm{UL}}^{(j)\frac{1}{2}}. \tag{4.3}$$

Similarly, assuming that the combining matrix $\mathbf{U}_{\mathrm{UL},2}^{(k)}$ is of full rank, the receive filter $\begin{bmatrix}\mathbf{U}_{\mathrm{UL},1}^{(k)}\\ \mathbf{U}_{\mathrm{UL},2}^{(k)}\end{bmatrix}$ of the k-th BS can be factorized as

$$\begin{bmatrix}\mathbf{U}_{\mathrm{UL},1}^{(k)}\\ \mathbf{U}_{\mathrm{UL},2}^{(k)}\end{bmatrix} = \begin{bmatrix}\mathbf{U}_{\mathrm{UL}}^{(k)}\\ \mathbf{I}_N\end{bmatrix}\mathbf{W}_{\mathrm{UL}}^{(k)}, \tag{4.4}$$

where $\mathbf{W}_{\mathrm{UL}}^{(k)}$ is a full-rank $N \times N$ matrix. With the aforementioned factorization of the transmit and receive filters, the effective channel matrix $\mathbf{H}_{\mathrm{UL,eff}}^{(k,j)}$ from the MSs of the j-th cell to the k-th BS, which has been defined in (2.12), can be reformulated as

$$\begin{aligned}\mathbf{H}_{\mathrm{UL,eff}}^{(k,j)} &= \left[\mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \;\; \mathbf{U}_{\mathrm{UL},2}^{(k)*\mathrm{T}}\right]\begin{bmatrix}\mathbf{H}_{\mathrm{BM}}^{(k,j)} & \mathbf{0}\\ \sum\limits_{q=1}^{Q}\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,j)} & \mathbf{H}_{\mathrm{BM}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{\mathrm{UL},1}^{(j)}\\ \mathbf{V}_{\mathrm{UL},2}^{(j)}\end{bmatrix}\\
&= \mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\left[\mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}} \;\; \mathbf{I}_N\right]\begin{bmatrix}\mathbf{H}_{\mathrm{BM}}^{(k,j)} & \mathbf{0}\\ \sum\limits_{q=1}^{Q}\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,j)} & \mathbf{H}_{\mathrm{BM}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{V}_{\mathrm{UL}}^{(j)}\end{bmatrix}\mathbf{Q}_{\mathrm{UL}}^{(j)\frac{1}{2}}\\
&= \mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,j)} + \mathbf{H}_{\mathrm{BM}}^{(k,j)}\mathbf{V}_{\mathrm{UL}}^{(j)} + \mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{BM}}^{(k,j)}\right)\mathbf{Q}_{\mathrm{UL}}^{(j)\frac{1}{2}}\\
&= \mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,j)}\mathbf{Q}_{\mathrm{UL}}^{(j)\frac{1}{2}}. \end{aligned} \tag{4.5}$$

The above reformulation of the effective channel matrix $\mathbf{H}_{\mathrm{UL,eff}}^{(k,j)}$ has a similar form as the linearization approach that has been proposed for relay-aided IA in fully connected ad-hoc networks in Section 3.3, except that the matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$ and $\mathbf{Q}_{\mathrm{UL}}^{(j)\frac{1}{2}}$ are now diagonal matrices. Hence, the relay processing filters $\mathbf{G}_{\mathrm{UL}}^{(q)}$ and the matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$ and $\mathbf{U}_{\mathrm{UL}}^{(k)}$ shall be cooperatively designed to satisfy the following $K(K-1)N^2$ inter-cell IN conditions given in the matrix form

$$\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,j)} + \mathbf{H}_{\mathrm{BM}}^{(k,j)}\mathbf{V}_{\mathrm{UL}}^{(j)} + \mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{BM}}^{(k,j)} = \mathbf{0}, \quad \forall j \neq k, \tag{4.6}$$

while not violating any of the following K validity conditions

$$\det\left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,k)} + \mathbf{H}_{\mathrm{BM}}^{(k,k)}\mathbf{V}_{\mathrm{UL}}^{(k)} + \mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{BM}}^{(k,k)}\right) \neq 0, \quad \forall k. \tag{4.7}$$

A solution and the solution space of the inter-cell IN conditions (4.6) will be referred to as an inter-cell IN solution and the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$,

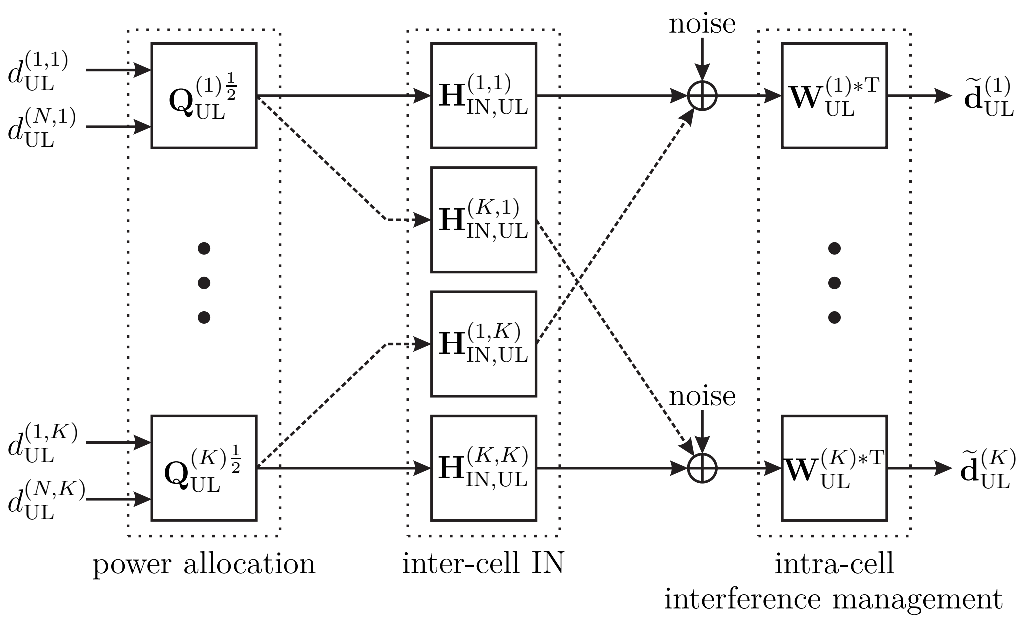


Figure 4.1.: In the uplink, the two-hop transmission scheme converts a considered fully connected cellular network into an interfering MAC. With a valid inter-cell IN solution, the inter-cell interferences can be perfectly nullified, i.e., the network can be converted into K non-interfering SIMO MACs. Then, the beamforming matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$ can be designed for intra-cell interference management and the power allocation specified by $\mathbf{Q}_{\mathrm{UL}}^{(k)}$ shall be designed to meet sum power constraints.

respectively. An inter-cell IN solution is invalid with respect to the k-th cell if it violates the k-th validity condition of (4.7), which means that the N data symbols intended for the k-th BS are not separable. Therefore, the DoF of the considered fully connected cellular networks can only be achieved by valid inter-cell IN solutions. Specifically, with a valid inter-cell IN solution, the considered fully connected cellular network can be converted into K non-interfering SIMO MACs, which correspond to the resulting intra-cell channels of the K cells. The channel matrix of the resulting intra-cell channel of the k-th cell is $\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}$. Thus, the matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$ can be considered as beamforming matrices, which shall be designed to handle the intra-cell interferences. The matrices $\mathbf{Q}_{\mathrm{UL}}^{(j)}$ specify a power allocation, which shall be designed to meet the considered sum power constraints. This approach to achieve relay-aided IA in the uplink of the considered cellular networks is illustrated in Figure 4.1.

The above approach is based on the assumption that all the pre-coding matrices

$\mathbf{V}_{\mathrm{UL},1}^{(j)}$ and the combining matrices $\mathbf{U}_{\mathrm{UL},2}^{(k)}$ are of full rank. That is to say, the optimality of this approach depends on whether or not all IA solutions, i.e., all the choices of the relay processing matrices, the transmit filters of the MSs, and the receive filters of the BSs that achieve the DoF of the network, fulfill this assumption. This will be argued as follows.

Firstly, suppose one of the pre-coding matrices $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ is rank deficient. This means one of the MSs of the j-th cell is muted in the first time slot, and hence the MS must transmit in the second time slot. In fully connected cellular networks with $K \geq 3$ cells, this results in that the interference subspace at each BS must be the N dimensional space spanned by the received signals in the second time slot, and that all the other MSs can only transmit in the second time slot in order to align the interferences. Furthermore, all the BSs must be shut down in the second time slot, i.e., $\mathbf{U}_{\mathrm{UL},2}^{(k)}$ must be zero, in order to nullify the interferences. Consequently, no data symbol can be successfully transmitted through the network.

Secondly, suppose one of the combining matrices $\mathbf{U}_{\mathrm{UL},2}^{(k)}$ is rank deficient. In order to nullify the inter-cell interferences between the MSs of the j-th cell and the k-th BS,

$$\mathbf{U}_{\mathrm{UL},2}^{(k)*\mathrm{T}} \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{H}_{\mathrm{RM}}^{(q,j)} \mathbf{V}_{\mathrm{UL},1}^{(j)} + \mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \mathbf{H}_{\mathrm{BM}}^{(k,j)} \mathbf{V}_{\mathrm{UL},1}^{(j)} + \mathbf{U}_{\mathrm{UL},2}^{(k)*\mathrm{T}} \mathbf{H}_{\mathrm{BM}}^{(k,j)} \mathbf{V}_{\mathrm{UL},2}^{(j)} = \mathbf{0} \tag{4.8}$$

must be satisfied. Following from (4.8),

$$\mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \mathbf{H}_{\mathrm{BM}}^{(k,j)} \mathbf{V}_{\mathrm{UL},1}^{(j)} = -\mathbf{U}_{\mathrm{UL},2}^{(k)*\mathrm{T}} \left(\sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{H}_{\mathrm{RM}}^{(q,j)} \mathbf{V}_{\mathrm{UL},1}^{(j)} + \mathbf{H}_{\mathrm{BM}}^{(k,j)} \mathbf{V}_{\mathrm{UL},2}^{(j)} \right) \tag{4.9}$$

holds. Therefore, if $\mathbf{U}_{\mathrm{UL},2}^{(k)}$ is rank deficient, the matrix $\mathbf{U}_{\mathrm{UL},1}^{(k)*\mathrm{T}} \mathbf{H}_{\mathrm{BM}}^{(k,j)} \mathbf{V}_{\mathrm{UL},1}^{(j)}$ must be rank deficient. In other words, either $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ or $\mathbf{U}_{\mathrm{UL},1}^{(k)}$ is rank deficient. If $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ is rank deficient, no data symbol can be successfully transmitted through the network as discussed earlier. If $\mathbf{V}_{\mathrm{UL},1}^{(j)}$ is of full rank and $\mathbf{U}_{\mathrm{UL},1}^{(k)}$ is rank deficient, then $\mathbf{U}_{\mathrm{UL},1}^{(k)}$ and $\mathbf{U}_{\mathrm{UL},2}^{(k)}$ must have identical column spaces. That is to say, the receive filter $\begin{bmatrix} \mathbf{U}_{\mathrm{UL},1}^{(k)} \\ \mathbf{U}_{\mathrm{UL},2}^{(k)} \end{bmatrix}$ of the k-th BS must be rank deficient. Consequently, the N data symbols intended for the k-th BS are not separable and the sum DoF of $KN/2$ is not achievable.

The above discussion shows that all IA solutions in fully connected cellular network can be mapped to valid inter-cell IN solutions in the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$. Hence, the conditions for the existence of valid inter-cell IN solutions, as will be discussed in the next subsection, form a necessary and sufficient feasibility condition for relay-aided IA in the considered fully connected cellular networks.

4.2.2. Inter-cell interference nulling

In the uplink, let the variables of the inter-cell IN conditions (4.6), i.e., the relay processing filters $\mathbf{G}_{\mathrm{UL}}^{(q)}$, the diagonal matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$, and the matrices $\mathbf{U}_{\mathrm{UL}}^{(k)}$, be stacked in a $(\sum_{q=1}^{Q} M_q^2 + KN + KN^2) \times 1$ vector as

$$\mathbf{x} = \begin{bmatrix} \mathrm{vec}\left(\left[\mathbf{G}_{\mathrm{UL}}^{(1)} \ \cdots \ \mathbf{G}_{\mathrm{UL}}^{(Q)}\right]\right) \\ \mathrm{diag}\left(\mathbf{V}_{\mathrm{UL}}^{(1)}\right) \\ \vdots \\ \mathrm{diag}\left(\mathbf{V}_{\mathrm{UL}}^{(K)}\right) \\ \mathrm{vec}\left(\left[\mathbf{U}_{\mathrm{UL}}^{(1)*\mathrm{T}} \ \cdots \ \mathbf{U}_{\mathrm{UL}}^{(K)*\mathrm{T}}\right]\right) \end{bmatrix}, \tag{4.10}$$

where $\mathrm{diag}(\cdot)$ denotes a column vector consisting of the diagonal entries of a matrix. Thus the matrix form of the system of linear equations consisting of all the inter-cell IN conditions (4.6) can be written as

$$\mathbf{A}_{\mathrm{IN}}\mathbf{x} = \left[\mathbf{A}_{\mathrm{RL}} \ \ \mathbf{A}_{\mathrm{DL},1} \ \ \mathbf{A}_{\mathrm{DL},2}\right]\mathbf{x} = \mathbf{0}, \tag{4.11}$$

where $\mathbf{A}_{\mathrm{IN}}$ is interpreted as a partitioned matrix consisting of the $K(K-1)N^2 \times \sum_{q=1}^{Q} M_q^2$ matrix

$$\mathbf{A}_{\mathrm{RL}} = \begin{bmatrix} \mathbf{H}_{\mathrm{RM}}^{(1,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(1,2)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RM}}^{(Q,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(Q,2)*\mathrm{T}} \\ \mathbf{H}_{\mathrm{RM}}^{(1,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(1,3)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RM}}^{(Q,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(Q,3)*\mathrm{T}} \\ \vdots & & \vdots \\ \mathbf{H}_{\mathrm{RM}}^{(1,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(1,K)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RM}}^{(Q,1)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(Q,K)*\mathrm{T}} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \mathbf{H}_{\mathrm{RM}}^{(1,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(1,1)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RM}}^{(Q,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(Q,1)*\mathrm{T}} \\ \mathbf{H}_{\mathrm{RM}}^{(1,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(1,2)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RM}}^{(Q,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(Q,2)*\mathrm{T}} \\ \vdots & & \vdots \\ \mathbf{H}_{\mathrm{RM}}^{(1,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(1,K-1)*\mathrm{T}} & \cdots & \mathbf{H}_{\mathrm{RM}}^{(Q,K)\mathrm{T}} \otimes \mathbf{H}_{\mathrm{RB}}^{(Q,K-1)*\mathrm{T}} \end{bmatrix}, \tag{4.12}$$

the $K(K-1)N^2 \times KN$ matrix

$$\mathbf{A}_{\mathrm{DL},1} = \begin{bmatrix} \mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(2,1)} & 0 & \cdots & 0 \\ \mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(3,1)} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(K,1)} & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & 0 & \mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(1,K)} \\ 0 & \cdots & 0 & \mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(2,K)} \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & \mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(K-1,K)} \end{bmatrix}, \tag{4.13}$$

and the $K(K-1)N^2 \times KN^2$ matrix

$$\mathbf{A}_{\mathrm{DL},2} = \begin{bmatrix} 0 & \mathbf{H}_{\mathrm{BM}}^{(2,1)\mathrm{T}} \otimes \mathbf{I}_N & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{H}_{\mathrm{BM}}^{(3,1)\mathrm{T}} \otimes \mathbf{I}_N & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \mathbf{H}_{\mathrm{BM}}^{(K,1)\mathrm{T}} \otimes \mathbf{I}_N \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \mathbf{H}_{\mathrm{BM}}^{(1,K)\mathrm{T}} \otimes \mathbf{I}_N & 0 & \cdots & 0 & 0 \\ 0 & \mathbf{H}_{\mathrm{BM}}^{(2,K)\mathrm{T}} \otimes \mathbf{I}_N & \cdots & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathbf{H}_{\mathrm{BM}}^{(K-1,K)\mathrm{T}} \otimes \mathbf{I}_N & 0 \end{bmatrix}. \tag{4.14}$$

Specially, in matrix $\mathbf{A}_{\mathrm{DL},1}$ as given in (4.13), $\odot$ denotes a column-wise Kronecker product, see Appendix A, i.e.,

$$\mathbf{I}_N \odot \mathbf{H}_{\mathrm{BM}}^{(k,j)} = \begin{bmatrix} \mathbf{h}_{\mathrm{BM},1}^{(k,j)} & \cdots & \mathbf{0}_{N\times 1} \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{N\times 1} & \cdots & \mathbf{h}_{\mathrm{BM},N}^{(k,j)} \end{bmatrix} \tag{4.15}$$

holds, where $\mathbf{h}_{\mathrm{BM},n}^{(k,j)}$ denotes the n-th column of $\mathbf{H}_{\mathrm{BM}}^{(k,j)}$. Then, the inter-cell IN solution space can be denoted by

$$\mathbb{S}_{\mathrm{IN}} = \mathrm{null}\left(\mathbf{A}_{\mathrm{IN}}\right), \tag{4.16}$$

and its dimension reads

$$\dim \mathbb{S}_{\mathrm{IN}} = \sum_{q=1}^{Q} M_q^2 + KN + KN^2 - \mathrm{rank}\left(\mathbf{A}_{\mathrm{IN}}\right). \tag{4.17}$$

Comparing the matrices $\mathbf{A}_{\mathrm{IN}}$ in fully connected cellular networks and in fully connected ad-hoc networks as introduced in Subsection 3.4.2, only the matrices $\mathbf{A}_{\mathrm{DL},1}$ have different structures, because the matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$ are now diagonal matrices. Hence, the rank of $\mathbf{A}_{\mathrm{IN}}$ in the fully connected cellular networks can be determined in a similar way.

Proposition 4.1. In a considered fully connected cellular network with $K \geq 3$ cells, if the channel coefficients are independently drawn from a continuous distribution, then

$$\mathrm{rank}\left(\mathbf{A}_{\mathrm{IN}}\right) = \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2 + KN + KN^2 - 1\right\} \tag{4.18}$$

holds with probability one, where the matrix $\mathbf{A}_{\mathrm{IN}}$ is defined in (4.11).

Proof. See Appendix D. □

In the considered cellular networks, the validity conditions (4.7) for inter-cell IN are non-linear in the variables $\mathbf{G}_{\mathrm{UL}}^{(q)}$, $\mathbf{V}_{\mathrm{UL}}^{(j)}$, and $\mathbf{U}_{\mathrm{UL}}^{(k)}$. Thus the invalid inter-cell IN solution subsets are in general not linear subspaces of the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$. However, the results stated in Proposition 3.7 can be easily extended to cellular networks. That is to say, in a considered fully connected cellular network with $K \geq 3$ cells, if the channel coefficients are independently drawn from a continuous distribution, then the following two statements hold true:

(1) If the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$ is one-dimensional, every inter-cell IN solution is an invalid inter-cell IN solution.
(2) If the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$ has at least two dimensions, a randomly picked inter-cell IN solution is almost surely not an invalid inter-cell IN solution with respect to any cell.

The proof is almost identical to the proof of Proposition 3.7 except that the notations shall be replaced correspondingly, and is therefore omitted in this thesis.

Following from the above results on the rank of the matrix $\mathbf{A}_{\mathrm{IN}}$ and the properties of the invalid inter-cell IN solution subsets, the feasibility conditions for relay-aided IA in the considered fully connected cellular networks can be given.

Proposition 4.2. In a fully connected cellular network with $K \geq 3$ cells, if the channel coefficients are independently drawn from a continuous distribution, then valid inter-cell IN solutions almost surely exist, i.e., relay-aided IA is almost surely feasible, if and only if

$$\dim \mathbb{S}_{\mathrm{IN}} \geq 2, \tag{4.19}$$

or equivalently

$$\sum_{q=1}^{Q} M_q^2 + KN - K(K-2)N^2 - 2 \geq 0, \tag{4.20}$$

holds.

Proof. This can be directly deduced from the above discussions. □

A few analogies between the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$ in the uplink of the considered fully connected cellular networks and that in fully connected ad-hoc networks shall be noticed.

Remark 4.1. Analogous to the single-antenna case in ad-hoc networks, every IA solution in the uplink of the considered cellular networks, if there exists one, can be constructed from a valid inter-cell IN solution $\mathbb{S}_{\mathrm{IN}}$, along with properly designed beamforming matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$ and a power allocation specified by $\mathbf{Q}_{\mathrm{UL}}^{(j)}$.

Remark 4.2. Analogous to ad-hoc networks, the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$ in the uplink of the considered cellular networks always has a one-dimensional subspace which is spanned by the non-trivial invalid inter-cell IN solution with $\mathbf{G}_{\mathrm{UL}}^{(q)} = \mathbf{0}_{M_q}$, $\forall q$, and $\mathbf{V}_{\mathrm{UL}}^{(j)} = -\mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}} = \mathbf{I}_N$, $\forall j, k$.

Remark 4.3. Analogous to the multi-antenna case in ad-hoc networks, the invalid inter-cell IN solution subsets are in general not linear subspaces of the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$ in the uplink of the considered cellular networks. Nevertheless, if the inter-cell IN solution space $\mathbb{S}_{\mathrm{IN}}$ has at least two dimensions, a randomly picked inter-cell IN solution is almost surely a valid one.

4.2.3. Intra-cell interference management

Given a valid inter-cell IN solution, i.e., a proper selection of the relay processing matrices $\mathbf{G}_{\mathrm{UL}}^{(q)}$ and the matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$ and $\mathbf{U}_{\mathrm{UL}}^{(k)}$ which nullifies the inter-cell interferences while not violating any of the validity conditions, the resulting channel matrix $\mathbf{H}_{\mathrm{IN,UL}}^{(k,j)}$ is zero for any $j \neq k$ and is of full rank for $j = k$. Thus, the output data symbols $\widetilde{\mathbf{d}}_{\mathrm{UL}}^{(k)}$ of the k-th BS, which have been defined in (2.14), can be rewritten as

$$\widetilde{\mathbf{d}}_{\mathrm{UL}}^{(k)} = \mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}} \mathbf{H}_{\mathrm{IN,UL}}^{(k,k)} \mathbf{Q}_{\mathrm{UL}}^{(k)\frac{1}{2}} \mathbf{d}_{\mathrm{UL}}^{(k)} + \mathbf{W}^{(k)*\mathrm{T}} \widetilde{\mathbf{n}}_{\mathrm{UL}}^{(k)}, \quad \forall k, \tag{4.21}$$

where $\widetilde{\mathbf{n}}_{\mathrm{UL}}^{(k)}$ is the effective noise at the input of $\mathbf{W}_{\mathrm{UL}}^{(k)}$ with the covariance matrix given by

$$\mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\mathrm{UL}}^{(k)} = \sigma_{\mathrm{BS}}^2 \left(\mathbf{I}_N + \mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}} \mathbf{U}_{\mathrm{UL}}^{(k)}\right) + \sigma_{\mathrm{R}}^2 \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}} \mathbf{H}_{\mathrm{RB}}^{(q,k)}, \quad \forall k. \tag{4.22}$$

Equation (4.21) models the resulting intra-cell channels of the given inter-cell IN solution, which are essentially Gaussian vector MACs, also known as space-division MACs [TV05]. Then, the beamforming matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$ can be designed to handle the intra-cell interferences of the corresponding cells. However, if the beamforming matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$ are designed such that the intra-cell interferences cannot be nullified, the DoF of the considered fully connected cellular network still cannot be achieved. In this thesis, two beamforming techniques will be considered. Firstly, ZF will be considered, which always perfectly nullifies the intra-cell interferences at all SNRs. Secondly, MMSE will be considered, which yields the optimum linear receive filters with respect to the individual intra-cell channels and is able to asymptotically nullify the intra-cell interferences in the high-SNR regime. Furthermore, the matrices $\mathbf{Q}_{\mathrm{UL}}^{(k)}$, which specify a transmit power allocation, shall be designed to meet the considered sum power constraints. In the uplink, the sum transmit power of all N MSs in the k-th cell is given by

$$P_{\mathrm{MS,UL}}^{(k)} = \mathrm{tr}\left(\left(\mathbf{I}_N + \mathbf{V}_{\mathrm{UL}}^{(k)*\mathrm{T}} \mathbf{V}_{\mathrm{UL}}^{(k)}\right) \mathbf{Q}_{\mathrm{UL}}^{(k)}\right), \tag{4.23}$$

and the transmit power of the q-th relay is given by

$$P_{\mathrm{R,UL}}^{(q)} = \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{H}_{\mathrm{RM}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{H}_{\mathrm{RM}}^{(q,k)} \mathbf{Q}_{\mathrm{UL}}^{(k)}\right) + \sigma_{\mathrm{R}}^2 \mathrm{tr}\left(\mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\right). \tag{4.24}$$

Hence, the total sum transmit power reads

$$\begin{aligned}
P_{\text{tot,UL}} &= \sum_{k=1}^{K} P_{\text{MS,UL}}^{(k)} + \sum_{q=1}^{Q} P_{\text{R,UL}}^{(q)} \\
&= \sum_{k=1}^{K} \operatorname{tr}\left(\left(\mathbf{I}_N + \mathbf{V}_{\text{UL}}^{(k)*\text{T}}\mathbf{V}_{\text{UL}}^{(k)} + \sum_{q=1}^{Q} \mathbf{H}_{\text{RM}}^{(q,k)*\text{T}}\mathbf{G}_{\text{UL}}^{(q)*\text{T}}\mathbf{G}_{\text{UL}}^{(q)}\mathbf{H}_{\text{RM}}^{(q,k)}\right)\mathbf{Q}_{\text{UL}}^{(k)}\right) \\
&\quad + \sigma_{\text{R}}^2 \sum_{q=1}^{Q} \operatorname{tr}\left(\mathbf{G}_{\text{UL}}^{(q)}\mathbf{G}_{\text{UL}}^{(q)*\text{T}}\right) \\
&= \sum_{k=1}^{K}\sum_{n=1}^{N}\left[\mathbf{I}_N + \mathbf{V}_{\text{UL}}^{(k)*\text{T}}\mathbf{V}_{\text{UL}}^{(k)} + \sum_{q=1}^{Q} \mathbf{H}_{\text{RM}}^{(q,k)*\text{T}}\mathbf{G}_{\text{UL}}^{(q)*\text{T}}\mathbf{G}_{\text{UL}}^{(q)}\mathbf{H}_{\text{RM}}^{(q,k)}\right]_{nn} q_{\text{UL}}^{(k,n)} \\
&\quad + \sigma_{\text{R}}^2 \sum_{q=1}^{Q} \operatorname{tr}\left(\mathbf{G}_{\text{UL}}^{(q)}\mathbf{G}_{\text{UL}}^{(q)*\text{T}}\right),
\end{aligned} \tag{4.25}$$

where $[\cdot]_{nn}$ denotes the n-th diagonal entry of a matrix. In this subsection, the power allocation in the uplink specified by the matrices $\mathbf{Q}_{\text{UL}}^{(k)}$ will be optimized under a total sum transmit power constraint $P_{\text{tot,max}}$ either to maximize the achievable sum rate for uplink ZF or to minimize the sum MSE for uplink MMSE.

Uplink ZF

Let the beamforming matrix $\mathbf{W}_{\text{UL}}^{(k)}$ of the k-th BS be chosen as the unnormalized ZF filter

$$\mathbf{W}_{\text{ULZF}}^{(k)} = \mathbf{H}_{\text{IN,UL}}^{(k,k)-*\text{T}}. \tag{4.26}$$

Since the intra-cell interferences are always perfectly nullified by $\mathbf{W}_{\text{ULZF}}^{(k)}$, the SNR $\gamma_{\text{UL}}^{(k,n)}$ of the n-th data symbol of the k-th cell can be given by

$$\gamma_{\text{UL}}^{(k,n)} = \frac{q_{\text{UL}}^{(k,n)}}{\left[\mathbf{H}_{\text{IN,UL}}^{(k,k)-1}\mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\text{UL}}^{(k)}\mathbf{H}_{\text{IN,UL}}^{(k,k)-*\text{T}}\right]_{nn}}, \tag{4.27}$$

where $q_{\text{UL}}^{(k,n)}$ is the n-th diagonal entry of $\mathbf{Q}_{\text{UL}}^{(k)}$. The power allocation specified by $q_{\text{UL}}^{(k,n)}$ will then be optimized aiming at maximizing the achieved sum rate under a total sum transmit power constraint $P_{\text{tot,max}}$. Thus, the power allocation problem can be formulated as

$$\underset{q_{\mathrm{UL}}^{(k,n)},\ \forall k,n}{\text{maximize}} \quad \frac{1}{2}\sum_{k=1}^{K}\sum_{n=1}^{N}\log_2\left(1+\frac{q_{\mathrm{UL}}^{(k,n)}}{\left[\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-1}\mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\mathrm{UL}}^{(k)}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-*\mathrm{T}}\right]_{nn}}\right), \tag{4.28}$$

$$\text{subject to} \quad P_{\mathrm{tot,UL}} \leq P_{\mathrm{tot,max}}, \tag{4.29}$$

$$q_{\mathrm{UL}}^{(k,n)} \geq 0, \quad \forall k,n, \tag{4.30}$$

where $P_{\mathrm{tot,UL}}$ has been given in (4.25). Using the method of Lagrangian multipliers, a water-filling-like closed-form solution can be obtained as

$$q_{\mathrm{ULZF}}^{(k,n)} = \max\left\{0, s^{(k,n)}\right\}, \tag{4.31}$$

where $s^{(k,n)}$ reads

$$s^{(k,n)} = \frac{S_{\mathrm{W}}}{\left[\mathbf{I}_N + \mathbf{V}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{V}_{\mathrm{UL}}^{(k)} + \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RM}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,k)}\right]_{nn}} - \left[\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-1}\mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\mathrm{UL}}^{(k)}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-*\mathrm{T}}\right]_{nn}, \tag{4.32}$$

and S_{W} is chosen such that the total sum transmit power constraint is satisfied with equality.

Uplink MMSE

Let the beamforming matrix $\mathbf{W}_{\mathrm{UL}}^{(k)}$ of the k-th BS be chosen as the unmormalized MMSE filter

$$\mathbf{W}_{\mathrm{ULMMSE}}^{(k)} = \left(\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{UL}}^{(k)}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}} + \mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\mathrm{UL}}^{(k)}\right)^{-1}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{UL}}^{(k)\frac{1}{2}} \tag{4.33}$$

to minimize the MSE of the corresponding cell. Then the sum MSE of all K cells can be given by

$$\mathrm{MSE}_{\mathrm{sum}} = \sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{I}_N - \mathbf{W}_{\mathrm{ULMMSE}}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{UL}}^{(k)\frac{1}{2}}\right). \tag{4.34}$$

Consider a total sum transmit power constraint $P_{\mathrm{tot,max}}$. Let the power allocation specified by $q_{\mathrm{UL}}^{(k,n)}$ be optimized to minimize the sum MSE given in (4.34).

Equivalently, the optimum power allocation $q_{\mathrm{ULMMSE}}^{(k,n)}$ can be obtained by solving the optimization problem formulated as

$$\underset{q_{\mathrm{UL}}^{(k,n)},\ \forall k,n}{\text{minimize}} \quad \sum_{k=1}^{K} \operatorname{tr}\left(\left(\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{UL}}^{(k)}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}} + \mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\mathrm{UL}}^{(k)}\right)^{-1}\mathbf{S}_{\tilde{\mathbf{n}}\tilde{\mathbf{n}},\mathrm{UL}}^{(k)}\right), \tag{4.35}$$

$$\text{subject to} \quad P_{\mathrm{tot,UL}} \leq P_{\mathrm{tot,max}}, \tag{4.36}$$

$$q_{\mathrm{UL}}^{(k,n)} \geq 0, \quad \forall k, n. \tag{4.37}$$

The above optimization problem is a convex problem, and can be readily solved using standard convex optimization tools. For the numerical simulations in this thesis, this optimization problem is solved using CVX, a package for specifying and solving convex programs [CVX12, GB08].

Remark 4.4. Although only linear filters, i.e., the beamforming matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$, are investigated in this thesis for intra-cell interference management, it is also possible to combine inter-cell IN with non-linear filters at the BSs. For instance, given any valid inter-cell IN solution, it is well-known that the sum capacities of the individual resulting intra-cell channels, as modeled in (4.21), can be achieved using SIC at the BSs [VG97]. In this case, the optimum power allocation under a total sum transmit power constraint as well as the achievable sum rate can be computed numerically [TV05]. Combining inter-cell IN with SIC will be considered as a reference scheme in the numerical simulations in Section 4.4.

4.3. Uplink-downlink duality

4.3.1. Duality of inter-cell interference nulling

In this section, relay-aided IA in the downlink of the considered fully connected cellular networks will be investigated. Specifically, the reciprocal downlink channels, as being modeled in Section 2.2, will be exploited and a relay-aided IA scheme that is dual to the one in the uplink will be considered. The uplink-downlink duality for relay-aided IA in the considered fully connected cellular networks has the following two implications. Firstly, the inter-cell IN solutions in the uplink and downlink are dual, as will be discussed in this subsection. That is to say, every valid inter-cell IN solution in the uplink corresponds to a unique valid inter-cell IN solution in the downlink, and vice versa. Secondly, based on the duality of inter-cell IN solutions, the beamforming matrices designed for intra-cell interference management as well as the achievable performances in the uplink and downlink are also dual, as will be discussed in the next subsection. More precisely, given a pair of dual valid inter-cell IN solutions, the achievable rate regions in the uplink and downlink are dual under a total sum transmit power constraint.

Consider the downlink transmission. Let the transmit filter $\begin{bmatrix}\mathbf{V}_{\mathrm{DL},1}^{(k)}\\ \mathbf{V}_{\mathrm{DL},2}^{(k)}\end{bmatrix}$ of the k-th BS be factorized as

$$\begin{bmatrix}\mathbf{V}_{\mathrm{DL},1}^{(k)}\\ \mathbf{V}_{\mathrm{DL},2}^{(k)}\end{bmatrix} = \begin{bmatrix}\mathbf{I}_N\\ \mathbf{V}_{\mathrm{DL}}^{(k)}\end{bmatrix}\mathbf{W}_{\mathrm{DL}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)\frac{1}{2}}, \tag{4.38}$$

where $\mathbf{W}_{\mathrm{DL}}^{(k)}$ is an $N \times N$ beamforming matrix for intra-cell interference management, and $\mathbf{Q}_{\mathrm{DL}}^{(k)}$ is an $N \times N$ diagonal matrix with positive real-valued diagonal entries $q_{\mathrm{DL}}^{(k,n)}$ specifying a power allocation. Let the receive filters $\begin{bmatrix}\mathbf{U}_{\mathrm{DL},1}^{(j)}\\ \mathbf{U}_{\mathrm{DL},2}^{(j)}\end{bmatrix}$ of the MSs in the j-th cell be factorized as

$$\begin{bmatrix}\mathbf{U}_{\mathrm{DL},1}^{(j)}\\ \mathbf{U}_{\mathrm{DL},2}^{(j)}\end{bmatrix} = \begin{bmatrix}\mathbf{U}_{\mathrm{DL}}^{(j)}\\ \mathbf{I}_N\end{bmatrix}\mathbf{D}^{(j)}. \tag{4.39}$$

Since the MSs cannot perform joint receiving in the downlink, the combining matrices $\mathbf{U}_{\mathrm{DL},1}^{(j)}$ and $\mathbf{U}_{\mathrm{DL},2}^{(j)}$ are diagonal matrices. Therefore, $\mathbf{U}_{\mathrm{DL}}^{(j)}$ and $\mathbf{D}^{(j)}$ are also diagonal matrices. Furthermore, since $\mathbf{D}^{(j)}$ has no impact on the achievable rates of the individual MSs, it can be simply fixed as the identity matrix. Exploiting the above factorization of the transmit and receive filters in the downlink, the effective channel matrix $\mathbf{H}_{\mathrm{DL,eff}}^{(j,k)}$ from the k-th BS to the MSs in the j-th cell can be formulated as

$$\begin{aligned}
\mathbf{H}_{\mathrm{DL,eff}}^{(j,k)} &= \begin{bmatrix}\mathbf{U}_{\mathrm{DL},1}^{(j)*\mathrm{T}} & \mathbf{U}_{\mathrm{DL},2}^{(j)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RM}}^{(q,j)*\mathrm{T}}\mathbf{G}_{\mathrm{DL}}^{(q)}\mathbf{H}_{\mathrm{RB}}^{(q,k)} & \mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{\mathrm{DL},1}^{(k)}\\ \mathbf{V}_{\mathrm{DL},2}^{(k)}\end{bmatrix}\\
&= \begin{bmatrix}\mathbf{U}_{\mathrm{DL}}^{(j)*\mathrm{T}} & \mathbf{I}_N\end{bmatrix}\begin{bmatrix}\mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}} & \mathbf{0}\\ \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RM}}^{(q,j)*\mathrm{T}}\mathbf{G}_{\mathrm{DL}}^{(q)}\mathbf{H}_{\mathrm{RB}}^{(q,k)} & \mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{V}_{\mathrm{DL}}^{(k)}\end{bmatrix}\mathbf{W}_{\mathrm{DL}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)\frac{1}{2}}\\
&= \left(\sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RM}}^{(q,j)*\mathrm{T}}\mathbf{G}_{\mathrm{DL}}^{(q)}\mathbf{H}_{\mathrm{RB}}^{(q,k)} + \mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}}\mathbf{V}_{\mathrm{DL}}^{(k)} + \mathbf{U}_{\mathrm{DL}}^{(j)*\mathrm{T}}\mathbf{H}_{\mathrm{BM}}^{(k,j)*\mathrm{T}}\right)\mathbf{W}_{\mathrm{DL}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)\frac{1}{2}}\\
&= \mathbf{H}_{\mathrm{IN,DL}}^{(j,k)}\mathbf{W}_{\mathrm{DL}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)\frac{1}{2}}.
\end{aligned} \tag{4.40}$$

Then, the uplink-downlink duality of inter-cell IN solutions can be shown as follows. Suppose the relay processing filters $\mathbf{G}_{\mathrm{UL}}^{(q)}$ and the matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$ and $\mathbf{U}_{\mathrm{UL}}^{(k)}$

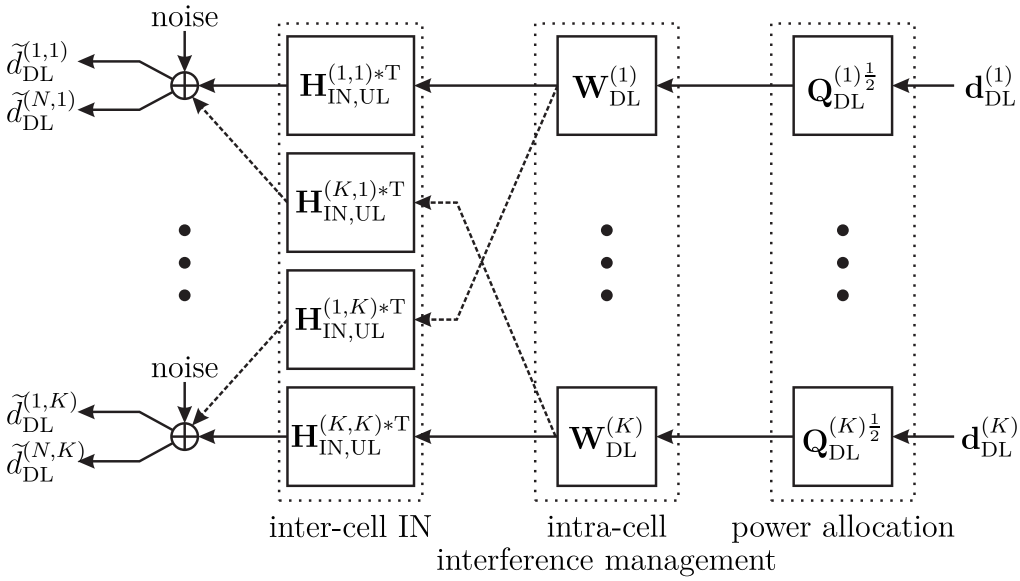


Figure 4.2.: Considering dual valid inter-cell IN solutions, the resulting channel matrix $\mathbf{H}_{\mathrm{IN,DL}}^{(j,k)} = \mathbf{H}_{\mathrm{IN,UL}}^{(k,j)*\mathrm{T}}$ in the downlink is zero for any $j \neq k$ and is of full rank for $j = k$.

form a valid inter-cell IN solution in the uplink, then substituting

$$\mathbf{G}_{\mathrm{DL}}^{(q)} = \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}, \quad \mathbf{V}_{\mathrm{DL}}^{(k)} = \mathbf{U}_{\mathrm{UL}}^{(k)}, \quad \text{and} \quad \mathbf{U}_{\mathrm{DL}}^{(j)} = \mathbf{V}_{\mathrm{UL}}^{(j)}, \quad \forall q,k,j \tag{4.41}$$

into (4.40) yields that the resulting channel matrix $\mathbf{H}_{\mathrm{IN,DL}}^{(j,k)} = \mathbf{H}_{\mathrm{IN,UL}}^{(k,j)*\mathrm{T}}$ is zero for any $j \neq k$ and is of full rank for $j = k$. That is to say, the relay processing filters $\mathbf{G}_{\mathrm{DL}}^{(q)}$ and the matrices $\mathbf{V}_{\mathrm{DL}}^{(j)}$ and $\mathbf{U}_{\mathrm{DL}}^{(k)}$ given by (4.41) form a valid inter-cell IN solution in the downlink. This is illustrated in Figure 4.2. Furthermore, the optimality of the aforementioned factorizations of the transmit filters and receive filters in the downlink depends on whether or not all IA solutions in the downlink contain full rank pre-coding matrices $\mathbf{V}_{\mathrm{DL},1}^{(k)}$ and combining matrices $\mathbf{U}_{\mathrm{DL},2}^{(j)}$. This can be argued in the same way as in the uplink, see Subsection 4.2.1. Hence, the duality of inter-cell IN solutions in the uplink and downlink follows. Based on the duality of inter-cell IN solutions, the results on the dimension of the inter-cell IN solution space and on the feasibility conditions in the uplink also apply in the downlink.

4.3.2. Duality of intra-cell interference management

In this subsection, a duality of the achievable rate regions under a total sum transmit power constraint in the uplink and the downlink will be derived first. Exploiting this duality of achievable rate regions, the beamforming matrices of the BSs for intra-cell management as well as the corresponding optimum power allocations in the downlink which are dual to the uplink ZF and dual to the uplink MMSE will be addressed, respectively. In order to derive the duality of achievable rate regions, it is assumed that the noise at the relays, the noise at the BSs, and the noise at the MSs have a common variance σ^2, i.e.,

$$\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{BS}}^2 = \sigma_{\mathrm{MS}}^2 = \sigma^2 \tag{4.42}$$

holds. Furthermore, some expressions will be frequently used. Hence, the following matrices will be introduced for notational simplicity. Let $\mathbf{A}^{(k)}$ be an $N \times N$ positive-definite diagonal matrix, whose n-th diagonal entry $\left[\mathbf{A}^{(k)}\right]_{nn}$ is given by

$$\left[\mathbf{A}^{(k)}\right]_{nn} = \left[\mathbf{I}_N + \mathbf{V}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{V}_{\mathrm{UL}}^{(k)} + \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RM}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{H}_{\mathrm{RM}}^{(q,k)}\right]_{nn}. \tag{4.43}$$

Let $\mathbf{B}^{(k)}$ be an $N \times N$ positive-definite Hermitian matrix given by

$$\mathbf{B}^{(k)} = \mathbf{I}_N + \mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{U}_{\mathrm{UL}}^{(k)} + \sum_{q=1}^{Q}\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\mathbf{H}_{\mathrm{RB}}^{(q,k)}. \tag{4.44}$$

First consider the uplink. Suppose the relay processing filters $\mathbf{G}_{\mathrm{UL}}^{(q)}$ and the matrices $\mathbf{V}_{\mathrm{UL}}^{(j)}$ and $\mathbf{U}_{\mathrm{UL}}^{(k)}$ form a valid inter-cell IN solution in the uplink, the matrices $\mathbf{W}_{\mathrm{UL}}^{(k)}$ are arbitrary beamforming matrix for intra-cell management in the uplink, and the matrices $\mathbf{Q}_{\mathrm{UL}}^{(k)}$ specify an arbitrary power allocation in the uplink. Then, the resulting intra-cell channel of the k-th cell can be modeled as

$$\tilde{\mathbf{d}}_{\mathrm{UL}}^{(k)} = \mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{UL}}^{(k)\frac{1}{2}}\mathbf{d}_{\mathrm{UL}}^{(k)} + \mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\tilde{\mathbf{n}}_{\mathrm{UL}}^{(k)}, \tag{4.45}$$

with the covariance matrix of the noise $\tilde{\mathbf{n}}_{\mathrm{UL}}^{(k)}$ being $\sigma^2\mathbf{B}^{(k)}$, where $\mathbf{B}^{(k)}$ has been introduced in (4.44). Furthermore, using the diagonal matrix $\mathbf{A}^{(k)}$ introduced in (4.43), the total sum transmit power constraint in the uplink reads

$$\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{A}^{(k)}\mathbf{Q}_{\mathrm{UL}}^{(k,n)}\right) + \sigma^2\sum_{q=1}^{Q}\mathrm{tr}\left(\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\right) \le P_{\mathrm{tot,max}}, \tag{4.46}$$

see (4.23) – (4.25). In order to compare the uplink and the downlink transmissions, let the intra-cell channel of the k-th cell (4.45) and the total sum transmit power constraint (4.46) be reformulated. Introduce $\widetilde{\mathbf{W}}^{(k)} = \mathbf{B}^{(k)\frac{1}{2}}\mathbf{W}_{\mathrm{UL}}^{(k)}$ and $\widetilde{\mathbf{Q}}_{\mathrm{UL}}^{(k)} = \mathbf{A}^{(k)}\mathbf{Q}_{\mathrm{UL}}^{(k,n)}$. Then, (4.45) can be reformulated as

$$\widetilde{\mathbf{d}}_{\mathrm{UL}}^{(k)} = \widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{A}^{(k)-\frac{1}{2}}\widetilde{\mathbf{Q}}_{\mathrm{UL}}^{(k)\frac{1}{2}}\mathbf{d}_{\mathrm{UL}}^{(k)} + \widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\mathbf{n}, \tag{4.47}$$

with $\mathbf{n}\sim\mathcal{N}(0,\sigma^2\mathbf{I}_N)$. In other words, each one of the K non-interfering intra-cell channels in the uplink is equivalent to, in terms of the achievable rate region under a total sum transmit power constraint for the cell, a SIMO MAC with the channel matrix $\mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{A}^{(k)-\frac{1}{2}}$ and i.i.d. white Gaussian noise, as illustrated in Figure 4.3a. The matrices $\widetilde{\mathbf{Q}}_{\mathrm{UL}}^{(k)}$ specify a power allocation and the matrices $\widetilde{\mathbf{W}}^{(k)}$ are the receive beamforming matrices. Furthermore, the total sum transmit power constraint in the uplink can be equivalently reformulated as

$$\sum_{k=1}^{K}\mathrm{tr}\left(\widetilde{\mathbf{Q}}_{\mathrm{UL}}^{(k)}\right) \leq P_{\mathrm{tot,max}} - \sigma^2\sum_{q=1}^{Q}\mathrm{tr}\left(\mathbf{G}_{\mathrm{UL}}^{(q)}\mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\right). \tag{4.48}$$

In the downlink, consider a dual valid inter-cell IN solution as given by (4.41). Furthermore, let the same beamforming matrix $\mathbf{W}_{\mathrm{UL}}^{(k)}$ for intra-cell management in the uplink be also used as the beamforming matrix in the downlink, i.e.,

$$\mathbf{W}_{\mathrm{DL}}^{(k)} = \mathbf{W}_{\mathrm{UL}}^{(k)} \tag{4.49}$$

is considered. Then, for a power allocation specified by $\mathbf{Q}_{\mathrm{DL}}^{(k)}$, the intra-cell channel of the k-th cell in the downlink can be modeled as

$$\widetilde{\mathbf{d}}_{\mathrm{DL}}^{(k)} = \mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}}\mathbf{W}_{\mathrm{UL}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)\frac{1}{2}}\mathbf{d}_{\mathrm{DL}}^{(k)} + \widetilde{\mathbf{n}}_{\mathrm{DL}}^{(k)}, \quad \forall k, \tag{4.50}$$

where the relation $\mathbf{H}_{\mathrm{IN,DL}}^{(k,k)} = \mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}}$ resulting from the duality of inter-cell IN solutions is considered, and the variance of the effective noise $\widetilde{n}_{\mathrm{DL}}^{(k,n)}$ at the n-th MS of the k-th cell reads

$$\mathrm{E}\left\{\widetilde{n}_{\mathrm{DL}}^{(k,n)}\widetilde{n}_{\mathrm{DL}}^{(k,n)*}\right\} = \sigma^2\left[\mathbf{A}^{(k)}\right]_{nn}, \tag{4.51}$$

where $\mathbf{A}^{(k)}$ has been introduced in (4.43). Furthermore, the sum transmit power $P_{\mathrm{BS,DL}}^{(k)}$ of the k-th BS in the downlink is

$$P_{\mathrm{BS,DL}}^{(k)} = \mathrm{tr}\left(\mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\left(\mathbf{I}_N + \mathbf{U}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{U}_{\mathrm{UL}}^{(k)}\right)\mathbf{W}_{\mathrm{UL}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)}\right), \tag{4.52}$$

and the transmit power of the q-th relay is

$$P_{\mathrm{R,DL}}^{(q)} = \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}} \mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}} \mathbf{H}_{\mathrm{RB}}^{(q,k)} \mathbf{W}_{\mathrm{UL}}^{(k)} \mathbf{Q}_{\mathrm{DL}}^{(k)}\right) + \sigma^2 \mathrm{tr}\left(\mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\right). \tag{4.53}$$

Hence, the total sum transmit power in the downlink reads

$$\begin{aligned} P_{\mathrm{tot,DL}} &= \sum_{k=1}^{K} P_{\mathrm{BS,DL}}^{(k)} + \sum_{q=1}^{Q} P_{\mathrm{R,DL}}^{(q)} \\ &= \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}} \mathbf{B}^{(k)} \mathbf{W}_{\mathrm{UL}}^{(k)} \mathbf{Q}_{\mathrm{DL}}^{(k)}\right) + \sigma^2 \sum_{q=1}^{Q} \mathrm{tr}\left(\mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\right), \end{aligned} \tag{4.54}$$

where $\mathbf{B}^{(k)}$ has been introduced in (4.44). Then, using $\widetilde{\mathbf{W}}^{(k)} = \mathbf{B}^{(k)\frac{1}{2}} \mathbf{W}_{\mathrm{UL}}^{(k)}$, the intra-cell channel of the k-th cell in the downlink, as given by (4.50), can be equivalently reformulated as

$$\widetilde{\mathbf{d}}_{\mathrm{DL}}^{(k)} = \mathbf{A}^{(k)-\frac{1}{2}} \mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}} \mathbf{B}^{(k)-\frac{1}{2}} \widetilde{\mathbf{W}}^{(k)} \mathbf{Q}_{\mathrm{DL}}^{(k)\frac{1}{2}} \mathbf{d}_{\mathrm{DL}}^{(k)} + \mathbf{n}, \quad \forall k, \tag{4.55}$$

with $\mathbf{n} \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_N)$. That is to say, each one of the K non-interfering intra-cell channels in the downlink is equivalent to, in terms of the achievable rate region under a total sum transmit power constraint for the cell, a MISO BC with the channel matrix $\mathbf{A}^{(k)-\frac{1}{2}} \mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}} \mathbf{B}^{(k)-\frac{1}{2}}$ and i.i.d. white Gaussian noise, as illustrated in Figure 4.3b. The matrices $\widetilde{\mathbf{Q}}_{\mathrm{DL}}^{(k)}$ specify a power allocation and the matrices $\widetilde{\mathbf{W}}^{(k)}$ are the transmit beamforming matrices. Moreover, the total sum transmit power constraint in the downlink can be equivalently reformulated as

$$\sum_{k=1}^{K} \mathrm{tr}\left(\widetilde{\mathbf{W}}^{(k)*\mathrm{T}} \widetilde{\mathbf{W}}^{(k)} \mathbf{Q}_{\mathrm{DL}}^{(k)}\right) \leq P_{\mathrm{tot,max}} - \sigma^2 \sum_{q=1}^{Q} \mathrm{tr}\left(\mathbf{G}_{\mathrm{UL}}^{(q)} \mathbf{G}_{\mathrm{UL}}^{(q)*\mathrm{T}}\right). \tag{4.56}$$

Obviously, for each individual cell, the equivalent intra-cell channels in the uplink (4.47) and in the downlink (4.55), as illustrated in Figure 4.3, are a pair of dual MAC and BC in terms of the achievable rate regions under a sum transmit power constraint for the cell[1]. More specifically, by properly choosing the power allocation specified by $\mathbf{Q}_{\mathrm{DL}}^{(k)}$, the same SINRs can be achieved in both uplink and

[1] The capacity regions of these channels, if non-linear filters are considered at the BSs, are also dual according to the well-known MAC-BC duality [VJG03, VT03].

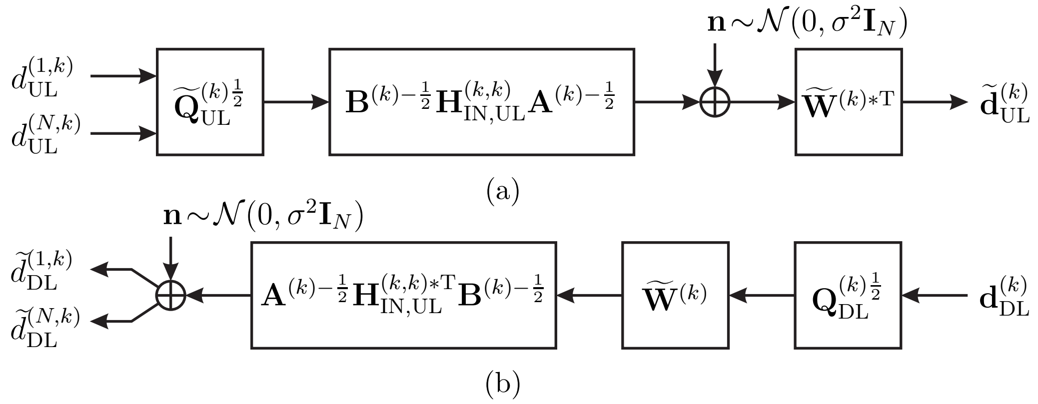


Figure 4.3.: Equivalent intra-cell channel of the k-th cell in (a) the uplink and (b) the downlink

downlink, i.e.,

$$\frac{\left|\left[\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{A}^{(k)-\frac{1}{2}}\right]_{nn}\right|^2\widetilde{q}_{\mathrm{UL}}^{(k,n)}}{\sum\limits_{m\neq n}\left|\left[\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{A}^{(k)-\frac{1}{2}}\right]_{nm}\right|^2\widetilde{q}_{\mathrm{UL}}^{(k,n)}+\sigma^2\left[\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\widetilde{\mathbf{W}}^{(k)}\right]_{nn}}$$
$$=\frac{\left|\left[\mathbf{A}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}}\mathbf{B}^{(k)-\frac{1}{2}}\widetilde{\mathbf{W}}^{(k)}\right]_{nn}\right|^2 q_{\mathrm{DL}}^{(k,n)}}{\sum\limits_{m\neq n}\left|\left[\mathbf{A}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}}\mathbf{B}^{(k)-\frac{1}{2}}\widetilde{\mathbf{W}}^{(k)}\right]_{nm}\right|^2 q_{\mathrm{DL}}^{(k,m)}+\sigma^2},\quad\forall k,n,\quad(4.57)$$

holds, and the total sum transmit powers are equal in both uplink and downlink, i.e.,

$$\sum_{k=1}^{K}\mathrm{tr}\left(\widetilde{\mathbf{Q}}_{\mathrm{UL}}^{(k)}\right)=\sum_{k=1}^{K}\mathrm{tr}\left(\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\widetilde{\mathbf{W}}^{(k)}\mathbf{Q}_{\mathrm{DL}}^{(k)}\right)\quad(4.58)$$

holds. Furthermore, for the special case where $\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{A}^{(k)-\frac{1}{2}}$ is a Hermitian matrix, e.g., for ZF and MMSE, the dual downlink power allocation can be obtained in closed form as

$$q_{\mathrm{DL}}^{(k,n)}=\frac{\widetilde{q}_{\mathrm{UL}}^{(k,n)}}{\left[\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}\widetilde{\mathbf{W}}^{(k)}\right]_{nn}}=\frac{\left[\mathbf{A}^{(k)}\right]_{nn}}{\left[\mathbf{W}_{\mathrm{UL}}^{(k)*\mathrm{T}}\mathbf{B}^{(k)}\mathbf{W}_{\mathrm{UL}}^{(k)}\right]_{nn}}q_{\mathrm{UL}}^{(k,n)},\quad\forall k,n.\quad(4.59)$$

Remark 4.5. In simple words, the duality of the achievable rate regions in uplink and downlink can be understood as follows. On the one hand, the transmit power consumed by the relays in the uplink, which depends on the channels $\mathbf{H}_{\mathrm{RM}}^{(q,k)}$ from the MSs to the relays, is compensated by the noise amplified by the relays in the downlink, which depends on the reciprocal channels $\mathbf{H}_{\mathrm{RM}}^{(q,k)*\mathrm{T}}$ from the relays to the MSs. On the other hand, the transmit power consumed by the relays in the downlink, which depends on the channels $\mathbf{H}_{\mathrm{RB}}^{(q,k)}$ from the BSs to the relays, is compensated by the noise amplified by the relays in the uplink, which depends on the reciprocal channels $\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}$ from the relays to the BSs.

Dual downlink ZF

Exploiting the aforementioned uplink-downlink duality, the beamforming matrices $\mathbf{W}_{\mathrm{DL}}^{(k)}$ in the downlink that achieve a dual rate region as the uplink ZF shall be chosen as

$$\mathbf{W}_{\mathrm{DLZF}}^{(k)} = \mathbf{W}_{\mathrm{ULZF}}^{(k)} = \mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-*\mathrm{T}} = \mathbf{H}_{\mathrm{IN,DL}}^{(k,k)-1}, \quad \forall k, \tag{4.60}$$

which are ZF transmit filters with respect to the individual intra-cell channels in the downlink. The corresponding optimum power allocation $q_{\mathrm{DLZF}}^{(k,n)}$ in the downlink, which maximizes the achieved sum rate under a total sum transmit power constraint follows from (4.59) and reads

$$q_{\mathrm{DLZF}}^{(k,n)} = \max\left\{0, \frac{S_{\mathrm{W}}}{\left[\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-1}\mathbf{B}^{(k)}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)-*\mathrm{T}}\right]_{nn}} - \left[\mathbf{A}^{(k)}\right]_{nn}\right\}, \tag{4.61}$$

where S_{W} is chosen such that the total sum transmit power constraint $P_{\mathrm{tot,max}}$ is met with equality. Alternatively, the optimum power allocation in the downlink, as given by (4.61), can also be obtained by solving the corresponding sum rate maximization problem in the downlink following the same approach as being discussing in Subsection 4.2.3. This also verifies that the sum rates achieved by ZF in the uplink and the downlink are dual under a total sum transmit power constraint.

Dual downlink MMSE

Exploiting the aforementioned uplink-downlink duality, the beamforming matrices $\mathbf{W}_{\mathrm{DL}}^{(k)}$ in the downlink that achieve a dual rate region as the uplink MMSE shall be chosen as

$$\begin{aligned}\mathbf{W}_{\mathrm{DLMMSE}}^{(k)} &= \mathbf{W}_{\mathrm{ULMMSE}}^{(k)} \\ &= \left(\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{ULMMSE}}^{(k)}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)*\mathrm{T}} + \sigma^2\mathbf{B}^{(k)}\right)^{-1}\mathbf{H}_{\mathrm{IN,UL}}^{(k,k)}\mathbf{Q}_{\mathrm{ULMMSE}}^{(k)\frac{1}{2}}.\end{aligned} \tag{4.62}$$

Note that in (4.62), the covariance matrix $\mathbf{S}^{(k)}_{\hat{\mathbf{n}}\hat{\mathbf{n}},\mathrm{UL}}$ of the uplink noise is replaced by $\sigma^2\mathbf{B}^{(k)}$, as introduced in (4.44), because this matrix shall be interpreted differently in the downlink. Furthermore, the optimum power allocation which minimizes the sum MSE in the downlink reads

$$q^{(k,n)}_{\mathrm{DLMMSE}} = \frac{\left[\mathbf{A}^{(k)}\right]_{nn}}{\left[\mathbf{W}^{(k)*\mathrm{T}}_{\mathrm{DLMMSE}}\mathbf{B}^{(k)}\mathbf{W}^{(k)}_{\mathrm{DLMMSE}}\right]_{nn}} q^{(k,n)}_{\mathrm{ULMMSE}}. \tag{4.63}$$

The dual downlink MMSE transmit filters and the corresponding sum MSE minimizing power allocation can be understood as follows. Suppose $\mathbf{T}^{(k)}$ is an $N\times N$ diagonal matrix with the n-th diagonal entry being $\left[\mathbf{W}^{(k)*\mathrm{T}}_{\mathrm{DLMMSE}}\mathbf{B}^{(k)}\mathbf{W}^{(k)}_{\mathrm{DLMMSE}}\right]_{nn} q^{(k,n)}_{\mathrm{DLMMSE}}$. Therefore, $\mathbf{Q}^{(k)}_{\mathrm{ULMMSE}} = \mathbf{A}^{(k)-\frac{1}{2}}\mathbf{T}^{(k)}\mathbf{A}^{(k)-\frac{1}{2}}$ holds. Then, the compound transmit filter $\mathbf{W}^{(k)}_{\mathrm{DLMMSE}}\mathbf{Q}^{(k,n)\frac{1}{2}}_{\mathrm{DLMMSE}}$ can be reformulated as

$$\begin{aligned}
&\mathbf{W}^{(k)}_{\mathrm{DLMMSE}}\mathbf{Q}^{(k,n)\frac{1}{2}}_{\mathrm{DLMMSE}} \\
&= \left(\mathbf{H}^{(k,k)}_{\mathrm{IN,UL}}\mathbf{Q}^{(k)}_{\mathrm{ULMMSE}}\mathbf{H}^{(k,k)*\mathrm{T}}_{\mathrm{IN,UL}} + \sigma^2\mathbf{B}^{(k)}\right)^{-1}\mathbf{H}^{(k,k)}_{\mathrm{IN,UL}}\mathbf{Q}^{(k)\frac{1}{2}}_{\mathrm{ULMMSE}}\mathbf{Q}^{(k,n)\frac{1}{2}}_{\mathrm{DLMMSE}} \\
&= \mathbf{B}^{(k)-\frac{1}{2}}\widetilde{\mathbf{W}}^{(k)}_{\mathrm{MMSE}}\mathbf{\Lambda}^{(k)},
\end{aligned} \tag{4.64}$$

where

$$\begin{aligned}
\widetilde{\mathbf{W}}^{(k)}_{\mathrm{MMSE}} = &\left(\mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}^{(k,k)}_{\mathrm{IN,UL}}\mathbf{A}^{(k)-\frac{1}{2}}\mathbf{T}^{(k)}\mathbf{A}^{(k)-\frac{1}{2}}\mathbf{H}^{(k,k)*\mathrm{T}}_{\mathrm{IN,UL}}\mathbf{B}^{(k)-\frac{1}{2}} + \sigma^2\mathbf{I}_N\right)^{-1} \\
&\cdot \mathbf{B}^{(k)-\frac{1}{2}}\mathbf{H}^{(k,k)}_{\mathrm{IN,UL}}\mathbf{A}^{(k)-\frac{1}{2}}\mathbf{T}^{(k)\frac{1}{2}}
\end{aligned} \tag{4.65}$$

is the MMSE transmit filter under a total sum transmit power constraint for the equivalent intra-cell channel (4.55) in the downlink, and $\mathbf{\Lambda}^{(k)}$ is a diagonal matrix with the diagonal entries being the normalization factors

$$\left[\mathbf{\Lambda}^{(k)}\right]_{nn} = \sqrt{\frac{\left[\mathbf{T}^{(k)}\right]_{nn}}{\left[\widetilde{\mathbf{W}}^{(k)*\mathrm{T}}_{\mathrm{MMSE}}\widetilde{\mathbf{W}}^{(k)}_{\mathrm{MMSE}}\right]_{nn}}}. \tag{4.66}$$

Remark 4.6. It shall be noted that applying the dual MMSE filters given by (4.62) in the downlink does not directly yield the same MSEs as in the uplink. In order to achieve the dual MSE region in the downlink, a scaling factor shall be considered at each MS [SSB07, HJU09].

4.4. Numerical simulations and results

In this section, the sum rate achieved by relay-aided IA in the considered fully connected cellular networks will be investigated using numerical simulations and compared with a few other interference management approaches. Such a fully connected cellular network has been modeled in Section 2.2. If reciprocal channels are assumed, the same sum rate can be achieved in both the uplink and the downlink under a total sum transmit power constraint, as discussed in the previous section. Therefore, it suffices to only consider the uplink transmission. For the scenario considered in the following, the entries of the channel matrices $\mathbf{H}_{\mathrm{BM}}^{(k,j)}$, $\mathbf{H}_{\mathrm{RM}}^{(q,j)}$, and $\mathbf{H}_{\mathrm{RB}}^{(q,k)*\mathrm{T}}$ are assumed to be independently drawn from the circularly symmetric complex Gaussian distribution with unit variance, i.e., i.i.d. Rayleigh channels with unit average channel gain are considered. The noises at the relays and at the BSs in both time slots are assumed to be additive i.i.d. circularly symmetric complex Gaussian noise with a common variance $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{BS}}^2 = \sigma^2$. For a fair comparison among the difference approaches, the PSNR, which is defined as

$$\gamma_{\mathrm{PSNR}} = \frac{P_{\mathrm{tot}}}{K\sigma^2}, \tag{4.67}$$

will be considered. For the comparison, the following interference management approaches will be considered as references.

- **TDMA without relays + SIC:** A total number of K time slots will be used. In each time slot, the N MSs in one of the K cells directly transmits to the corresponding BS without the help of relays under a sum transmit power constraint of $P_{\mathrm{tot}}/2K$. Furthermore, SIC is considered at the BSs for sum rate maximization in the individual cells. The optimum power allocation at the MSs and the corresponding achievable sum rate in each cell have been given in [VG97]. In this simulation, they are computed using CVX.

- **Sum MSE minimization:** The two-hop transmission scheme is applied. The transmit filters, the receive filters, and the relay processing filters are alternately adapted aiming at minimizing the sum MSE across the BSs under a total sum transmit power constraint P_{tot}, see Appendix C.1.

- **Sum rate maximization:** The two-hop transmission scheme is applied. The transmit filters, the receive filters, and the relay processing filters are alternately adapted aiming at maximizing the sum rate under a total sum transmit power constraint P_{tot}, see Appendix C.2.

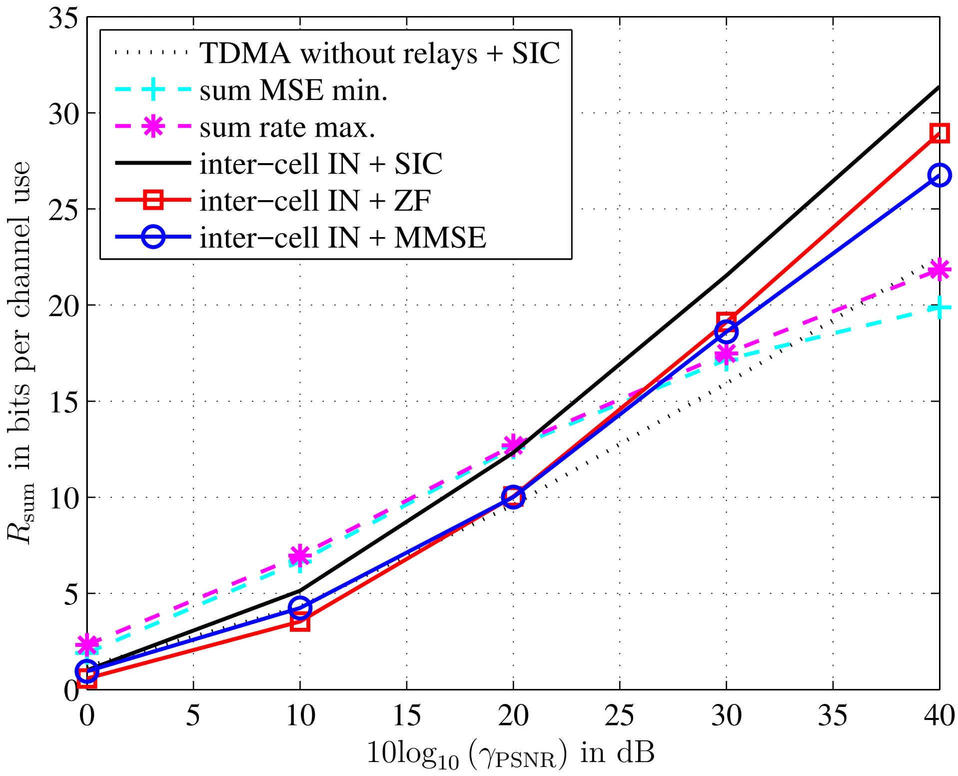


Figure 4.4.: The average achievable sum rate R_{sum} in bits per channel use as a function of the PSNR γ_{PSNR} in dB in a scenario with $K = 3$ cells, where each cell includes $N = 2$ single-antenna MSs and a BS with $N = 2$ antennas, and $Q = 2$ relays with $M_q = 2$ antennas each

The considered scenario consists of $K = 3$ cells, where each cell includes $N = 2$ single-antenna MSs and a BS with $N = 2$ antennas, and $Q = 2$ relays with $M_q = 2$ antennas each. The sum DoF of this network is 3. When using TDMA without relays, a sum DoF of 2/3 per cell can be achieved, which corresponds to a sum DoF 2 in the entire network, as shown by the dotted curve in Figure 4.4. Similar to fully connected ad-hoc networks, both sum MSE minimization and sum rate maximization are able to achieve satisfactory performances in the low-PSNR regime, as shown by the dashed curves in Figure 4.4. However, in the high-PSNR regime, it is difficult for both of them to converge to a solution which perfectly nullifies all the interferences, especially when the algorithms are randomly initialized. Consequently, the average achievable sum DoFs of these two approaches are smaller than the sum DoF of the network. In contrast to this, relay-aided IA is always able to perfectly nullify the inter-cell interferences, and is also able to nullify the intra-cell interferences if the filters of the BSs are properly designed. Therefore, a sum DoF of 3 is achievable, as shown by the solid curves in Figure 4.4. The solid curve without marks indicates the achievable sum rate of a randomly picked valid inter-cell IN solution followed by SIC for sum rate maximization. This can be considered as an upper bound of the sum rates achieved by the given inter-cell IN solution under a total sum transmit power constraint. The solid curves marked by squares and circles indicate the achievable sum rates of a randomly picked valid IN solution followed by ZF and MMSE, respectively, as been proposed in Subsection 4.2.3. It is understandable that the sum rates achieved by linear beamforming are inferior to those achieved by SIC. Moreover, MMSE achieves higher sum rates in the low-PSNR regime as compared to ZF, because the noise is jointly considered with the intra-cell interferences. However, in this thesis, the power allocation for MMSE is optimized aiming at minimizing the sum MSE, rather than directly maximizing the sum rate. For this reason, MMSE achieves slightly lower sum rate as compared to ZF in the high-PSNR regime.

Chapter 5.

Relay-aided interference alignment in partially connected ad-hoc networks

5.1. Overview

In this chapter, relay-aided IA in a class of partially connected ad-hoc networks will be investigated. The considered partially connected ad-hoc networks consist of multiple subnetworks, as introduced in Section 2.3. Each subnetwork is a relatively small fully connected ad-hoc network. The different subnetworks are partially connected to each other by a few present inter-subnetwork direct links. Except for the intra-subnetwork links and the present inter-subnetwork direct links, the other links are considered to be absent in the system model, i.e., the corresponding channel coefficients are set to zero, due to their negligibly small channel gains. In the considered partially connected ad-hoc networks, relay-aided IA can be achieved even without full channel knowledge. Before discussing the proposed relay-aided IA scheme with partial channel knowledge, the case with full channel knowledge will be reviewed first and a few important concepts will be introduced.

If full channel knowledge is assumed, i.e., if the global, perfect, and instantaneous CSI is assumed to be known by a central unit or by every node in the network, an IA solution can be obtained in the same way as in fully connected ad-hoc networks as discussed in Chapter 3. That is to say, the linearization approach can be applied to obtain an IN solution space. If relay-aided IA is feasible, a randomly picked IN solution is almost surely valid. Then an IA solution can be constructed based on any given valid IN solution. Therefore, the discussions in this chapter focus on the feasibility conditions. The difficulty is mainly due to the irregular partial connectivity between different subnetworks, i.e., due to the fact that the presence of inter-subnetwork direct links does not follow a certain pattern. For this reason, the IN conditions in the entire network will be classified into intra- and inter-subnetwork IN conditions and considered separately. The intra-subnetwork IN conditions in each subnetwork are the same as the IN conditions in a fully connected ad-hoc network. In contrast, the inter-subnetwork IN conditions do not depend on the channel realization of inter-subnetwork direct links, but only depend on the presence of them. In this thesis, the inter-subnetwork IN conditions will be transformed to a set of equivalent linear conditions called external constraints employing graph theory. The intra-subnetwork IN conditions and the external constraints can then be jointly considered to obtain the feasibility conditions, which can be summarised as follows. In the considered partially connected ad-hoc networks, every part of the entire network, i.e., every subset of subnetworks, must have enough free variables to satisfy its intra-subnetwork IN conditions and external constraints while not violating the validity conditions.

For relay-aided IA in fully connected ad-hoc and cellular networks, as discussed in the previous two chapter, full channel knowledge is always assumed. However, this assumption, which is in fact also a requirement for achieving the DoF of these networks, is difficult to fulfil in practice, especially in large networks with many nodes. In this chapter, a relay-aided IA scheme with partial channel knowledge will be proposed. The considered partial channel knowledge includes the intra-subnetwork CSI, the network topology, and some side information obtained from other subnetworks. Using the intra-subnetwork CSI and the network topology, each subnetwork is able to obtain a solution space defined by the intra-subnetwork IN conditions and the external constraints for it. Following a certain order, each subnetwork shall properly select a solution from the solution space using the side information. Then, all these solutions selected by the individual subnetworks form a valid IN solution for the entire network. The feasibility conditions for the proposed relay-aided IA with partial channel knowledge will be addressed as well. In the considered partially connected ad-hoc networks, the feasibility conditions for relay-aided IA will be interpreted as the required numbers of relay antennas. It will be shown that the proposed scheme with partial channel knowledge is also able to achieve relay-aided IA without additional relay antennas as compared to that with full channel knowledge. Finally, the proposed scheme with partial channel knowledge usually requires that the individual subnetworks select their solutions one after another. However, a parallelization approach which allows several subnetworks to select their solutions simultaneous can be applied to speed up this process.

5.2. Feasibility conditions with full channel knowledge

5.2.1. Intra- and inter-subnetwork interference-nulling

In this section, the feasibility conditions for relay-aided IA in the considered partially connected ad-hoc networks will be first investigated under the assumption of full channel knowledge. This helps to understand the specialty of the considered partially connected ad-hoc networks, and it also provides a benchmark for the proposed relay-aided IA scheme with partial channel knowledge, which will be discussed in the next section.

A considered partially connected ad-hoc networks result from setting the channel coefficients of certain links in a fully connected ad-hoc network to zero, as introduced in Section 2.3. Therefore, the linearization approach which has been proposed for fully connected ad-hoc networks in Chapter 3 can be used to obtain the set of IA solutions. Recall that the source and destination nodes in the considered partially connected ad-hoc networks have a single antenna each and that the transmit filters $\left[v_1^{(j)} \;\; v_2^{(j)}\right]^{\mathrm{T}}$ of the source nodes and the receive filters $\left[u_1^{(j)} \;\; u_2^{(j)}\right]^{\mathrm{T}}$ of the destination nodes can be factorized as

$$\begin{bmatrix} v_1^{(j)} \\ v_2^{(j)} \end{bmatrix} = \begin{bmatrix} 1 \\ v^{(j)} \end{bmatrix} v_1^{(j)} \quad \text{and} \quad \begin{bmatrix} u_1^{(k)} \\ u_2^{(k)} \end{bmatrix} = \begin{bmatrix} u^{(k)} \\ 1 \end{bmatrix} u_2^{(k)}, \quad \forall j, k, \tag{5.1}$$

respectively. Then, the IN solution space $\mathbb{S}_{\mathrm{IN}}$ in the considered partially connected ad-hoc networks is the solution space of the system of linear equations consisting of all the IN conditions, which can be generally given by

$$\sum_{q=1}^{Q} \mathbf{h}_{\mathrm{RD}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RS}}^{(q,j)} + h_{\mathrm{DS}}^{(k,j)} v^{(j)} + h_{\mathrm{DS}}^{(k,j)} u^{(k)*} = 0, \quad \forall j \neq k. \tag{5.2}$$

Taking the partial connectivity into account, i.e., the channel coefficients of the absent links are zero, the general form of the IN conditions (5.2) can be simplified depending on the locations of the j-th source node and the k-th destination node. Specially, if both the j-th source node and the k-th destination node belong to the same subnetwork, say the q-th subnetwork, (5.2) represents an intra-subnetwork IN condition, which can be simplified as

$$\mathbf{h}_{\mathrm{RD}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RS}}^{(q,j)} + h_{\mathrm{DS}}^{(k,j)} v^{(j)} + h_{\mathrm{DS}}^{(k,j)} u^{(k)*} = 0. \tag{5.3}$$

Note that as shown in (5.3), only the q-th relay participates in nullifying the intra-subnetwork interferences of the q-th subnetwork. If the j-th source node and the k-th destination node do not belong to the same subnetwork, but they are connected by a present inter-subnetwork direct link, i.e., $h_{\text{DS}}^{(k,j)} \neq 0$, (5.2) represents an intra-subnetwork IN condition, which can be simplified as

$$v^{(j)} + u^{(k)*} = 0. \tag{5.4}$$

Note that as shown in (5.4), no relays participate in nullifying the inter-subnetwork interferences. If the j-th source node and the k-th destination node do not belong to the same subnetwork, and they are not connected by any present inter-subnetwork direct link, i.e., $h_{\text{DS}}^{(k,j)} = 0$, (5.2) trivially holds.

Furthermore, the invalid IN solution subset $\mathbb{S}_{\text{inv}}^{(k)}$ with respect to the k-th node pair, which belongs to the q-th subnetwork, is formed by the IN solutions which violate the validity condition

$$\mathbf{h}_{\text{RD}}^{(q,k)*\text{T}} \mathbf{G}^{(q)} \mathbf{h}_{\text{RS}}^{(q,k)} + h_{\text{DS}}^{(k,k)} v^{(k)} + h_{\text{DS}}^{(k,k)} u^{(k)*} \neq 0, \quad \forall k. \tag{5.5}$$

Since the above validity condition is a linear inequality condition, each $\mathbb{S}_{\text{inv}}^{(k)}$ must either be identical to the IN solution space $\mathbb{S}_{\text{IN}}$ or be a hyperplane of $\mathbb{S}_{\text{IN}}$, see the discussions in Chapter 3.

However, it is difficult to investigate the IN solution space $\mathbb{S}_{\text{IN}}$ and the invalid IN solution subspaces $\mathbb{S}_{\text{inv}}^{(k)}$ in the considered partially connected ad-hoc networks following the same line as Chapter 3. This is mainly due to the irregular partial connectivity between different subnetworks, i.e., due to the fact that the presence of inter-subnetwork direct links does not follow a certain pattern. Resulting from this, the dimensions of the different $\mathbb{S}_{\text{inv}}^{(k)}$ may be different in a considered partially connected ad-hoc network. More specifically, some $\mathbb{S}_{\text{inv}}^{(k)}$ may be hyperplanes of $\mathbb{S}_{\text{IN}}$ whereas the others are identical to $\mathbb{S}_{\text{IN}}$. Consequently, it may occur that a randomly picked IN solution in $\mathbb{S}_{\text{IN}}$ is almost surely valid with respect to some node pairs but invalid with respect to other node pairs. This can be seen from the following example.

Example 5.1. Consider a partially connected ad-hoc network consisting of two subnetworks, where each subnetwork is of size three. The two subnetworks are connected by a single present inter-subnetwork direct link between a source node in one subnetwork and a destination node in the other subnetwork. Furthermore, assume that the relays in the first and the second subnetworks have a single antenna and two antennas, respectively. One can numerically verify that the dimension of $\mathbb{S}_{\mathrm{IN}}$ in this network is almost surely 4. Moreover, the dimension of $\mathbb{S}_{\mathrm{inv}}^{(k)}$ with respect to every node pair in the first subnetwork is also 4, whereas the dimension of $\mathbb{S}_{\mathrm{inv}}^{(k)}$ with respect to every node pair in the second subnetwork is 3, almost surely. That is to say, every IN solution in $\mathbb{S}_{\mathrm{IN}}$ must be invalid with respect to every node pair in the first subnetwork, but a randomly picked IN solution in $\mathbb{S}_{\mathrm{IN}}$ is almost surely valid with respect to every node pair in the second subnetwork.

In this thesis, the intra-subnetwork IN conditions (5.3) and the inter-subnetwork IN conditions (5.4) for partially connected ad-hoc networks will be investigated separately. Consider the intra-subnetwork IN conditions first. Let a subset of subnetworks be denoted by its index set $\Phi \subseteq \{1, \ldots, Q\}$. Let $\mathbb{S}_{\mathrm{intra}}^{\{q\}}$ denote the solution space of the intra-subnetwork IN conditions (5.3) of the q-th subnetwork[1]. In other words, $\mathbb{S}_{\mathrm{intra}}^{\{q\}}$ can be considered as the IN solution space in a fully connected ad-hoc network, as discussed in Chapter 3. Furthermore, define $\mathbb{S}_{\mathrm{intra}}^{\Phi}$ to be the Cartesian product of all $\mathbb{S}_{\mathrm{intra}}^{\{q\}}$ with $q \in \Phi$, i.e.,

$$\mathbb{S}_{\mathrm{intra}}^{\Phi} = \prod_{q \in \Phi} \mathbb{S}_{\mathrm{intra}}^{\{q\}} \tag{5.6}$$

holds. In this thesis, $\mathbb{S}_{\mathrm{intra}}^{\Phi}$ is referred to as the intra-subnetwork IN solution space of the subset Φ of subnetworks, which can be interpreted as the solution space of the intra-subnetwork IN conditions (5.3) of every subnetwork in the subset Φ. The dimension of the intra-subnetwork IN solution space $\mathbb{S}_{\mathrm{intra}}^{\Phi}$ can be given by

$$\begin{aligned} \dim \mathbb{S}_{\mathrm{intra}}^{\Phi} &= \sum_{q \in \Phi} \dim \mathbb{S}_{\mathrm{intra}}^{\{q\}} \\ &= \sum_{q \in \Phi} \max \left\{ M_q^2 - K_q(K_q - 3),\ 1 \right\}, \end{aligned} \tag{5.7}$$

almost surely, exploiting the results in Subsection 3.4.1. Moreover, every IN solution in $\mathbb{S}_{\mathrm{IN}}$, which nullifies all the intra- and inter-subnetwork interferences in the entire network, must lie in the intra-subnetwork IN solution space $\mathbb{S}_{\mathrm{intra}}^{\{1,\ldots,Q\}}$ of the set of all Q subnetworks, i.e.,

$$\mathbb{S}_{\mathrm{IN}} \subseteq \mathbb{S}_{\mathrm{intra}}^{\{1,\ldots,Q\}} = \prod_{q=1}^{Q} \mathbb{S}_{\mathrm{intra}}^{\{q\}} \tag{5.8}$$

[1] It is important to emphasize that the system of the intra-subnetwork IN conditions in the q-th subnetwork involves only $M_q^2 + 2K_q$ variables. Then, the results on the dimension of the IN solution space in fully connected ad-hoc networks can be directly exploited.

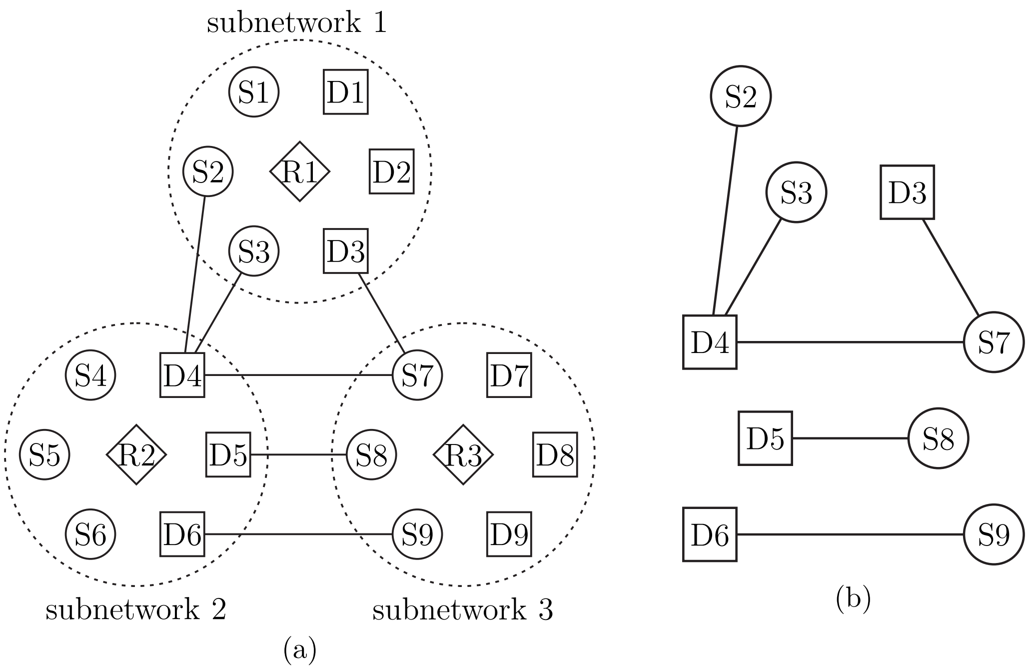


Figure 5.1.: (a) A partially connected ad-hoc network consisting of three subnetworks, and (b) the corresponding graph $\mathcal{G}_{\text{inter}}$

holds.

The inter-subnetwork IN conditions (5.4) will not be investigated directly. Instead, they will be transformed to a set of equivalent conditions called external constraints using graphs. To begin with, define a graph to describe the presence of inter-subnetwork direct links. Consider a bipartite graph $\mathcal{G}_{\text{inter}}$ for a given partially connected ad-hoc network. The edges of $\mathcal{G}_{\text{inter}}$ correspond to the present inter-subnetwork direct links in the network, and the vertices of $\mathcal{G}_{\text{inter}}$ correspond to the source and destination nodes being connected by the present inter-subnetwork direct links. An example of such a graph is given as follows.

Example 5.2. Consider a partially connected ad-hoc network consisting of three subnetworks, where each subnetwork is of size three, as depicted in Figure 5.1a. The corresponding graph $\mathcal{G}_{\text{inter}}$ is depicted in Figure 5.1b.

In fact, every edge of $\mathcal{G}_{\text{inter}}$ also implies an inter-subnetwork IN condition that has to be satisfied. Based on this, the external constraints can then be defined. Before giving the definition of external constraints, consider the following two examples, which help to understand the concept.

Example 5.3. In the partially connected ad-hoc network shown in Figure 5.1a, the destination node D4 in subnetwork 2 receives inter-subnetwork interferences from both source nodes S2 and S3 in subnetwork 1. In order to nullify these inter-subnetwork interferences, the inter-subnetwork IN conditions

$$v^{(2)} + u^{(4)*} = 0 \quad \text{and} \quad v^{(3)} + u^{(4)*} = 0 \tag{5.9}$$

have to be satisfied. Consequently, $v^{(2)} = v^{(3)}$ follows. In other words, $v^{(2)} = v^{(3)}$ can be considered as a constraint that subnetwork 1 must satisfy, in addition to its intra-subnetwork IN conditions, in order to nullify the inter-subnetwork interferences. In the corresponding graph $\mathcal{G}_{\text{inter}}$ as depicted in Figure 5.1b, the constraint $v^{(2)} = v^{(3)}$ follows from a path with both ends, i.e., S2 and S3, belonging to subnetwork 1. Similarly, the source node S3 and the destination node D3 in subnetwork 1 are connected by a path of length three in $\mathcal{G}_{\text{inter}}$. Hence, $v^{(3)}+u^{(3)*} = 0$, which results from the inter-subnetwork IN conditions implied by the edges of the path, is another constraint that subnetwork 1 must satisfy.

Example 5.4. In the same network shown in Figure 5.1a, the destination node D3 receives inter-subnetwork interference from the source node S7. The corresponding inter-subnetwork IN condition is

$$v^{(7)} + u^{(3)*} = 0. \tag{5.10}$$

Alternatively, $v^{(7)} + u^{(3)*} = 0$ can be considered as a constraint that subnetwork 1 and subnetwork 3 must satisfy together, in addition to the intra-subnetwork IN conditions in both subnetworks. In the graph $\mathcal{G}_{\text{inter}}$ as depicted in Figure 5.1b, the constraint $v^{(7)} + u^{(3)*} = 0$ follows from an edge, i.e., a path of length one, between the two subnetworks. Similarly, S3 and S7 are connected by a path of length two, which results in the constraint $v^{(3)} = v^{(7)}$ that subnetwork 1 and subnetwork 3 must satisfy together.

The above two examples reveal a relation between the present inter-subnetwork direct links, which are described by the graph $\mathcal{G}_{\text{inter}}$, and some constraints which need to be satisfied in order to nullify the inter-subnetwork interferences. These constraints result from a sequence of inter-subnetwork IN conditions, and can be graphically determined by paths in $\mathcal{G}_{\text{inter}}$. Unlike the inter-subnetwork IN conditions which only exist between a source node and a destination node, these constraints may also exist between two source nodes or two destination nodes. Furthermore, such a constraint does not depend on the intermediate nodes of the path, but only depends on the end nodes. That is to say, if multiple paths exist between two nodes, they result in the same constraint. These constraints are called external constraints[2] in this thesis, and a formal definition is given as follows.

Definition 5.1 (External constraint). In a considered partially connected ad-hoc network, an external constraint for the subset $\Phi \subseteq \{1, \ldots, Q\}$ of subnetworks results from a path of $\mathcal{G}_{\text{inter}}$ with both ends in the subset Φ of subnetworks. In particular, if such a path exists between the j-th source node and the k-th destination node, the external constraint $v^{(j)} + u^{(k)*} = 0$ follows. If such a path exists between two source nodes or two destination nodes, the external constraint $v^{(j)} = v^{(k)}$ or $u^{(j)} = u^{(k)}$ follows, respectively.

The following properties of external constraints can be directly deduced from the definition.

Remark 5.1. If Φ' is a subset of Φ, every external constraint for the subset Φ' of subnetworks is an external constraint for the subset Φ of subnetworks.

Remark 5.2. The external constraints are equivalent to the inter-subnetwork IN conditions in the sense that all the inter-subnetwork IN conditions are satisfied if and only if the external constraints for the set $\{1, \ldots, Q\}$ of all subnetworks are satisfied.

Note that the concept of external constraints is not introduced for the purpose of removing linear dependency of the inter-subnetwork IN conditions. On the contrary, it usually introduces more linear dependency. In order to obtain a set of linearly independent external constraints, graphs of external constraints can be exploited. Let $\mathcal{G}_{\text{EC}}^{\Phi}$ be a graph, whose edges are the external constraints for the subset Φ of subnetworks and the vertices are the nodes being involved in the external constraints. For instance, the graphs $\mathcal{G}_{\text{EC}}^{\{1\}}$, $\mathcal{G}_{\text{EC}}^{\{2,3\}}$, and $\mathcal{G}_{\text{EC}}^{\{1,2,3\}}$ for the network shown in Figure 5.1a are depicted in Figure 5.2. Thus, a set of linearly independent external constraints for the subset Φ of subnetworks can be represented by a maximal forest in $\mathcal{G}_{\text{EC}}^{\Phi}$, see Appendix B. Then, the number of linearly independent external constraints, which will be denoted by N_{EC}^{Φ} in the following part of this chapter, is equal to the rank of the incidence matrix of $\mathcal{G}_{\text{EC}}^{\Phi}$. Concerning the number N_{EC}^{Φ} of linearly independent external constraints, the following properties shall be noticed.

[2] The word "external" emphasizes that these constraints are derived from the inter-subnetwork IN conditions, rather than the intra-subnetwork IN conditions. It does not suggest that these constraints are from the outside of a subset of subnetworks.

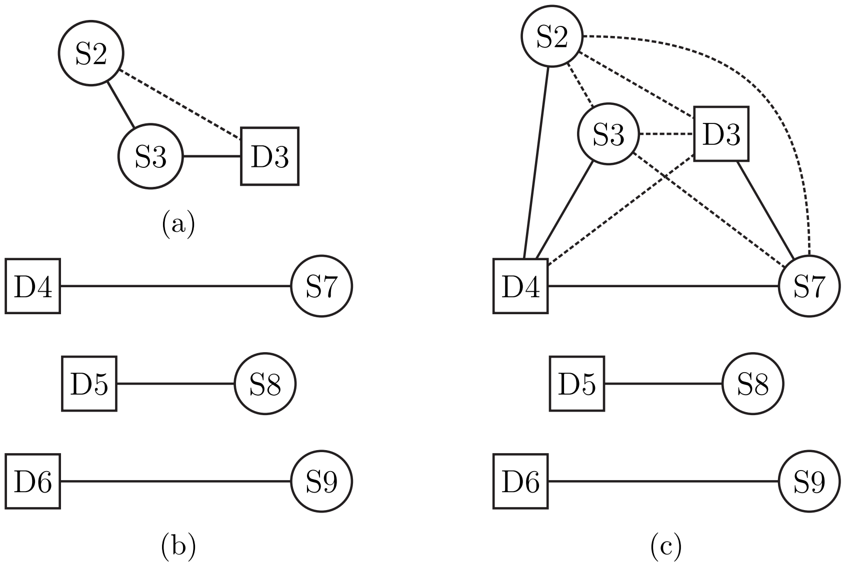


Figure 5.2.: Graphs of external constraints (a) $\mathcal{G}_{\mathrm{EC}}^{\{1\}}$, (b) $\mathcal{G}_{\mathrm{EC}}^{\{2,3\}}$, and (c) $\mathcal{G}_{\mathrm{EC}}^{\{1,2,3\}}$ for the network shown in Figure 5.1a. Solid lines represent a maximal forest in each graph.

Remark 5.3. A subset Φ of subnetworks has $2\sum_{q\in\Phi} K_q$ nodes. Thus, the graph $\mathcal{G}_{\mathrm{EC}}^{\Phi}$ of external constraints for the subset Φ has at most $2\sum_{q\in\Phi} K_q$ vertices. Consequently, the number N_{EC}^{Φ} of linearly independent external constraints for Φ must satisfy $N_{\mathrm{EC}}^{\Phi} \leq 2\sum_{q\in\Phi} K_q - 1$.

Remark 5.4. If the subset Φ of subnetworks consists of two disjoint subsets Φ' and Φ'' of subnetworks, i.e., Φ' and Φ'' do not have any common subnetwork, the numbers of linearly independent external constraints for these subsets of subnetworks satisfy $N_{\mathrm{EC}}^{\Phi} \geq N_{\mathrm{EC}}^{\Phi'} + N_{\mathrm{EC}}^{\Phi''}$. This results from Remark 5.1. Furthermore, $N_{\mathrm{EC}}^{\Phi} - N_{\mathrm{EC}}^{\Phi'} - N_{\mathrm{EC}}^{\Phi''}$ is the number of linearly independent external constraints between the subsets Φ' and Φ''.

Remark 5.5. Exploiting graph theory, there is a simpler way to determine N_{EC}^{Φ}, without the need to find the graph $\mathcal{G}_{\mathrm{EC}}^{\Phi}$ of external constraints. One can first count the number of vertices of $\mathcal{G}_{\mathrm{inter}}$ which belong to the subset Φ. Then count the number of connected components in $\mathcal{G}_{\mathrm{inter}}$ which have common vertices with the subset Φ. Finally, N_{EC}^{Φ} is given by the difference of these two numbers. For instance, the graph $\mathcal{G}_{\mathrm{inter}}$ as depicted in Figure 5.1b has three vertices in subnetwork 1, i.e., S2, S3, and D3, and only one connected component in $\mathcal{G}_{\mathrm{inter}}$ has common vertices with subnetwork 1, i.e., the subgraph consisting of the vertices

S2, S3, D3, D4, S7, and the edges connecting them. This yields $N_{\text{EC}}^{\{1\}} = 2$, which can be seen from Figure 5.2a as well.

Finally, the intra-subnetwork IN conditions and the external constraints for a subset Φ of subnetworks can be considered jointly. Define $\mathbb{S}_{\text{intra+EC}}^{\Phi}$ as the solution space of the system of linear equations consisting of all the intra-subnetwork IN conditions and the external constraints for the subset Φ of subnetworks. Then,

$$\mathbb{S}_{\text{IN}} = \mathbb{S}_{\text{intra+EC}}^{\{1,\ldots,Q\}} \tag{5.11}$$

can be immediately observed, since the external constraints for the set $\{1, \ldots, Q\}$ of all subnetworks are equivalent to all the inter-subnetwork IN conditions as mentioned in Remark 5.2. Furthermore, the dimension of $\mathbb{S}_{\text{intra+EC}}^{\Phi}$ can be almost surely given by

$$\begin{aligned} \dim \mathbb{S}_{\text{intra+EC}}^{\Phi} &= \max \left\{ \dim \mathbb{S}_{\text{intra}}^{\Phi} - N_{\text{EC}}^{\Phi},\ 1 \right\} \\ &= \max \left\{ \sum_{q \in \Phi} \max \left\{ M_q^2 - K_q(K_q - 3), 1 \right\} - N_{\text{EC}}^{\Phi},\ 1 \right\}, \end{aligned} \tag{5.12}$$

where $\dim \mathbb{S}_{\text{intra}}^{\Phi}$ has been given in (5.7). In simple words, this can be understood as follows[3]. Firstly, the N_{EC}^{Φ} linearly independent external constraints for the subset Φ reduce the dimension of $\mathbb{S}_{\text{intra}}^{\Phi}$ by N_{EC}^{Φ}. Secondly, the invalid solution with $\mathbf{G}^{(q)} = \mathbf{0}_{M_q}$, $\forall q \in \Phi$, and $v^{(k)} = -u^{(j)*} = 1$, $\forall j, k \in \Phi$ is always able to satisfy all intra-subnetwork IN conditions, see Remark 3.2, as well as all the external constraints for Φ. Therefore, $\mathbb{S}_{\text{intra}}^{\Phi}$ has at least one dimension.

5.2.2. Feasible relay antenna configurations

With the help of external constraints, the feasibility conditions for relay-aided IA with full channel knowledge in the considered partially connected networks can be addressed.

Proposition 5.1. In a considered partially connected ad-hoc network with subnetwork sizes $K_q \geq 3$, if the channel coefficients are independently drawn from a continuous distribution and full channel knowledge is assumed, then relay-aided IA is feasible with probability one if and only if

$$\dim \mathbb{S}_{\text{intra+EC}}^{\Phi} = \max \left\{ \sum_{q \in \Phi} \max \left\{ M_q^2 - K_q(K_q - 3), 1 \right\} - N_{\text{EC}}^{\Phi},\ 1 \right\} \geq 2, \tag{5.13}$$

[3] A strict proof of this follows the same line as the proof of Proposition 3.4, and is omitted.

or equivalently,

$$\sum_{q\in\Phi} M_q^2 - K_q(K_q - 3) - N_{\text{EC}}^{\Phi} - 2 \geq 0, \tag{5.14}$$

holds for every non-empty subset $\Phi \subseteq \{1, \ldots, Q\}$ of subnetworks, where N_{EC}^{Φ} denotes the number of linearly independent external constraints for the subset Φ of subnetworks.

Proof. See Appendix D. □

In simple words, Proposition 5.1 shows that the feasibility conditions for relay-aided IA in the considered partially connected ad-hoc networks depend on the numbers of node pairs, relays, and relay antennas in every part, i.e., every subset of subnetworks, of the entire network. This results from the irregular partial connectivity between different subnetworks. That is to say, every subnetwork shall provide enough free variables[4] such that there is a valid intra-subnetwork IN solution which is also able to, in cooperation with other subnetworks, nullify all the inter-subnetwork interferences.

Remark 5.6. In a partially connected ad-hoc network consisting of Q subnetworks, the total number of non-empty subsets of subnetworks is $2^Q - 1$. Therefore, generally speaking, $2^Q - 1$ inequalities as (5.14) have to be checked in order to determine whether relay-aided IA is almost surely feasible or infeasible in a partially connected ad-hoc network. That is to say, the feasibility problem for relay-aided IA in the considered partially connected ad-hoc networks is an NP problem.

In the following, the feasibility conditions given in (5.14) will be interpreted as the required numbers of relay antennas in a partially connected ad-hoc network with fixed topology, i.e., with fixed subnetwork sizes and present inter-subnetwork direct links. Let a tuple $(M_1^2, \ldots, M_Q^2)$ be referred to as a feasible relay antenna configuration[5] if it satisfies

$$\sum_{q\in\Phi} M_q^2 \geq \sum_{q\in\Phi} K_q(K_q - 3) + N_{\text{EC}}^{\Phi} + 2, \quad \forall\Phi \subseteq \{1, \ldots, Q\}. \tag{5.15}$$

[4] In this context, the number of free variables is better to be understood as the degrees of freedom, which is the number of parameters of a system that may vary independently. However, the term “degrees of freedom” has been used in this thesis to represent another quantity.

[5] In this thesis, it is assumed that each subnetwork only has a single relay for simplicity. Hence, M_q must be an integer resulting that not every feasible relay antenna configuration is meaningful. Alternatively, one can also consider M_q^2 single-antenna relays for a subnetwork.

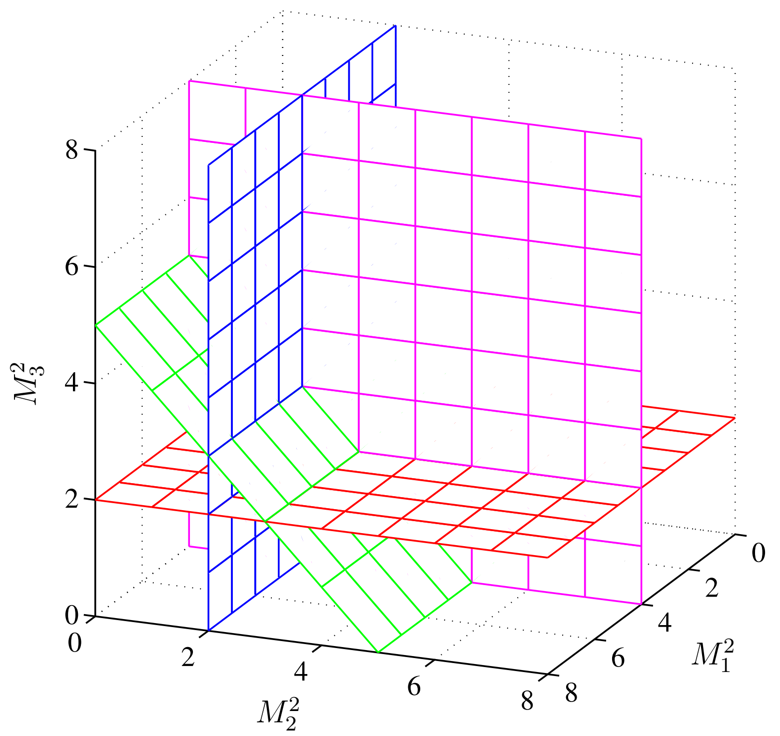


Figure 5.3.: Boundaries of feasible relay antenna configurations in the network shown in Figure 5.1a

Example 5.5. Recall the network shown in Figure 5.1a as an example. By (5.15), the feasible relay antenna configurations in this network can be characterized by

$$\begin{array}{lll} M_1^2 \geq 4, & M_2^2 \geq 2, & M_3^2 \geq 2, \\ M_1^2 + M_2^2 \geq 5, & M_1^2 + M_3^2 \geq 5, & M_2^2 + M_3^2 \geq 5, \\ M_1^2 + M_2^2 + M_3^2 \geq 8. & & \end{array}$$

Specially, only

$$M_1^2 \geq 4, \ M_2^2 \geq 2, \ M_3^2 \geq 2, \text{ and } M_1^2 + M_2^2 \geq 5 \tag{5.16}$$

are active in this network, as depicted in Figure 5.3.

Remark 5.7. Note that the number N_{EC}^{Φ} of linearly independent external constraints for a subset Φ of subnetworks must be upper bounded by $2\sum_{q\in\Phi} K_q - 1$, see Remark 5.3. Substituting $N_{\text{EC}}^{\Phi} = 2\sum_{q\in\Phi} K_q - 1$ in (5.15) yields that

$$M_q^2 = K_q^2 - K_q + 1, \quad \forall q, \tag{5.17}$$

must form a feasible relay antenna configuration. That is to say, even in the worst case where all the inter-subnetwork direct links are present, the required number of antennas at a relay only depends on the size of the subnetwork it belongs to, regardless of the sizes of other subnetworks and the total number of subnetworks in the entire network.

5.3. Relay-aided interference alignment with partial channel knowledge

5.3.1. Toy example

In this section, a relay-aided IA scheme which achieves the DoF of the considered partially connected ad-hoc networks with partial channel knowledge will be proposed. Before discussing the proposed relay-aided IA scheme with partial channel knowledge in the next subsection, the possibilities of achieving relay-aided IA in partially connected ad-hoc networks without full channel knowledge will be first illustrated by a toy example in this subsection.

Consider a partially connected ad-hoc network consisting of two subnetworks, where each subnetwork is of size three. Specially, assume that 5 inter-subnetwork direct links are present, as shown by the corresponding graph $\mathcal{G}_{\text{inter}}$ in Figure 5.4. Note that in Figure 5.4, the source nodes S1 and S6 and the destination node D1 are not depicted since they are not end nodes of any present inter-subnetwork direct link. In this toy example, the feasible relay antenna configurations with full channel knowledge are bounded, according to (5.15), by

$$M_1^2 \geq 2, \quad M_2^2 \geq 3, \quad \text{and} \quad M_1^2 + M_2^2 \geq 7, \tag{5.18}$$

where M_1 and M_2 denote the numbers of antennas of the first and the second relay, respectively. In the following, three other schemes with different types of channel knowledge will be introduced. All of them are also able to achieve the DoF of the network. The corresponding feasible relay antenna configurations will be compared.

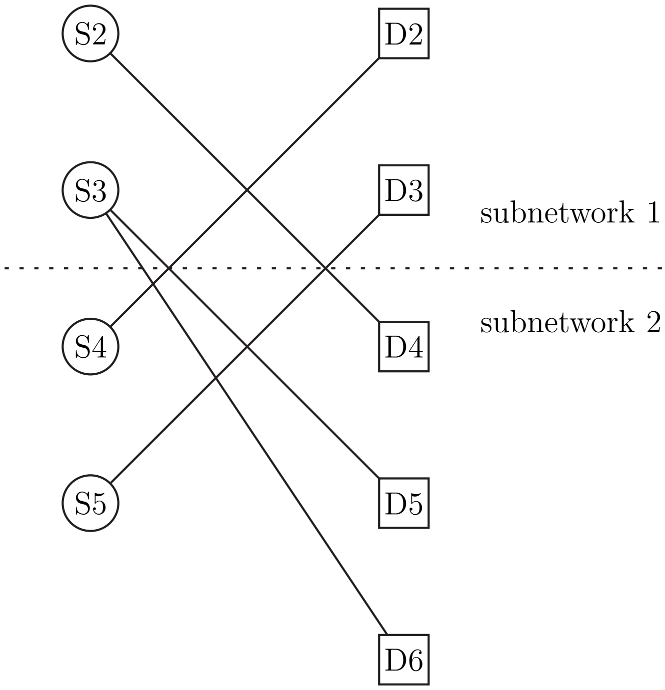


Figure 5.4.: Graph $\mathcal{G}_{\text{inter}}$ of a partially connected ad-hoc network consisting of two subnetworks, where each subnetwork is of size three and 5 inter-subnetwork direct links are present. The source nodes S1 and S6 and the destination node D1 are not vertices of $\mathcal{G}_{\text{inter}}$ since they are not end nodes of any present inter-subnetwork direct link.

For the first scheme, assume that each subnetwork only knows perfect and instantaneous CSI of the intra-subnetwork links of itself, but does not know the CSI of the intra-subnetwork links of the other subnetwork nor the present inter-subnetwork direct links. In this thesis, this type of CSI will be referred to as the intra-subnetwork CSI. Note that in order to acquire the intra-subnetwork CSI, the amount of time, as well as energy, that shall be used for channel estimation only depends on the sizes of the individual subnetworks, rather than the size of the entire network. In this case, the difficulty lies in nulling the inter-subnetwork interferences. Fortunately, the inter-subnetwork IN condition (5.4) suggests that an inter-subnetwork interference can be nullified by simply choosing the variables $v^{(j)}$ and $u^{(k)}$ of the corresponding source and destination nodes as $v^{(j)} + u^{(k)*} = 0$, without the need to know the channel realization of the present inter-subnetwork direct link $h_{\text{DS}}^{(k,j)}$. Therefore, every subnetwork can fix the variables $v^{(j)}$ and $u^{(k)}$ of its source and destination nodes as $v^{(j)} = 1$ and $u^{(k)} = -1$ to nullify the inter-subnetwork interferences. Since the subnetworks do not know which source and destination nodes are connected by present inter-subnetwork direct links, each subnetwork has to fix the variables $v^{(j)}$ and $u^{(k)}$ of all the source and destination nodes. Substituting this into the intra-subnetwork IN conditions (5.3) yields

$$\mathbf{h}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{h}_{\mathrm{RS}}^{(q,j)} = 0. \tag{5.19}$$

That is to say, the intra-subnetwork interferences in each subnetwork have to be nullified by the relay alone. This is possible using the orthogonalize-and-forward relaying scheme [RW07]. To this end, the required numbers of relay antennas, i.e., the feasible relay antenna configurations, are given by

$$M_1^2 \geq 7 \quad \text{and} \quad M_2^2 \geq 7 \tag{5.20}$$

in order to obtain a non-trivial solution, i.e., in order to ensure that the data symbols can be successfully transmitted through the network.

For the second scheme, assume that each subnetwork knows, in addition to the intra-subnetwork CSI, the subnetwork sizes and the presence of inter-subnetwork direct links in the entire network. In this thesis, this type of CSI is referred to as the network topology. Note that even in large partially connected ad-hoc networks with many source-destination node pairs, the network topology is still relatively easy to acquire, since the presence of each inter-subnetwork direct link contains only one bit information [Jaf14]. In this case, each subnetwork knows which source and destination nodes are connected by present inter-subnetwork direct links. Therefore, only the variables $v^{(j)}$ and $u^{(k)}$ of these nodes have to be fixed. In the toy example, the variables of the source nodes S1 and S6 and the destination node D1 must not be fixed. Instead, they can be chosen to help nullifying the intra-subnetwork interferences. Consequently, the required numbers of free variables provided by the relays can be reduced as compared to the case with only intra-subnetwork CSI. In this toy example, the feasible relay antenna configurations with intra-subnetwork CSI and network topology are bounded by

$$M_1^2 \geq 5 \quad \text{and} \quad M_2^2 \geq 6, \tag{5.21}$$

which can be numerically verified.

Finally, a certain form of cooperation between the two subnetworks can be taken into consideration. Assume that each subnetwork knows the intra-subnetwork CSI and the network topology. Using the intra-subnetwork CSI and the network topology, each subnetwork can determine its intra-subnetwork IN solution space $\mathbb{S}_{\mathrm{intra}}^{\{q\}}$ and the external constraints for it, respectively. That is to say, each subnetwork can determine its solution space $\mathbb{S}_{\mathrm{intra+EC}}^{\{q\}}$, as discussed in the previous section.

Thus, any valid solution $\mathbf{x}_1 \in \mathbb{S}^{\{1\}}_{\text{intra+EC}}$ is able to nullify the intra-subnetwork interferences in subnetwork 1; any valid solution $\mathbf{x}_2 \in \mathbb{S}^{\{2\}}_{\text{intra+EC}}$ is able to nullify the intra-subnetwork interferences in subnetwork 2 and satisfy the external constraint $u^{(5)} = u^{(6)}$ for subnetwork 2. Consequently, if $\mathbf{x}_1$ and $\mathbf{x}_2$ are selected in such a way that all the inter-subnetwork interferences can be nullified, combining $\mathbf{x}_1$ and $\mathbf{x}_2$ yields a valid IN solution for the entire network. In particular, $\mathbf{x}_1$ and $\mathbf{x}_2$ only have to satisfy the inter-subnetwork IN conditions

$$v^{(2)} + u^{(4)*} = 0, \tag{5.22}$$
$$v^{(3)} + u^{(5)*} = 0, \tag{5.23}$$
$$v^{(4)} + u^{(2)*} = 0, \tag{5.24}$$
$$v^{(5)} + u^{(3)*} = 0, \tag{5.25}$$

because the inter-subnetwork IN condition

$$v^{(3)} + u^{(6)*} = 0 \tag{5.26}$$

is implied by (5.23) and the external constraint $u^{(5)} = u^{(6)}$ for subnetwork 2 and can be automatically satisfied. The following two-step scheme can be employed to select such $\mathbf{x}_1$ and $\mathbf{x}_2$. In the first step, let subnetwork 1 randomly pick a solution $\mathbf{x}_1$ from $\mathbb{S}^{\{1\}}_{\text{intra+EC}}$. In order to guarantee that the picked solution $\mathbf{x}_1$ is almost surely valid with respect to every node pair in subnetwork 1, $\dim \mathbb{S}^{\{1\}}_{\text{intra+EC}} \geq 2$ must be satisfied, i.e., the number of antennas of the first relay shall satisfy

$$M_1^2 \geq 2. \tag{5.27}$$

Furthermore, subnetwork 1 forwards the variables $v^{(2)}$, $v^{(3)}$, $u^{(2)}$, and $u^{(3)}$ in the selected solution $\mathbf{x}_1$ to subnetwork 2. In the second step, let subnetwork 2 use the knowledge of these variables to select a solution $\mathbf{x}_2$ from $\mathbb{S}^{\{2\}}_{\text{intra+EC}}$ such that the inter-subnetwork IN conditions (5.22) – (5.25) are satisfied. The existence of such a solution $\mathbf{x}_2 \in \mathbb{S}^{\{2\}}_{\text{intra+EC}}$ requires $\dim \mathbb{S}^{\{2\}}_{\text{intra+EC}} \geq 4$, i.e., the number of antennas of the second relay being

$$M_2^2 \geq 5. \tag{5.28}$$

Since the solution $\mathbf{x}_1$ is randomly picked by subnetwork 1 and $\mathbf{x}_2$ is selected based on $\mathbf{x}_1$, the selection of $\mathbf{x}_2$ is independent from the channel realization of the intra-subnetwork links in subnetwork 2. That is to say, $\mathbf{x}_2$ can be considered as a randomly picked solution from $\mathbb{S}^{\{2\}}_{\text{intra+EC}}$. Therefore, $\mathbf{x}_2$ is almost surely valid with respect to every node pair in subnetwork 2. Hence, $\mathbf{x}_1$ and $\mathbf{x}_2$ form a valid IN solution for the entire network. Alternatively, $\mathbf{x}_1$ and $\mathbf{x}_2$ can be selected in

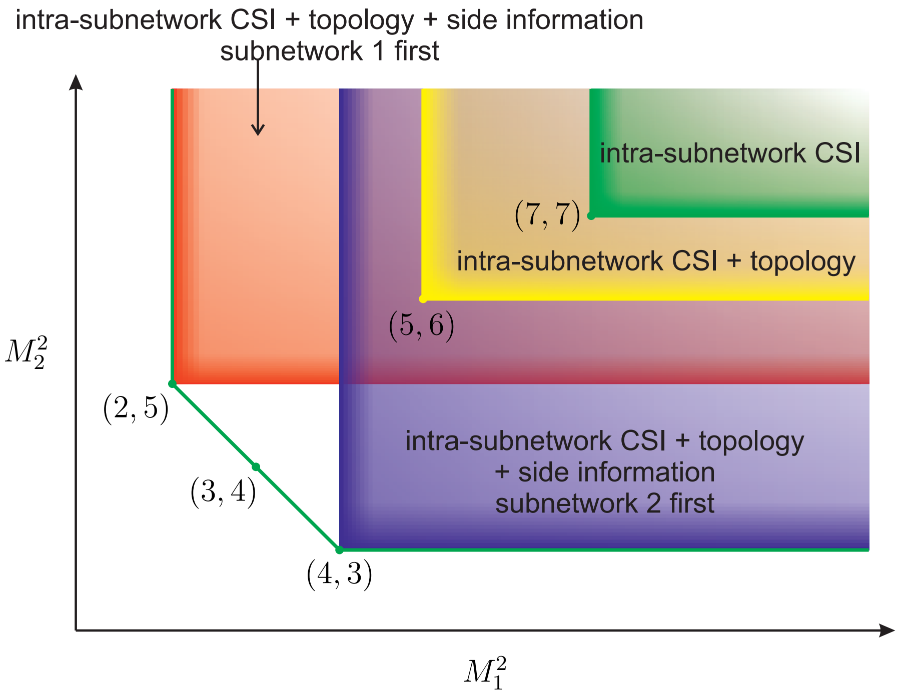


Figure 5.5.: Comparison of feasible relay antenna configurations in the toy example

reverse order. In the first step, let subnetwork 2 randomly pick a solution $\mathbf{x}_2$ from $\mathbb{S}^{\{2\}}_{\text{intra+EC}}$ and forward the variables $v^{(4)}$, $v^{(5)}$, $u^{(4)}$, and $u^{(5)}$ in the selected solution $\mathbf{x}_2$ to subnetwork 1. In the second step, let subnetwork 1 use the knowledge of these variables to select a solution $\mathbf{x}_1$ from $\mathbb{S}^{\{1\}}_{\text{intra+EC}}$ such that the inter-subnetwork IN conditions (5.22) – (5.25) are satisfied. In this case, the required numbers of relays antennas are given by

$$M_1^2 \geq 4 \quad \text{and} \quad M_2^2 \geq 3, \tag{5.29}$$

for the same reasons as mentioned before.

In the third scheme discussed above, the cooperation between the two subnetworks is realized by forwarding some variables $v^{(j)}$ and $u^{(k)}$ from one subnetwork to another. These variables will be referred to as side information in this thesis. The side information can be considered as some "compressed" CSI, which contains the most relevant information for nullifying inter-subnetwork interferences. More precisely, the number of variables $v^{(j)}$ and $u^{(k)}$ that shall be forwarded from one subnetwork to another as side information is determined by the number of linearly independent external constraints between the two subnetworks. In this toy example, four variables shall be forwarded as side information, regardless of the order of selecting $\mathbf{x}_1$ and $\mathbf{x}_2$. The feasible relay antenna configurations for the aforementioned three schemes are compared in Figure 5.5. Among the three schemes, the third one, i.e., the one with intra-subnetwork CSI, the network topology, and the side information, requires only few relay antennas as compared to the other two. Furthermore, the third scheme can achieve relay-aided IA if the relay antennas are configured as $(M_1^2, M_2^2) = (2, 5)$ and $(M_1^2, M_2^2) = (4, 3)$, depending on the order of selecting $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. These two relay antenna configurations are indeed Pareto points of the feasible relay antenna configurations with full channel knowledge. However, if the relay antennas are configured as $(M_1^2, M_2^2) = (3, 4)$, relay-aided IA cannot be achieved using the third scheme. This can be considered as a penalty for the lack of full channel knowledge.

5.3.2. Generalization to multiple subnetworks

In this subsection, the third scheme discussed in the previous subsection will be generalized. The considered partial channel knowledge includes three types of CSI: the intra-subnetwork CSI, the network topology, and the side information. Roughly speaking, the proposed relay-aided IA scheme with partial channel knowledge is able to find a valid IN solution $\mathbf{x}$ from the IN solution space $\mathbb{S}_{\text{IN}}$ for the entire network within Q steps. Following a certain order, each subnetwork will select a solution $\mathbf{x}_q$ from the solution space $\mathbb{S}_{\text{intra+EC}}^{\{q\}}$ and forwards some of the variables $v^{(j)}$ and $u^{(k)}$ from the selected solution $\mathbf{x}_q$ to other subnetworks as side information in one of the Q steps. Then, the valid IN solution $\mathbf{x}$ for the entire network is formed by the solutions $\mathbf{x}_1, \ldots, \mathbf{x}_Q$ selected by the individual subnetworks. The answers to the following three questions further explain the details of the proposed scheme.

Q1. In which order shall the Q subnetworks select their solutions $\mathbf{x}_q$? Generally speaking, every permutation of the Q subnetworks corresponds to an order to perform the proposed relay-aided IA scheme with partial channel knowledge. However, different permutations of the Q subnetworks yield different feasible relay antenna configurations, which will be further discussed at the end of this subsection. In order to facilitate the following discussions, the natural permutation of the subnetworks will be considered, i.e., the q-th subnetwork selects its solution in the q-th step, unless specifically mentioned otherwise.

Q2. Which variables $v^{(j)}$ and $u^{(k)}$ shall be forwarded from one subnet-

work to another as side information? First of all, one subnetwork does not have to forward variables to every other subnetwork as side information. Specifically, if the q-th subnetwork selects its solution in the q-th step, then only the subnetworks in the subset $\{1, \ldots, q-1\}$ may need to forward variables to the q-th subnetwork as side information and the q-th subnetwork may need to forward variables to only the subnetworks in the subset $\{q+1, \ldots, Q\}$ as side information. Secondly, if one subnetwork needs to forward side information to another, it does not have to forward all the variables $v^{(j)}$ and $u^{(k)}$. Intuitively, if the r-th subnetwork selects its solution before the q-th subnetwork and there is an external constraint between them, the r-th subnetwork needs to forward the variable $v^{(j)}$ or $u^{(k)}$ of the involved source or destination node, respectively, to the q-th subnetwork as side information. Using this intuitive approach, whether or not a variable $v^{(j)}$ or $u^{(k)}$ shall be forwarded to the q-th subnetwork can be decided by the r-th subnetwork alone, without coordination with other subnetworks. However, this intuitive approach is usually inefficient. Note that the side information for the q-th subnetwork is used to satisfy the external constraints between the subset $\{1, \ldots, q-1\}$ and the q-th subnetwork. For this reason, the q-th subnetwork only needs $N_{\text{EC}}^{\{1,\ldots,q\}} - N_{\text{EC}}^{\{1,\ldots,q-1\}} - N_{\text{EC}}^{\{q\}}$ variables as side information, see Remark 5.4. In this thesis, it is simply assumed that every subnetwork is able to acquire the required side information. The problem of how to optimize this procedure will not be further discussed.

Q3. How does the q-th subnetwork select its solution $\mathbf{x}_q$ using partial channel knowledge? Firstly, the q-th subnetwork determines its intra-subnetwork IN solution space $\mathbb{S}_{\text{intra}}^{\{q\}}$ using the intra-subnetwork CSI. Secondly, the q-th subnetwork determines the external constraints for it using the network topology. That is to say, the q-th subnetwork can determine the solution space $\mathbb{S}_{\text{intra+EC}}^{\{q\}}$. Finally, the q-th subnetwork uses the side information acquired from the subnetworks in the subset $\{1, \ldots, q-1\}$ to select a solution $\mathbf{x}_q \in \mathbb{S}_{\text{intra+EC}}^{\{q\}}$ which satisfies all the external constraints between the subset $\{1, \ldots, q-1\}$ and the q-th subnetwork. Specially, the first subnetwork can randomly pick a solution from $\mathbb{S}_{\text{intra+EC}}^{\{1\}}$.

For the proposed relay-aided IA scheme with partial channel knowledge, the feasibility conditions guarantee that the solution $\mathbf{x}_q \in \mathbb{S}_{\text{intra+EC}}^{\{q\}}$ selected by each subnetwork is almost surely valid with respect to every node pair in the subnetwork. As illustrated by the toy example in the previous subsection, the feasibility conditions depend on the order to perform the propose relay-aided IA scheme. Consider the natural permutation of the subnetworks without loss of generality, i.e., the q-th subnetwork will select its solution $\mathbf{x}_q$ from $\mathbb{S}_{\text{intra+EC}}^{\{q\}}$ in the q-th step. Then the feasibility conditions can be found as follows. Firstly, for every subnetwork, $\mathbb{S}_{\text{intra+EC}}^{\{q\}}$ must have at least two dimensions otherwise every solution $\mathbf{x}_q \in \mathbb{S}_{\text{intra+EC}}^{\{q\}}$ is an invalid solution with respect to every node pair in the subnetwork, i.e.,

$$\dim \mathbb{S}_{\text{intra+EC}}^{\{q\}} = \max\left\{\max\left\{M_q^2 - K_q(K_q-3), 1\right\} - N_{\text{EC}}^{\{q\}},\ 1\right\} \geq 2, \quad \forall q, \tag{5.30}$$

must be satisfied, where the expression (5.12) for $\dim \mathbb{S}_{\text{intra+EC}}^{\{q\}}$ is considered. Secondly, for the q-th subnetwork, where $q \geq 2$ holds, the dimension of $\mathbb{S}_{\text{intra+EC}}^{\{q\}}$ must also be sufficiently large such that there exists a solution $\mathbf{x}_q \in \mathbb{S}_{\text{intra+EC}}^{\{q\}}$ which is able to satisfy all the external constraints between the subset $\{1, \ldots, q-1\}$ and the q-th subnetwork. Since the number of linearly independent external constraints between the subset $\{1, \ldots, q-1\}$ and the q-th subnetwork is given by $N_{\text{EC}}^{\{1,\ldots,q\}} - N_{\text{EC}}^{\{1,\ldots,q-1\}} - N_{\text{EC}}^{\{q\}}$,

$$\begin{aligned}\dim \mathbb{S}_{\text{intra+EC}}^{\{q\}} &= \max\left\{\max\left\{M_q^2 - K_q(K_q-3), 1\right\} - N_{\text{EC}}^{\{q\}},\ 1\right\} \\ &\geq N_{\text{EC}}^{\{1,\ldots,q\}} - N_{\text{EC}}^{\{1,\ldots,q-1\}} - N_{\text{EC}}^{\{q\}}, \quad \forall q \geq 2,\end{aligned} \tag{5.31}$$

must also be satisfied. Alternatively, the feasibility conditions given by (5.30) and (5.31) can be interpreted as the feasible relay antenna configurations, which are given by

$$M_1^2 \geq K_1(K_1-3) + N_{\text{EC}}^{\{1\}} + 2 \tag{5.32}$$

and

$$M_q^2 \geq K_q(K_q-3) + \max\left\{N_{\text{EC}}^{\{q\}} + 2,\ N_{\text{EC}}^{\{1,\ldots,q\}} - N_{\text{EC}}^{\{1,\ldots,q-1\}}\right\}, \quad \forall q \geq 2. \tag{5.33}$$

Intuitively, relay-aided IA with full channel knowledge shall be at least as capable as the proposed relay-aided IA scheme with partial channel knowledge. That is to say, the feasible relay antenna configurations with partial channel knowledge, as given by (5.32) and (5.33), must be a subset of the feasible relay antenna configurations with full channel knowledge, as given by (5.15). For completeness, a proof of this is given in this thesis.

Proposition 5.2. In a considered partially connected ad-hoc network, suppose the tuple $(M_{1,\min}^2, \ldots, M_{Q,\min}^2)$ is a Pareto point of the feasible relay antenna configurations for the proposed relay-aided IA scheme with partial channel knowledge, i.e., $(M_{1,\min}^2, \ldots, M_{Q,\min}^2)$ satisfies (5.32) and (5.33) with equality. Then, $(M_{1,\min}^2, \ldots, M_{Q,\min}^2)$ is also a feasible relay antenna configuration for relay-aided IA with full channel knowledge, i.e.,

$$\sum_{q\in\Phi} M_{q,\min}^2 \geq \sum_{q\in\Phi} K_q(K_q-3) + N_{\text{EC}}^{\Phi} + 2, \quad \forall \Phi \subseteq \{1, \ldots, Q\}, \tag{5.34}$$

holds.

Proof. See Appendix D. □

However, the question whether or not the tuple $(M^2_{1,\min},\ \ldots,\ M^2_{Q,\min})$ is also a Pareto point of the feasible relay antenna configurations with full channel knowledge is difficult to answer. This problem is equivalent to finding the Pareto points of the feasible relay antenna configurations with full channel knowledge, which requires to check $2^Q - 1$ inequalities and is an NP problem, see Remark 5.6. In simple words, the answer to this question depends on the network topology and the considered permutation of subnetworks, which can be illustrated by the following example.

Example 5.6. Consider a partially connected ad-hoc network as shown in Figure 5.6. In this particular network, the external constraints are simply the inter-subnetwork IN conditions for the 6 present inter-subnetwork direct links. The feasible relay antenna configurations with full channel knowledge can be found using (5.15), and they are bounded by $M^2_1 \geq 2$, $M^2_2 \geq 2$, and $M^2_3 \geq 2$ in this network. Furthermore, the three subnetworks have 6 permutations, which correspond to 6 different orders to perform the proposed relay-aided IA scheme with partial channel knowledge. Each permutation of subnetworks yields a tuple of $(M^2_{1,\min},\ M^2_{2,\min},\ M^2_{3,\min})$, as listed in Table 5.1. It can be seen that only two permutations of the subnetworks, i.e., $(2,\ 3,\ 1)$ and $(3,\ 2,\ 1)$, yield a tuple of $(M^2_{1,\min},\ M^2_{2,\min},\ M^2_{3,\min}) = (2,\ 2,\ 2)$, which is a Pareto point of the feasible relay antenna configurations with full channel knowledge.

Table 5.1.

permutation of subnetworks	$M^2_{1,\min}$	$M^2_{2,\min}$	$M^2_{3,\min}$
(1, 2, 3)	2	2	3
(1, 3, 2)	2	3	2
(2, 1, 3)	2	2	3
(2, 3, 1)	2	2	2
(3, 1, 2)	2	3	2
(3, 2, 1)	2	2	2

This will be briefly explained by looking at the natural permutation $(1,\ 2,\ 3)$. In the first step, subnetwork 1 selects its solution $\mathbf{x}_1 \in \mathbb{S}^{\{1\}}_{\text{intra+EC}}$. Since subnetwork 1 does not need to consider any external constraint, i.e., any inter-subnetwork IN condition in this particular network, the feasibility condition only requires

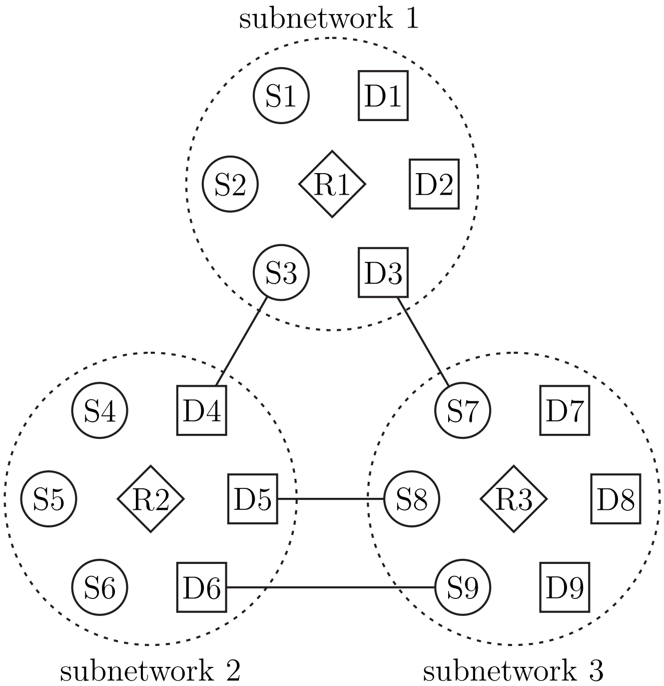


Figure 5.6.: A partially connected ad-hoc network consisting of three subnetworks, where the feasible relay antenna configurations for relay-aided IA with full channel knowledge are bounded by $M_1^2 \geq 2$, $M_2^2 \geq 2$, and $M_3^2 \geq 2$.

$\dim \mathbb{S}^{\{1\}}_{\text{intra+EC}} \geq 2$, or equivalently $M_1^2 \geq 2$, to guarantee that a randomly picked solution from $\mathbb{S}^{\{1\}}_{\text{intra+EC}}$ is almost surely valid. In the second step, subnetwork 2 selects its solution $\mathbf{x}_2 \in \mathbb{S}^{\{2\}}_{\text{intra+EC}}$. The selected solution $\mathbf{x}_2$ shall be able to nullify the inter-subnetwork interference between S3 and D4. This requires $\dim \mathbb{S}^{\{2\}}_{\text{intra+EC}} \geq 1$. However, in order to obtain a valid solution, $\mathbb{S}^{\{2\}}_{\text{intra+EC}}$ must be at least two dimensional. This yields $M_2^2 \geq 2$. In the third step, subnetwork 3 selects its solution $\mathbf{x}_3 \in \mathbb{S}^{\{3\}}_{\text{intra+EC}}$. In this step, subnetwork 3 shall consider three present inter-subnetwork direct links, i.e., the one between D3 and S7, the one between D5 and S8, and the one between D6 and S9. Thus, $\mathbb{S}^{\{3\}}_{\text{intra+EC}}$ must be at least three dimensional, which yields $M_3^2 \geq 3$. Looking back at the second step, subnetwork 2 should be able to exploit its two-dimensional solution space $\mathbb{S}^{\{2\}}_{\text{intra+EC}}$ to nullify two inter-subnetwork interferences instead of one. That is to say, one free variable in the entire network is wasted when subnetwork 2 selects its solution $\mathbf{x}_2 \in \mathbb{S}^{\{2\}}_{\text{intra+EC}}$. In contrast, if the proposed scheme is performed following the permutation $(2,\ 3,\ 1)$ or $(3,\ 2,\ 1)$, the free variables in every subnetwork can be efficiently exploited.

5.3.3. Parallelization

The proposed relay-aided IA scheme with partial channel knowledge can be performed following any permutation of the subnetworks, as discussed in the previous subsection. Nevertheless, the individual subnetworks shall always select their solution one after another. This is because one subnetwork cannot select its solution until it obtains all the required side information. In the worst case, the q-th subnetwork may require side information from all the previous $q-1$ subnetworks. Consequently, the q-th subnetwork has to wait until all the previous $q-1$ subnetworks have selected their solutions. If the individual subnetworks select their solution one after another, it is guaranteed that every subnetwork is able to obtain all the required side information. However, this causes significant delay if the entire network has lots of subnetworks. If more than one subnetwork can select their solutions simultaneously, the delay can be reduced.

Suppose no side information needs to be forwarded from one subnetwork to another. Then, the two subnetworks can select their solutions simultaneously. Extreme examples of this are the first and the second scheme discussed in Subsection 5.3.1, where the inter-subnetwork interferences are nullified by fixing some of the variables $v^{(j)}$ and $u^{(k)}$ without forwarding any side information. Therefore, the two subnetworks can perform intra-subnetwork IN simultaneously. However, such schemes usually require more relay antennas as compared to the proposed relay-aided IA scheme with partial channel knowledge. In the following, it will be investigated which subnetworks can select their solutions simultaneously in the proposed relay-aided IA scheme.

Considering the natural permutation of subnetworks, suppose that $q-1$ subnetworks have selected their solutions already. Then, the q-th and the $(q+1)$-th subnetworks are supposed to select their solutions in the following two steps, respectively. However, if the $(q+1)$-th subnetwork does not require any side information from the q-th subnetwork, i.e., if the side information that is required by the $(q+1)$-th subnetwork can already be provided by the subset $\{1, \ldots, q-1\}$, the q-th and the $(q+1)$-th subnetworks can select their solutions individually without influencing each other. That is to say, both the q-th and the $(q+1)$-th subnetworks can select their solutions simultaneously in the next step. In such a way, no variables in the q-th or the $(q+1)$-th subnetwork need to be fixed. Hence, the feasible relay antenna configurations will not be influenced. However, whether or not this is possible depends on the network topology. More precisely, it is possible only in the following two cases. The examples of these two cases are given in Example 5.7.

Case 1. If there are no external constraints between the q-th and the $(q+1)$-th subnetwork, no side information needs to be forwarded between them.

Case 2. Suppose there is an external constraint between node i in the q-th subnetwork and node j in the $(q+1)$-th subnetwork. Then the $(q+1)$-th subnetwork needs to know the variable of node i in order to satisfy the external constraint. However, suppose there is another external constraint between node i and node k in the subset $\{1, \ldots, q-1\}$ of subnetworks. Thus, there is also an external constraint between node k and node j, which linearly depends on the previous two. If the variable of node k would have been forwarded by the subset $\{1, \ldots, q-1\}$ to the $(q+1)$-th subnetwork as side information, the $(q+1)$-th subnetwork could select its solution without knowing the variable of node i.

Example 5.7. Consider two partially connected ad-hoc networks as depicted in Figure 5.7. In the network shown in Figure 5.7a, there are no external constraints between subnetwork 2 and subnetwork 3. Therefore, once subnetwork 1 has selected its solution and forwarded $v^{(3)}$ to subnetwork 2 and $u^{(3)}$ to subnetwork 3 as side information in the first step, subnetwork 2 and subnetwork 3 can select their solutions simultaneously in the second step. In the network shown in Figure 5.7b, there are three linearly dependent external constraints among S3, D4, and S7. Thus, once subnetwork 1 has selected its solution and forwarded $v^{(3)}$ to both subnetwork 2 and subnetwork 3 as side information in the first step, subnetwork 2 and subnetwork 3 can also select their solutions simultaneously in the second step.

The aforementioned parallelization approach can be very useful when implementing the proposed relay-aided IA scheme. In practice, only the inter-subnetwork direct links between nodes close to the common boundary of a few neighboring subnetworks may be sufficiently strong and need to be considered as present links. Therefore, there are no external constraints between non-neighboring subnetworks. Consequently, a subnetwork neither need to forward side information to nor requires side information from non-neighboring subnetworks. That is to say, non-neighboring subnetworks can select their solutions simultaneously. For instance, in a partially connected ad-hoc network as shown in Figure 5.8, where the subnetworks, as depicted by the circles, are arranged on a hexagonal grid and the external constraints, as depicted by the dashed lines, only exist between neighboring subnetworks, only three steps are required to perform the proposed relay-aided IA scheme with partial channel knowledge. As compared to selecting the solutions in the individual subnetworks one after another, the parallelization approach significantly reduces the delay.

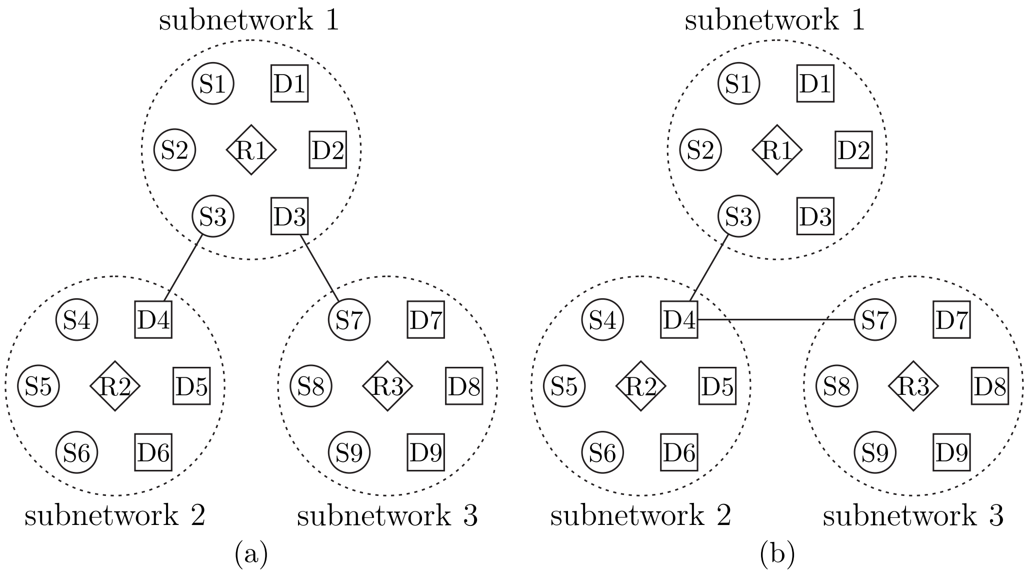


Figure 5.7.: Two partially connected ad-hoc networks where subnetwork 2 and subnetwork 3 can select their solutions simultaneously using the side information from subnetwork 1

5.4. Numerical simulations and results

In this section, the feasible relay antenna configurations and the sum rates achieved by relay-aided IA in partially connected ad-hoc networks will be investigated using numerical simulations and compared with those in fully connected ad-hoc networks.

First consider the feasible relay antenna configurations in partially ad-hoc networks consisting of Q subnetworks, where K_q is the size of the q-th subnetwork and M_q is the number of antennas of the q-th relay. Define the average number $\overline{M}_q$ of required antennas per relay for relay-aided IA as

$$\overline{M}_q = \sqrt{\arg\min \frac{1}{Q} \sum_{q=1}^{Q} M_q^2}, \tag{5.35}$$

where M_1^2, ..., M_Q^2 form a feasible relay antenna configuration. As discussed in the previous section, if partial channel knowledge is assumed, the feasible relay antenna configurations depend on the considered permutation of subnetworks. Therefore, in order to perform a fair comparison, full channel knowledge will be assumed, so that $\overline{M}_q$ only depends on the network topology and can be uniquely

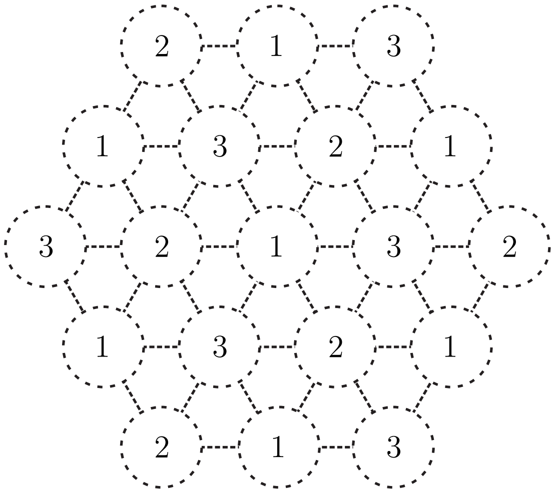


Figure 5.8.: A partially connected ad-hoc network where the subnetworks, as depicted by the circles, are arranged on a hexagonal grid and the external constraints, as depicted by the lines, only exist between neighboring subnetworks. In this network, only three steps are required to perform the proposed relay-aided IA scheme with partial channel knowledge since non-neighboring subnetworks can select their solutions simultaneously.

determined using (5.15). It will be investigated how the number Q of subnetworks and the sizes K_q of subnetworks influence $\overline{M}_q$, respectively. To this end, the following two cases will be considered. In the first case, the number Q of subnetworks is fixed to 5 and the size K_q of each subnetwork increases from 3 to 8. In the second case, the size K_q of each subnetwork is fixed to 5 and the number Q of subnetworks increases from 3 to 8. Furthermore, if an inter-subnetwork direct link is sufficiently strong, i.e., has a channel gain larger than a given threshold, it shall be considered as a present one. In this simulation, it is assume that every inter-subnetwork direct link is present with an equal probability p, where p is chosen to be either 0.9 or 0.1. In plain words, p represents the density of present inter-subnetwork direct links. As a reference, $\overline{M}_q$ for relay-aided IA in the fully connected ad-hoc networks with the same size K and the same number Q of relays is also considered. In fully connected ad-hoc networks, $\overline{M}_q$ can be computed using (3.33). The results are compared in Figure 5.9. In fully connected ad-hoc networks, the average number $\overline{M}_q$ of relay antennas linearly increases with both the number Q of subnetworks and the subnetwork size K_q. This is mainly because every relay has to process the signals from all the source nodes and to help nullifying the interferences at all destination nodes. In contrast, in partially connected

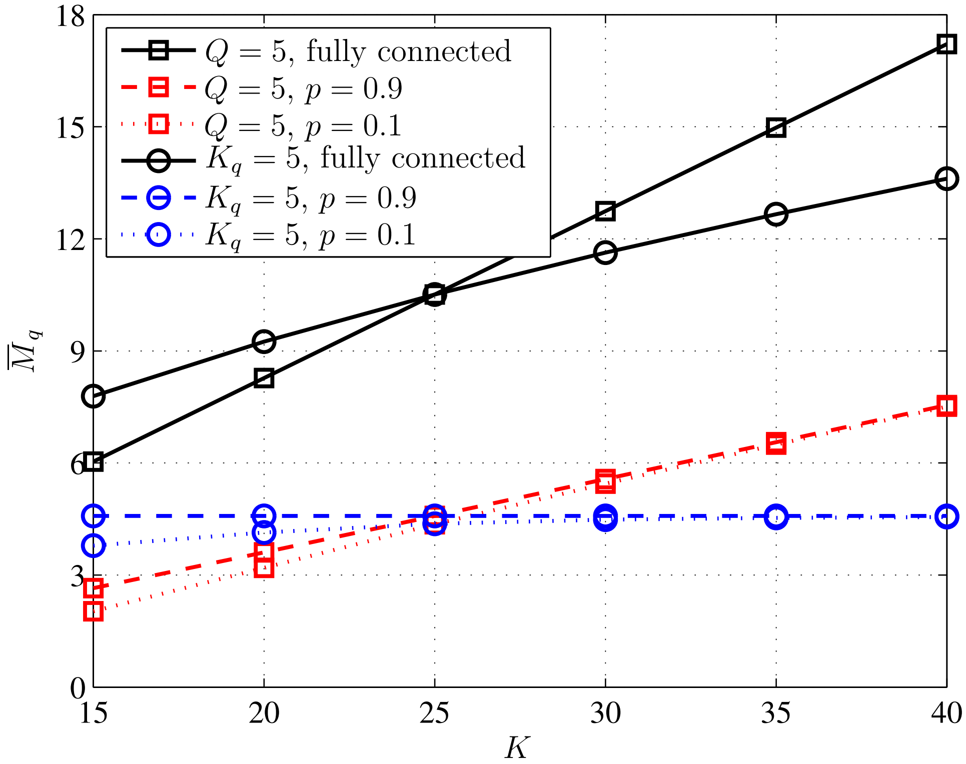


Figure 5.9.: Comparison of average numbers $\overline{M}_q$ of antennas per relay versus the total numbers K of source-destination node pairs in the entire network

ad-hoc networks, every relay only needs to help nullifying the intra-subnetwork interferences in its own subnetwork. Therefore, the average number $\overline{M}_q$ of relay antennas can be significantly reduced. Furthermore, in partially connected ad-hoc networks, $\overline{M}_q$ also linearly increases with the subnetwork size K_q, since more intra-subnetwork interferences have to be nullified as the subnetworks gets larger. However, if the subnetwork size K_q is fixed, $\overline{M}_q$ is also limited, regardless of the number Q of subnetworks. The reason for this has been discussed in Remark 5.7. Hence, it is more favorable to construct large networks using many relatively small subnetworks rather than a few relatively large ones, so that every subnetwork only needs a smaller number of relay antennas.

Next consider the sum rates achieved by the proposed relay-aided IA scheme with partial channel knowledge. Consider an ad-hoc network consisting of $Q = 3$

subnetworks, where each subnetwork has $K_q = 3$ single-antenna source-destination node pairs and a single relay. Independently distributed Rayleigh channels are assumed. However, the average channel gains are assumed to be different. In particular, the intra-subnetwork links are assumed to have unit average channel gains. The inter-subnetwork direct links, i.e., the channels between source and destination nodes in different subnetworks, are assumed to suffer from a path loss factor of $\beta_{\mathrm{DL}} = -20$ dB. The inter-subnetwork relay links, i.e., the channels between a source or destination node in one subnetwork and the relay in another subnetwork, are assumed to suffer from a path loss factor of either $\beta_{\mathrm{RL}} = -20$ dB or $\beta_{\mathrm{RL}} = -40$ dB. Such a network, despite that all the channel coefficients are almost surely non-zero, can be modeled as a partially connected ad-hoc network by ignoring comparatively weak interferences. In particular, for a certain channel realization, if an inter-subnetwork direct link has a channel gain larger than a given threshold, it is considered to be present. This threshold is chosen to be either -29.77 dB or -16.38 dB, such that every inter-subnetwork direct link is present with an equal probability of either $p = 0.9$ or $p = 0.1$, respectively. The inter-subnetwork relay links are always considered to be absent. The proposed relay-aided IA scheme with partial channel knowledge is applied to nullify the interferences propagating via the present links. The interferences propagating via the absent links, in spite of being ignored when modelling the network as a partially connected ad-hoc network, will be considered as residual interferences and be treated as noise when computing the achievable sum rate R_{sum}. In order to satisfy the feasibility conditions (5.32) and (5.33), each relay is assumed to have $M_q = 3$ antennas. Since full channel knowledge is not available, the transmit powers of the source nodes and the relay in each subnetwork will be optimized under a total sum transmit power constraint per subnetwork, which is assumed to be $P_{\mathrm{tot}}/3$ for each subnetwork. In order to perform a fair comparison, the PSNR

$$\gamma_{\mathrm{PSNR}} = \frac{P_{\mathrm{tot}}}{K\sigma^2} \tag{5.36}$$

will be considered, where σ^2 is the noise variance. As a reference, the sum rate achieved by relay-aided IA when the network is modeled as a fully connected ad-hoc network will be investigated as well. In this case, the desired IN solution shall nullify all the interferences in the entire network. In order to satisfy the feasibility condition (3.33), each relay is assumed to have $M_q = 5$ antennas in this case. Moreover, the transmit powers will be optimized under a total sum transmit power constraint P_{tot}.

The achieved sum rates averaged over a large number of channel realizations are shown in Figure 5.10. Firstly, if the network is considered as a fully connected ad-

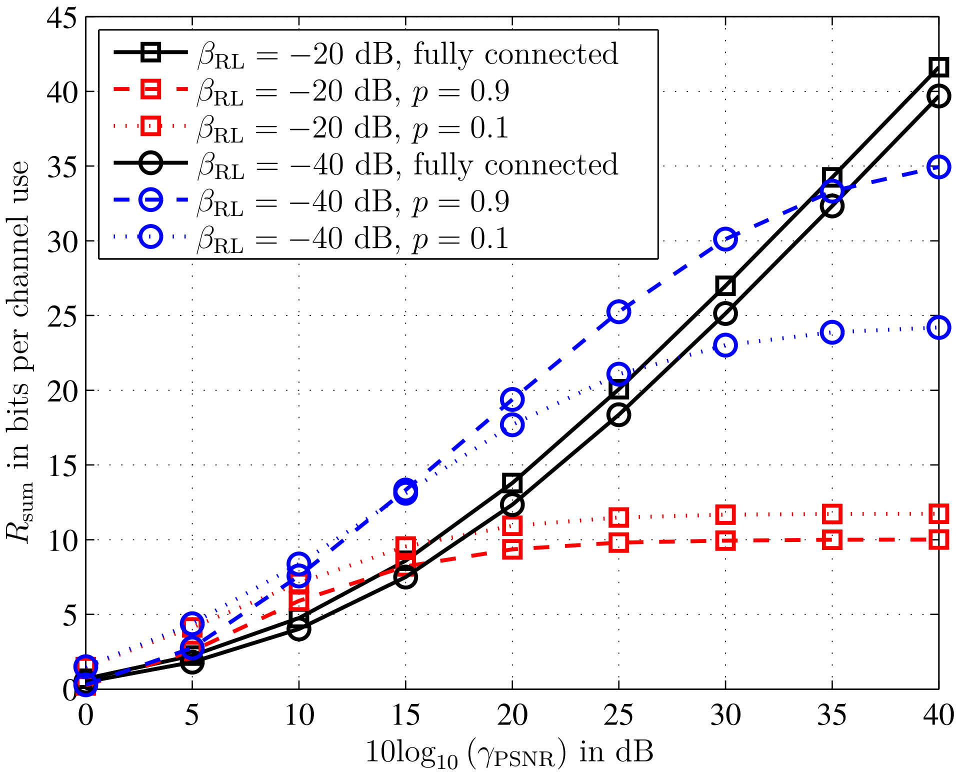


Figure 5.10.: The average achievable sum rate R_{sum} in bits per channel use as a function of the PSNR γ_{PSNR} in dB in the scenario with $Q = 3$ subnetworks, where each subnetwork is of size $K_q = 3$.

hoc network and relay-aided IA aims at perfectly nullifying all the interferences, the DoF of the network, i.e., a sum DoF of 9/2, can be achieved. Consequently, outstanding sum rates can be achieved in the high-PSNR regime, as shown by the solid curves. In contrast, if the network is considered as a partially connected ad-hoc network and relay-aided IA only aims at nullifying the non-negligible interferences, higher sum rates can be achieved in the low- to moderate-PSNR regime, not to mention that fewer relay antennas are required. However, the achievable sum rates will saturate due to the residual interferences. Secondly, comparing the two cases of $\beta_{\mathrm{RL}} = -20$ dB and $\beta_{\mathrm{RL}} = -40$ dB, respectively, significantly higher sum rates can be achieved in the later case if the network is considered as a partially connected ad-hoc network. This is simply because weaker inter-subnetwork relay links result in less residual interferences. Thirdly, the choice of the threshold also influences the achievable sum rate. Intuitively, if more inter-subnetwork direct links are considered to be present, less residual interferences remain. This usually improves the achievable sum rate, especially if the inter-subnetwork relay links are relatively weak and the residual interferences mainly come from the absent inter-subnetwork direct links, as shown by the dashed and dotted curves marked by circles in Figure 5.10. However, considering more inter-subnetwork direct links to be present also reduces the dimension of the IN solution space. Consequently, a randomly picked IN solution is more likely to be a "bad" one, i.e., to be close to an invalid solution, which achieves lower sum rate. This effect is observable if the inter-subnetwork relay links are relatively strong and the residual interferences mainly come from the inter-subnetwork relay links, as shown by the dashed and dotted curves marked by squares in Figure 5.10.

Chapter 6.

Summaries

6.1. Summary in English

Pursuing higher data rates is always one of the main objectives of the research in wireless radio communications. However, in current wireless radio communications systems, the multiuser interference is a major performance-limiting factor due to the scarceness of spectrum. Recently, it has been discovered that every user in a multiuser interference network is able to get “half the cake” using a novel interference management approach known as IA. IA is able to achieve the DoFs of many multiuser interference networks, leading to outstanding performances in the high-SNR regime as compared to other interference management approaches. As compared to a variety of IA schemes in literature, relay-aided IA has its own advantages. This thesis focuses on four related research topics of relay-aided IA, i.e., the IA solutions, the feasibility conditions, relay-aided IA with partial channel knowledge, and the achievable performances. These research topics are discussed in three types of relay networks, i.e., fully connected ad-hoc networks, fully connected cellular networks, and partially connected ad-hoc networks.

Ad-hoc networks have various applications in practice. The ad-hoc networks considered in this thesis consist of multiple source-destination node pairs and half-duplex AF relays which assist in the unidirectional communications. The fully connected ad-hoc networks are also the basic network of the three types of relay networks in this thesis. The conditions for achieving relay-aided IA are discussed first, which are classified into the IN conditions and the validity conditions. An IA solution is thus a set of cooperatively designed transmit filters, receive filters, and relay processing filters which satisfy all the IN conditions while not violating any of the validity conditions. Then, a linearization approach is proposed in order to analytically find all the IA solutions, or at least a set of the relevant ones.

Specifically, the invalid IN solutions with respect to each node pair form either a linear subspace or a non-linear algebraic subset of the linear IN solution space. Besides these invalid IN solutions, the other IN solutions, i.e., the valid ones, correspond to the IA solutions. The feasibility conditions for the proposed relay-aided IA scheme are also addressed, which ensure that valid IN solutions exist in the almost sure sense. For the single-antenna case, every invalid IN solution subspace shall be a hyperplane of the IN solution space. For the multi-antenna case, every invalid IN solution subset shall be a negligibly small strict subset of the IN solution space. Finally, how to construct the transmit and receive filters from any given valid IN solution aiming at sum rate maximization is studied. Both the total sum transmit power constraint and the individual sum power constraints are considered. In the former case, the optimization problem is convex and can be solved in closed-form. In the latter case, the optimization problem is non-convex. However, a suboptimal solution is proposed, which can be readily obtained using standard convex optimization tools.

Mobile cellular networks are the most successful commercial wireless radio communications networks worldwide. The considered cellular networks are extended from the ad-hoc networks in the sense that BSs and MSs play the roles of source and destination nodes. However, since the MSs cannot perform joint signal processing, both the intra- and inter-cell interferences have to be considered. In this thesis, the proposed relay-aided IA scheme for the cellular networks includes inter-cell IN and intra-cell interference management. The inter-cell IN is performed, with the help of relays, in order to find the valid inter-cell IN solutions, which convert the cellular network into multiple non-interfering MACs or BCs. For intra-cell interference management, two widely used linear beamforming techniques, i.e., ZF and MMSE, are considered at the BSs. The corresponding optimal power allocations under a total sum transmit power constraint are addressed. Furthermore, the uplink-downlink duality of relay-aided IA is also investigated, which has the following two implications. First, the inter-cell IN solutions in the uplink and the downlink are dual. Second, based on the duality of inter-cell IN solutions, the beamforming matrices designed for intra-cell interference management as well as the achievable performances in the uplink and the downlink are also dual.

In practical mobile radio communications networks, some significantly weak interferences may be comparable with noise and hence can be ignored. Motivated by this, a class of partially connected ad-hoc networks is considered, which consist of multiple subnetwork being partially connected to each other. Thus, only the interferences propagating via the present links need to be nullified, which include the intra-subnetwork interferences propagating via the intra-subnetwork links and the inter-subnetwork interferences propagating via a few inter-subnetwork direct links. Due to the fact that the presence of inter-subnetwork direct links does not follow a certain pattern, the intra- and inter-subnetwork IN conditions are investigated separately. Specially, the inter-subnetwork IN conditions are transformed to an equivalent set of external constraints employing graph theory. Then, every part of the entire network, i.e., every subset of subnetworks, shall be able to satisfy both its intra-subnetwork IN conditions and external constraints. Moreover, a relay-aided IA scheme with partial channel knowledge is proposed, where the considered partial channel knowledge includes the intra-subnetwork CSI, the network topology, and the side information. The side information can be considered as some “compressed” CSI, which enables cooperation between different subnetworks. Using the considered partial channel knowledge, every subnetwork can select its own solution following a certain order. All the solutions selected by the individual subnetworks together form a valid IN solution for the entire network. Finally, a parallelization approach which allows several subnetworks to select their solutions simultaneous can be applied to speed up this process.

6.2. Zusammenfassung auf Deutsch

Eines der Hauptziele der Forschung in der Mobilfunk-Kommunikation ist stets das Erreichen höherer Datenraten. In gegenwärtigen Funkkommunikationssystemen ist jedoch aufgrund der Begrenztheit des Spektrums die Mehrnutzer-Interferenz ein vornehmlicher performanzbegrenzenden Faktor. Vor kurzem wurde es entdeckt, dass jeder Benutzer in einem Mehrnutzer-Interferenznetzwerk mit einem neuartigen als Interference Alignment bezeichneten Interferenzreduktionsverfahren „den halben Kuchen“ gewissermaßen bekommen kann. IA ist in der Lage, von vielen Mehrnutzer-Interferenznetzwerken die DoFs zu erreichen, was im Vergleich zu anderen Interferenzreduktionsverfahren zu herausragenden Leistungen im Bereich hoher SNRs führt. Im Vergleich zu einer Vielzahl aus der Literatur bekannte IA-Schemata hat das relaisunterstützte IA ihre eigenen Vorteile. Die vorliegende Arbeit konzentriert sich auf vier verwandte Forschungsthemen auf dem Gebiet des relaisunterstützten IA, d.h., die IA-Lösungen, die Machbarkeitsbedingungen, das relaisgestützte IA mit teilweisen Kanalkenntnisse, und die erreichbaren Leistungen. Diese Forschungsthemen werden in drei Arten von Relaisnetzwerken diskutiert, d.h., vollständig verbundenen Ad-hoc-Netzwerken, vollständig verbundenen zellularen Netzwerken, und teilweise verbundenen Ad-hoc-Netzwerken.

Ad-hoc-Netzwerke haben in der Praxis verschiedene Anwendungen. Die in dieser Arbeit betrachteten Ad-hoc-Netzwerke bestehen aus mehreren Quelle-Ziel-Knotenpaaren und Halbduplex-AF-Relais, die die unidirektionalen Kommunikationen unterstützen. Die vollständig verbundenen Ad-hoc-Netzwerke sind auch das grundlegende Netzwerk der drei in dieser Arbeit betrachteten Arten von Relaisnetzwerken. Zuerst werden die Bedingungen für das Erreichen eines relaisgestützten IA betrachtet, die in die Interference-Nulling (IN)-Bedingungen und die Gültigkeitsbedingungen klassifiziert werden. Eine IA-Lösung ist somit eine Menge von gemeinsam entworfenen Sendefiltern, Empfangsfiltern und Verarbeitungsfiltern der Relais, die alle IN-Bedingungen erfüllen, während sie keine der Gültigkeitsbedingungen verletzen. Anschließend wird ein Linearisierungsansatz vorgeschlagen, um alle IA-Lösungen, oder zumindest eine Menge relevanter Lösungen, analytisch zu finden. Insbesondere bilden die ungültigen IN-Lösungen in Bezug auf jedes Knotenpaar entweder einen linearen Unterraum oder eine nichtlineare algebraische Teilmenge des linearen IN-Lösungsraums. Alle IN-Lösungen außer diesen ungültigen IN-Lösungen, d.h., die gültigen, entsprechen den IA-Lösungen. Die Machbarkeitsbedingungen für das vorgeschlagene relaisgestützte IA-Schema, die sicherstellen, dass eine gültige IN-Lösung im fast sicheren Sinn existiert, werden ebenfalls angesprochen. Für den Fall einer einzelnen Antenne sollte jeder ungültige IN-Lösungsunterraum eine Hyperebene des IN-Lösungsraums sein. Für den Fall mehrerer Antennen sollte jede ungültige IN-Lösungsuntermenge eine vernachlässigbar kleine strenge Untermenge des IN-Lösungsraums sein. Schließlich wird untersucht, wie die Sende- und Empfangsfilter, die auf die Maximierung der Summenrate abzielen, für jede gegebene gültige IN-Lösung konstruiert werden. Es werden sowohl eine gesamte Summenübertragungsleistungsbeschränkung als auch mehrere individuellen Summenleistungsbeschränkungen berücksichtigt. Im ersten Fall ist das Optimierungsproblem konvex und kann in geschlossener Form gelöst werden. Im letzteren Fall ist das Optimierungsproblem nicht konvex. Es wird jedoch eine suboptimale Lösung vorgeschlagen, die leicht unter Verwenden der Werkzeuge für konvexe Optimierung erhalten werden kann.

Mobilfunknetze sind weltweit die erfolgreichsten kommerziellen Funkkommunikationsnetze. Die betrachteten zellularen Netzwerke sind erweiterte Ad-hoc-Netzwerke in dem Sinne, dass BSs und MSs die Rolle von Quell- und Zielknoten spielen. Da die MSs jedoch keine gemeinsame Signalverarbeitung durchführen können, müssen sowohl die Intrazell- als auch die Interzell-Interferenzen berücksichtigt werden. Das in dieser Arbeit vorgeschlagene relaisunterstützte IA-Schema für die zellularen Netzwerke beinhaltet Interzell-IN und Intrazell-Interferenzmanagement. Das Interzell-IN wird mit Hilfe von Relais durchgeführt, um die gültigen Interzell-IN-Lösungen zu finden, die das zellulare Netzwerk in mehrere nicht interferende MACs oder BCs umwandeln. Für Intrazell-

Interferenzmanagement werden zwei häufig benutzte lineare Strahlformungstechniken, d.h., ZF und MMSE, bei den BS berücksichtigt. Die entsprechenden unter einer gesamten Summenüberstragungsleistungsbeschränkung optimalen Leistungszuteilungen werden angesprochen. Darüber hinaus wird auch die Uplink-Downlink-Dualität von relaisunterstütztem IA untersucht, die die folgenden zwei Implikationen hat. Erstens sind die Interzell-IN-Lösungen in der Aufwärtsstrecke und der Abwärtsstrecke dual. Zweitens, basierend auf der Dualität von IN-Lösungen sind die für das Intrazell-Interferenzmanagement entworfenden Strahlformungsmatrizen sowie die erreichbaren Leistungen in der Aufwärtsstrecke und der Abwärtsstrecke ebenfalls dual.

In praktischen Mobilfunk-Kommunikationsnetzen können einige vergleichsweise schwache Interferenzen mit Rauschen vergleichbar sein und können daher ignoriert werden. Motiviert dadurch wird eine Klasse von teilweise verbundenen Ad-hoc-Netzwerken betrachtet, die aus mehreren teilweise miteinander verbundenen Teilnetzwerken bestehen. Somit müssen nur die über vorhandenen Strecken ausbreitenden Interferenzen unterdrückt werden. Diese Interferenzen umfassen die Intra-Teilnetzwerk-Interferenzen, die sich über die Intra-Teilnetzwerk-Strecken ausbreiten, und die Inter-Teilnetzwerk-Interferenzen, die sich über wenige Inter-Teilnetzwerk-Direktstrecken ausbreiten. Aufgrund der Tatsache, dass die vorhandenen Inter-Teilnetzwerk-Direktstrecken keinem bestimmten Muster folgen, werden die IN-Bedingungen für Intra- und Inter-Teilnetzwerk-Interferenzen separat untersucht. Insbesondere werden die Inter-Teilnetzwerk-IN-Bedingungen unter Verwenden der Graphentheorie in eine äquivalente Menge von externen Einschränkungen transformiert. Dann muss jeder Teil des gesamten Netzwerks, d.h., jede Teilmenge von Teilnetzwerken, sowohl den Inter-Teilnetzwerk-IN-Bedingungen als auch den externen Einschränkungen genügen. Darüber hinaus wird ein relaisunterstütztes IA-Schema mit teilweisen Kanalkentnisse vorgeschlagen, wobei die betrachteten teilweisen Kanalkentnisse die CSI innerhalb des Teilnetzwerkes, die Netzwerktopologie, und die Nebeninformationen umfassen. Die Nebeninformationen können als die „komprimierte“ CSI betrachtet werden, die die Zusammenarbeit zwischen verschiedenen Teilnetzwerken ermöglicht. Unter Verwenden der betrachteten teilseisen Kanalkentnisse kann jedes Teilnetzwerk gemäß einer bestimmten Reihenfolge seine eigene Lösung auswählen. Alle von den einzelnen Teilnetzwerken ausgewählten Lösungen zusammen bilden eine gültige IN-Lösung für das gesamte Netzwerk. Schließlich kann ein Parallelisierungsansatz angewendet werden, bei dem mehrere Teilnetzwerke ihre Lösungen gleichzeitig auswählen können, um diesen Prozess zu beschleunigen.

Appendix A.

Kronecker product and Khatri-Rao product

The Kronecker product and the related matrix operation Khatri-Rao product are useful for solving wireless communication problems, especially in MIMO systems. They are also frequently used in this thesis. Given the $I \times J$ matrix $\mathbf{A} = [a_{ij}]$ and the $M \times N$ matrix $\mathbf{B} = [b_{mn}]$, their Kronecker product is denoted by $\mathbf{A} \otimes \mathbf{B}$ and is defined as

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1J}\mathbf{B} \\ \vdots & & \vdots \\ a_{I1}\mathbf{B} & \cdots & a_{IJ}\mathbf{B} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & \cdots & a_{11}b_{1N} & \cdots & a_{1J}b_{11} & \cdots & a_{1J}b_{1N} \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{11}b_{M1} & \cdots & a_{11}b_{MN} & \cdots & a_{1J}b_{M1} & \cdots & a_{1J}b_{MN} \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{I1}b_{11} & \cdots & a_{I1}b_{1N} & \cdots & a_{IJ}b_{11} & \cdots & a_{IJ}b_{1N} \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{I1}b_{M1} & \cdots & a_{I1}b_{MN} & \cdots & a_{IJ}b_{M1} & \cdots & a_{IJ}b_{MN} \end{bmatrix}. \tag{A.1}$$

The Khatri-Rao product [KR68] is related to the Kronecker product, and it operates on partitioned matrices. Given two partitioned matrices $\mathbf{C} = [\mathbf{C}_{ij}]$ and $\mathbf{D} = [\mathbf{D}_{ij}]$ which have the same number of row and column partitions, their Khatri-Rao product is denoted by $\mathbf{C} \odot \mathbf{D}$ and defined as

$$\mathbf{C} \odot \mathbf{D} = \begin{bmatrix} \mathbf{C}_{11} \otimes \mathbf{D}_{11} & \cdots & \mathbf{C}_{1J} \otimes \mathbf{D}_{1J} \\ \vdots & & \vdots \\ \mathbf{C}_{I1} \otimes \mathbf{D}_{I1} & \cdots & \mathbf{C}_{IJ} \otimes \mathbf{D}_{IJ} \end{bmatrix}. \tag{A.2}$$

The Khatri-Rao product $\mathbf{C} \odot \mathbf{D}$ is a submatrix of the Kronecker product $\mathbf{C} \otimes \mathbf{D}$. A special case of the Khatri-Rao product occurs when both constituent matrices $\mathbf{C}$ and $\mathbf{D}$ have the same number of columns and they are partitioned column-wise. In this case, the Khatri-Rao product $\mathbf{C} \odot \mathbf{D}$ is a column-wise Kronecker product and has the same number of columns as $\mathbf{C}$ and $\mathbf{D}$.

The Kronecker product can be used to obtain a convenient representation for some matrix equations. Let vec$(\mathbf{A})$ denote the the vectorization of the matrix $\mathbf{A}$ formed by stacking the columns of $\mathbf{A}$ into a single vector as

$$\mathrm{vec}\,(\mathbf{A}) = \begin{bmatrix} a_{11} & \cdots & a_{I1} & \cdots & a_{1J} & \cdots & a_{IJ} \end{bmatrix}^{\mathrm{T}}. \tag{A.3}$$

Then the well-known result

$$\mathrm{vec}\,(\mathbf{AXB}) = (\mathbf{B}^{\mathrm{T}} \otimes \mathbf{A})\mathrm{vec}\,(\mathbf{X}) \tag{A.4}$$

holds for any matrices $\mathbf{A}$, $\mathbf{X}$, and $\mathbf{B}$ whose dimensions are consistent.

Each singular value of the Kronecker product $\mathbf{A} \otimes \mathbf{B}$ is the product of a pair of singular values of $\mathbf{A}$ and $\mathbf{B}$. Thus,

$$\mathrm{rank}\,(\mathbf{A} \otimes \mathbf{B}) = \mathrm{rank}\,(\mathbf{A})\,\mathrm{rank}\,(\mathbf{B}) \tag{A.5}$$

holds. As compared to the Kronecker product, finding the rank of the Khatri-Rao product is more complicated due to the partitioning. However, a lower bound has been found for the special case where the constituent matrices are partitioned column-wise [SL01]. This result is obtained by employing the Kruskal-rank [Kru77], which is also known as the k-rank. The Kruskal-rank of a matrix $\mathbf{A}$ is k if any k columns of $\mathbf{A}$ are linearly independent and either $\mathbf{A}$ has k columns or $\mathbf{A}$ contains a set of $k+1$ linearly dependent columns. Let $\mathbf{A} \odot \mathbf{B}$ be the Khatri-Rao product of the $I \times F$ matrix $\mathbf{A}$ and the $J \times F$ matrix $\mathbf{B}$ where both $\mathbf{A}$ and $\mathbf{B}$ are partitioned column-wise into F blocks. Then it holds that

$$k_{\mathbf{A} \odot \mathbf{B}} \geq \min\,\{k_{\mathbf{A}} + k_{\mathbf{B}} - 1, F\} \tag{A.6}$$

if $k_{\mathbf{A}} \geq 1$ and $k_{\mathbf{B}} \geq 1$ hold, where $k_{\mathbf{A}}$, $k_{\mathbf{B}}$, and $k_{\mathbf{A} \odot \mathbf{B}}$ denote the Kruskal-ranks of $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{A} \odot \mathbf{B}$, respectively. Since a matrix's Kruskal-rank is always smaller than or equal to its rank, (A.6) also gives a lower bound for the rank of $\mathbf{A} \odot \mathbf{B}$.

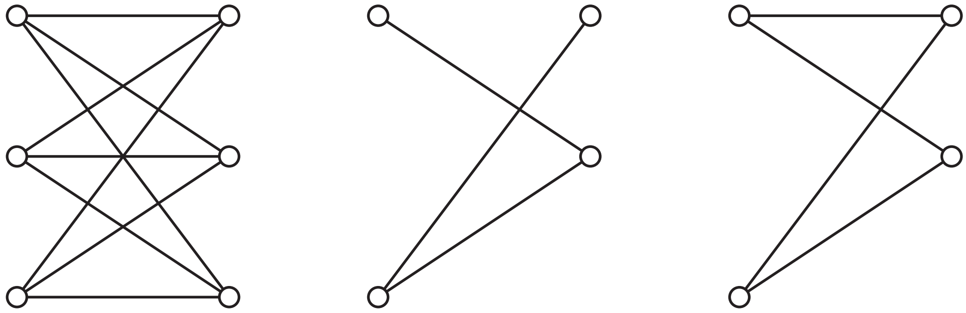
Appendix B.

Graph theory

Many practical problems can be described by graphs. In this thesis, tools and results from graph theory are used to describe the topology of the considered networks and to determine the feasibility conditions for relay-aided IA. These tools and results will be briefly introduced in this chapter. More information on graph theory can be found in [BM08].

In this thesis, only simple graphs will be considered. A simple graph $\mathcal{G}$ is an ordered pair $(\mathcal{V}, \mathcal{E})$ consisting of a set $\mathcal{V}$ of vertices and a set $\mathcal{E}$ of edges, such that any two vertices of $\mathcal{G}$ are associated with at most a single edge of $\mathcal{G}$, i.e., $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ holds. If e is an edge and u and v are vertices that are associated with the edge e, then the vertices u and v are adjacent and they are called the ends of e. Graphs can be represented graphically by their diagrams. For example, Figure B.1 shows the diagrams of three types of graphs, which play prominent roles in this thesis. A graph is bipartite if its vertex set $\mathcal{V}$ can be partitioned into two disjoint subsets $\mathcal{V}_1$ and $\mathcal{V}_2$ so that every edge has one end in $\mathcal{V}_1$ and the other end in $\mathcal{V}_2$. The diagram of a bipartite graph $\mathcal{G}_1$ is shown in Figure B.1a. A path is a graph whose vertices can be arranged in a linear sequence in such a way that two vertices are adjacent if they are consecutive in the sequence, and are nonadjacent otherwise. Likewise, a cycle is a graph whose vertices can be arranged in a cyclic sequence in such a way that two vertices are adjacent if they are consecutive in the sequence, and are nonadjacent otherwise. The length of a path or a cycle is the number of its edges. Figure B.1b and Figure B.1c show the diagrams of a path $\mathcal{G}_2$ of length three and a cycle $\mathcal{G}_3$ of length four, respectively. A graph is connected if for every partition of its vertex set into two nonempty sets $\mathcal{V}_1$ and $\mathcal{V}_2$, there is an edge with one end in $\mathcal{V}_1$ and the other end in $\mathcal{V}_2$. Otherwise the graph is disconnected. For instance, all the three graphs shown in Figure B.1 are connected graphs.

A subgraph of a graph $\mathcal{G}$ can be obtained either by deleting an edge from $\mathcal{G}$ but



(a) $\mathcal{G}_1$ (b) $\mathcal{G}_2$ (c) $\mathcal{G}_3$

Figure B.1.: (a) A bipartite graph $\mathcal{G}_1$, (b) a path $\mathcal{G}_2$ of length three, and (c) a cycle $\mathcal{G}_3$ of length four

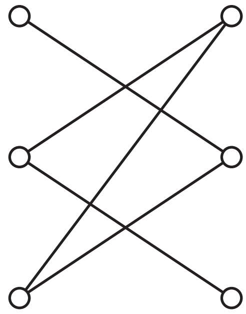


Figure B.2.: A spanning tree $\mathcal{G}_4$ of $\mathcal{G}_1$

leaving the vertices and the remaining edges intact or by deleting a vertex from $\mathcal{G}$ together with all the edges with the vertex as an end. For example, the graph $\mathcal{G}_2$ shown in Figure B.1b is a subgraph of the graph $\mathcal{G}_3$ shown in Figure B.1c, and both $\mathcal{G}_2$ and $\mathcal{G}_3$ are subgraphs of the graph $\mathcal{G}_1$ shown in Figure B.1a. Among the different types of subgraphs, two types of them, i.e., spanning trees and maximal forests, are of particular importance for the discussions in this thesis. A connected acyclic graph is called a tree. If a tree is a subgraph of a connected graph $\mathcal{G}$ and it is obtained by only deleting edges from $\mathcal{G}$, then this tree is a spanning tree of $\mathcal{G}$. It directly follows from the definition of spanning trees that, for a connected graph with n vertices, any spanning tree has $n-1$ edges. In contrast to a tree, an acyclic graph, which may be disconnected, is called a forest. In disconnected graphs, maximal forests play the role of spanning trees in connected graphs. For example, Figure B.2 shows the diagram of a spanning tree $\mathcal{G}_4$ of $\mathcal{G}_1$.

Besides the diagram, a graph can be alternatively specified by its incidence matrix. The incidence matrix of a graph $\mathcal{G}$ with m edges and n vertices is the

$m \times n$ matrix[1] $\boldsymbol{\Psi}_{\mathcal{G}} = [\psi_{ev}]$, where ψ_{ev} is the number of times that edge e and vertex v are incident. For instance, the graph $\mathcal{G}_1$, which is shown in Figure B.1a, can be alternatively specified by the incidence matrix

$$\boldsymbol{\Psi}_{\mathcal{G}_1} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}. \tag{B.1}$$

Note that the incidence matrix of a graph has only two non-zero entries in each row. Furthermore, the edges of a tree or a forest in a graph $\mathcal{G}$ correspond to a set of linearly independent rows of the incidence matrix $\boldsymbol{\Psi}_{\mathcal{G}}$. The edges of a spanning tree or a maximal forest of $\mathcal{G}$ correspond to a basis of the row space of $\boldsymbol{\Psi}_{\mathcal{G}}$. For instance, the incidence matrix of $\mathcal{G}_4$ can be written as

$$\boldsymbol{\Psi}_{\mathcal{G}_4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \tag{B.2}$$

whose rows are linearly independent and span the row space of $\boldsymbol{\Psi}_{\mathcal{G}_1}$. Hence, the rank of the incidence matrix of a graph is equal to the number of edges in its spanning trees or maximal forests. For any connected graph with n vertices, the rank of its incidence matrix is always equal to $n - 1$.

[1] This definition of the incidence matrix is used to accommodate the discussions in this thesis. In [BM08], the incidence matrix is alternatively defined such that its rows correspond to the vertices and its columns correspond to the edges.

Appendix C.

Reference schemes

C.1. Sum mean square error minimization

In the following, the sum MSE minimization algorithm will be introduced using the fully connected ad-hoc networks as an example. However, this algorithm can be extended to the fully connected cellular networks. To this end, the filters of the MSs shall be diagonal matrices due to the fact that they cannot perform joint signal processing.

The two-hop transmission scheme is applied in a fully connected ad-hoc network, as described in Section 2.1. The output data symbols $\widetilde{\mathbf{d}}^{(k)}$ of the k-th destination node can be represented as

$$\widetilde{\mathbf{d}}^{(k)} = \mathbf{H}_{\text{eff}}^{(k,k)}\mathbf{d}^{(k)} + \sum_{j\neq k}\mathbf{H}_{\text{eff}}^{(k,j)}\mathbf{d}^{(j)} + \mathbf{n}_{\text{eff}}^{(k)}, \tag{C.1}$$

where the effective channel matrix $\mathbf{H}_{\text{eff}}^{(k,j)}$ and the effective noise $\mathbf{n}_{\text{eff}}^{(k)}$ are given by (2.5) and (2.6), respectively. The transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$, the receive filters $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$, and the relay processing filters $\mathbf{G}^{(q)}$ are adapted aiming at minimizing the sum MSE across the destination nodes under a total sum transmit power constraint $P_{\text{tot,max}}$. Thus, the following optimization problem is considered:

$$\underset{\mathbf{V}_1^{(j)},\mathbf{V}_2^{(j)},\mathbf{U}_1^{(k)},\mathbf{U}_2^{(k)},\mathbf{G}^{(q)}}{\text{minimize}} \quad \sum_{k=1}^{K}\mathrm{E}\left\{\left\|\widetilde{\mathbf{d}}^{(k)}-\mathbf{d}^{(k)}\right\|^2\right\}, \tag{C.2}$$

$$\text{subject to} \quad \sum_{j=1}^{K}P_{\mathrm{S}}^{(j)} + \sum_{q=1}^{Q}P_{\mathrm{R}}^{(q)} \leq P_{\text{tot,max}}, \tag{C.3}$$

where $P_{\mathrm{S}}^{(j)}$ and $P_{\mathrm{R}}^{(q)}$ are the sum transmit power of the j-th source node and the transmit power of the q-th relay, as given in (3.49) and (3.50), respectively.

Since the above optimization problem is non-convex in the considered variables, Algorithm 1, which alternately adapts the relay processing filters, the transmit filters, and the receive filters, will be considered for the numerical simulations in this thesis. Note that Algorithm 1 only converges to a local optimum, which depends on the initialization. The possibility of using other initializations besides the random matrices considered in this thesis have been discussed in [ASLG+16, AS14] to further improve the performances. Finally, the achieved sum rate can be computed using (2.11).

C.2. Sum rate maximization

In the following, the sum rate maximization algorithm will be introduced using the fully connected ad-hoc networks as an example. However, this algorithm can be extended to the fully connected cellular networks. To this end, the filters of the MSs shall be diagonal matrices due to the fact that they cannot perform joint signal processing.

The sum rate maximization scheme introduced in the following was first proposed in [ASLG+16, AS14]. The two-hop transmission scheme is applied in a fully connected ad-hoc network, as described in Section 2.1. The output data symbols $\widetilde{\mathbf{d}}^{(k)}$ of the k-th destination node can be represented by

$$\widetilde{\mathbf{d}}^{(k)} = \mathbf{H}_{\text{eff}}^{(k,k)}\mathbf{d}^{(k)} + \sum_{j\neq k}\mathbf{H}_{\text{eff}}^{(k,j)}\mathbf{d}^{(j)} + \mathbf{n}_{\text{eff}}^{(k)}, \tag{C.4}$$

where the effective channel matrix $\mathbf{H}_{\text{eff}}^{(k,j)}$ and the effective noise $\mathbf{n}_{\text{eff}}^{(k)}$ are given by (2.5) and (2.6), respectively. The following sum rate maximization problem under a total sum transmit power constraint $P_{\text{tot,max}}$ is considered:

$$\underset{\mathbf{V}_1^{(j)},\mathbf{V}_2^{(j)},\mathbf{U}_1^{(k)},\mathbf{U}_2^{(k)},\mathbf{G}^{(q)}}{\text{maximize}} \quad R_{\text{sum}} = \frac{1}{2}\sum_{k=1}^{K}\sum_{n=1}^{N}\log_2\left(1+\gamma^{(k,n)}\right), \tag{C.5}$$

$$\text{subject to} \quad \sum_{j=1}^{K}P_{\text{S}}^{(j)} + \sum_{q=1}^{Q}P_{\text{R}}^{(q)} \leq P_{\text{tot,max}}, \tag{C.6}$$

where $\gamma^{(k,n)}$ is the received SINR of the n-th data symbol of the k-th destination node, and $P_{\text{S}}^{(j)}$ and $P_{\text{R}}^{(q)}$ are the sum transmit power of the j-th source node and the transmit power of the q-th relay, as given in (3.49) and (3.50), respectively. Since the above optimization problem is non-convex in the considered variables,

Algorithm 1 Sum MSE minimization

1: **Initialize** $\mathbf{V}_1^{(j)}$, $\mathbf{V}_2^{(j)}$, $\mathbf{U}_1^{(k)}$, $\mathbf{U}_2^{(k)}$, $\forall j, k = 1, \ldots, K$, with random $N \times N$ matrices satisfying

$$\operatorname{tr}\left(\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}} + \mathbf{V}_2^{(j)}\mathbf{V}_2^{(j)*\mathrm{T}}\right) = \frac{1}{2K}P_{\mathrm{tot,max}},$$

2: **repeat**

3: adapt $\mathbf{G}^{(q)}$, $\forall q = 1, \ldots, Q$, by solving the quadratically constrained quadratic optimization problem:

$$\begin{aligned} \underset{\mathbf{G}^{(q)},\forall q}{\text{minimize}} \quad & \sum_{k=1}^{K} \mathrm{E}\left\{\left\|\tilde{\mathbf{d}}^{(k)} - \mathbf{d}^{(k)}\right\|^2\right\}, \\ \text{subject to} \quad & \sum_{q=1}^{Q} P_{\mathrm{R}}^{(q)} \leq P_{\mathrm{tot,max}} - \sum_{j=1}^{K} P_{\mathrm{S}}^{(j)}, \end{aligned}$$

4: adapt $\mathbf{V}_1^{(j)}$ and $\mathbf{V}_2^{(j)}$, $\forall j = 1, \ldots, K$, by solving the quadratically constrained quadratic optimization problem:

$$\begin{aligned} \underset{\mathbf{V}_1^{(j)},\mathbf{V}_2^{(j)},\forall j}{\text{minimize}} \quad & \sum_{k=1}^{K} \mathrm{E}\left\{\left\|\tilde{\mathbf{d}}^{(k)} - \mathbf{d}^{(k)}\right\|^2\right\}, \\ \text{subject to} \quad & \sum_{j=1}^{K} P_{\mathrm{S}}^{(j)} + \sum_{q=1}^{Q} P_{\mathrm{R}}^{(q)} \leq P_{\mathrm{tot,max}}, \end{aligned}$$

5: adapt $\mathbf{U}_1^{(k)}$ and $\mathbf{U}_2^{(k)}$ using the linear MMSE receive filter

$$\begin{bmatrix} \mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)} \end{bmatrix} = \left(\sum_{j=1}^{K} \mathbf{H}^{(k,j)}\mathbf{H}^{(k,j)*\mathrm{T}} + \mathbf{S}_{\mathrm{noise}}^{(k)}\right)^{-1} \mathbf{H}^{(k,k)}, \quad \forall k,$$

where the $2N \times N$ matrix $\mathbf{H}^{(k,j)}$ is given by

$$\mathbf{H}^{(k,k)} = \begin{bmatrix} \mathbf{H}_{\mathrm{DS}}^{(k,j)} & \mathbf{0} \\ \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{H}_{\mathrm{RS}}^{(q,j)} & \mathbf{H}_{\mathrm{DS}}^{(k,j)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)} \end{bmatrix},$$

and $\mathbf{S}_{\mathrm{noise}}^{(k)}$ is the noise covariance matrix at the k-th destination node,

6: **until** required accuracy is met.

it was proposed in [ASLG+16, AS14] to consider a multi-convex optimization problem which is equivalent to the above problem in the sense that both have the same global and local maxima. In particular, consider the functions

$$f^{(k,n)} = \log_2\left(\left|w^{(k,n)}\right|^2\right) + \log_2\left(t^{(k,n)}\right) - \frac{t^{(k,n)}}{\ln(2)}\mathrm{E}\left\{\left|\widetilde{d}^{(k,n)} - w^{(k,n)}d^{(k,n)}\right|^2\right\}, \quad \text{(C.7)}$$

where $d^{(k,n)}$ and $\widetilde{d}^{(k,n)}$ are the n-th entries of $\mathbf{d}^{(k)}$ and $\widetilde{\mathbf{d}}^{(k)}$, respectively, and $w^{(k,n)}$ and $t^{(k,n)}$ are complex- and real-valued auxiliary variables, respectively. It has been shown that the functions $f^{(k,n)}$ in (C.7) have the following properties.

1. Given

$$w_{\text{opt}}^{(k,n)} = \frac{\left[\sum_{j=1}^{K}\mathbf{H}_{\text{eff}}^{(k,j)}\mathbf{H}_{\text{eff}}^{(k,j)*\mathrm{T}}\right]_{nn} + \left[\mathbf{S}_{\text{noise}}^{(k)}\right]_{nn}}{\left[\mathbf{H}_{\text{eff}}^{(k,k)}\right]_{nn}^{*}}, \quad \text{(C.8)}$$

where $[\cdot]_{nn}$ denotes the n-th diagonal entry of a matrix, and

$$t_{\text{opt}}^{(k,n)} = \frac{1}{\mathrm{E}\left\{\left|\widetilde{d}^{(k,n)} - w_{\text{opt}}^{(k,n)}d^{(k,n)}\right|^2\right\}}, \quad \text{(C.9)}$$

substituting $w_{\text{opt}}^{(k,n)}$ and $t_{\text{opt}}^{(k,n)}$ in (C.7) yields

$$f^{(k,n)}\Big|_{w_{\text{opt}}^{(k,n)},t_{\text{opt}}^{(k,n)}} = \log_2\left(1+\gamma^{(k,n)}\right) - \frac{1}{\ln(2)}. \quad \text{(C.10)}$$

2. The function $f^{(k,n)}$ is a concave function of the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$, if the receive filter $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$ of the k-th destination node, the relay processing filters $\mathbf{G}^{(q)}$, and the auxiliary variables $w^{(k,n)}$ and $t^{(k,n)}$ are fixed.

3. The function $f^{(k,n)}$ is a concave function of the relay processing filters $\mathbf{G}^{(q)}$, if the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$, the receive filter $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$ of the k-th destination node, and the auxiliary variables $w^{(k,n)}$ and $t^{(k,n)}$ are fixed.

4. The function $f^{(k,n)}$ is a concave function of the receive filter $\begin{bmatrix}\mathbf{U}_1^{(k)}\\ \mathbf{U}_2^{(k)}\end{bmatrix}$ of the k-th destination node, if the transmit filters $\begin{bmatrix}\mathbf{V}_1^{(j)}\\ \mathbf{V}_2^{(j)}\end{bmatrix}$, the relay processing filters $\mathbf{G}^{(q)}$, and the auxiliary variables $w^{(k,n)}$ and $t^{(k,n)}$ are fixed.

Based on these properties of $f^{(k,n)}$, the sum rate maximization problem considered in (C.5) and (C.6) can be solved by alternately adapting the transmit filters, the receive filters, the relay processing filters, and the auxiliary variables to maximize $\sum_{k=1}^{K}\sum_{n=1}^{N} f^{(k,n)}$. For the numerical simulations in this thesis, Algorithm 2 is considered. Note that Algorithm 2 only converges to a local optimum solution, which depends on the initialization. The possibilities of using other initializations besides the random matrices considered in this thesis have been discussed in [ASLG+16, AS14] to further improve the performances.

Algorithm 2 Sum rate maximization

1: **Initialize** $\mathbf{V}_1^{(j)}$, $\mathbf{V}_2^{(j)}$, $\mathbf{U}_1^{(k)}$, $\mathbf{U}_2^{(k)}$, $\forall j, k = 1, \ldots, K$, with random $N \times N$ matrices satisfying

$$\mathrm{tr}\left(\mathbf{V}_1^{(j)}\mathbf{V}_1^{(j)*\mathrm{T}} + \mathbf{V}_2^{(j)}\mathbf{V}_2^{(j)*\mathrm{T}}\right) = \frac{1}{2K}P_{\mathrm{tot,max}},$$

initialize $w^{(k,n)}$ and $t^{(k,n)}$ as ones,

2: **repeat**

3: adapt $\mathbf{G}^{(q)}$, $\forall q = 1, \ldots, Q$, by solving the quadratically constrained quadratic optimization problem:

$$\begin{aligned}\underset{\mathbf{G}^{(q)},\forall q}{\text{minimize}} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} t^{(k,n)}\mathrm{E}\left\{\left|\tilde{d}^{(k,n)} - w^{(k,n)}d^{(k,n)}\right|^2\right\}, \\ \text{subject to} \quad & \sum_{q=1}^{Q} P_{\mathrm{R}}^{(q)} \le P_{\mathrm{tot,max}} - \sum_{j=1}^{K} P_{\mathrm{S}}^{(j)},\end{aligned}$$

4: adapt $\mathbf{V}_1^{(j)}$ and $\mathbf{V}_2^{(j)}$, $\forall j = 1, \ldots, K$, by solving the quadratically constrained quadratic optimization problem:

$$\begin{aligned}\underset{\mathbf{V}_1^{(j)},\mathbf{V}_2^{(j)},\forall j}{\text{minimize}} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} t^{(k,n)}\mathrm{E}\left\{\left|\tilde{d}^{(k,n)} - w^{(k,n)}d^{(k,n)}\right|^2\right\}, \\ \text{subject to} \quad & \sum_{j=1}^{K} P_{\mathrm{S}}^{(j)} + \sum_{q=1}^{Q} P_{\mathrm{R}}^{(q)} \le P_{\mathrm{tot,max}},\end{aligned}$$

5: adapt $\mathbf{U}_1^{(k)}$ and $\mathbf{U}_2^{(k)}$ using the linear MMSE receive filter

$$\begin{bmatrix}\mathbf{U}_1^{(k)} \\ \mathbf{U}_2^{(k)}\end{bmatrix} = \left(\sum_{j=1}^{K} \mathbf{H}^{(k,j)}\mathbf{H}^{(k,j)*\mathrm{T}} + \mathbf{S}_{\mathrm{noise}}^{(k)}\right)^{-1} \mathbf{H}^{(k,k)}\mathbf{W}^{(k)*\mathrm{T}}\mathbf{T}^{(k)-1/2}, \quad \forall k,$$

where the $2N \times N$ matrix $\mathbf{H}^{(k,j)}$ is given by

$$\mathbf{H}^{(k,k)} = \begin{bmatrix}\mathbf{H}_{\mathrm{DS}}^{(k,j)} & \mathbf{0} \\ \sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,j)} & \mathbf{H}_{\mathrm{DS}}^{(k,j)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{(j)} \\ \mathbf{V}_2^{(j)}\end{bmatrix},$$

$\mathbf{S}_{\mathrm{noise}}^{(k)}$ is the noise covariance matrix at the k-th destination node, and the matrices $\mathbf{W}^{(k)}$ and $\mathbf{T}^{(k)}$ are $N \times N$ diagonal matrices with the n-th diagonal entry being $w^{(k,n)}$ and $t^{(k,n)}$, respectively,

6: update the auxiliary variables $w^{(k,n)}$ using (C.8),

7: update the auxiliary variables $t^{(k,n)}$ using (C.9),

8: **until** required accuracy is met.

Appendix D.

Proofs

Proof of Proposition 3.1. If (3.24) holds for each node pair, each invalid solution subset $\mathbb{S}_{\text{inv}}^{(k)}$ is a hyperplane of Lebesgue measure 0 in the IN solution space $\mathbb{S}_{\text{IN}}$. Then $\bigcup \mathbb{S}_{\text{inv}}^{(k)}$, which is the union of a finite number K of those hyperplanes, is also of Lebesgue measure 0 in $\mathbb{S}_{\text{IN}}$. It follows that the set $\mathbb{S}_{\text{IN}} \setminus \bigcup \mathbb{S}_{\text{inv}}^{(k)}$ of valid IN solutions is a non-empty set, i.e., relay-aided IA is feasible.

Conversely, if (3.24) does not hold for the k-th node pair,

$$\dim \mathbb{S}_{\text{inv}}^{(k)} = \dim \mathbb{S}_{\text{IN}} \tag{D.1}$$

holds. In other words, the invalid solution set $\mathbb{S}_{\text{inv}}^{(k)}$ is identical to the IN solution space $\mathbb{S}_{\text{IN}}$. Hence, every IN solution is an invalid IN solution with respect to the k-th node pair. Therefore, relay-aided IA is infeasible. □

Proof of Proposition 3.2. Define the block matrices $\mathbf{B} = [\mathbf{B}_1 \cdots \mathbf{B}_Q]$ and $\mathbf{C} = [\mathbf{C}_1 \cdots \mathbf{C}_Q]$, where the blocks $\mathbf{B}_q$ and $\mathbf{C}_q$ are $K \times M_q$ matrices and they are given by

$$\mathbf{B}_q = \begin{bmatrix} \mathbf{h}_{\text{RS}}^{(q,1)\text{T}} \\ \vdots \\ \mathbf{h}_{\text{RS}}^{(q,K)\text{T}} \end{bmatrix} \quad \text{and} \quad \mathbf{C}_q = \begin{bmatrix} \mathbf{h}_{\text{RD}}^{(q,1)*\text{T}} \\ \vdots \\ \mathbf{h}_{\text{RD}}^{(q,K)*\text{T}} \end{bmatrix}, \tag{D.2}$$

respectively. Then the Khatri-Rao product of $\mathbf{B}$ and $\mathbf{C}$ is the $K^2 \times \sum_{q=1}^{Q} M_q^2$ matrix

$$\mathbf{B} \odot \mathbf{C} = \begin{bmatrix} \mathbf{B}_1 \otimes \mathbf{C}_1 & \cdots & \mathbf{B}_Q \otimes \mathbf{C}_Q \end{bmatrix}, \tag{D.3}$$

where $\odot$ and $\otimes$ denote the Khatri-Rao product and the Kronecker product of two matrices, respectively. Corollary 1 in [JStB01] has shown that the column-wise Khatri-Rao product of two matrices whose entries are independently drawn from a continuous distribution is of full Kruskal-rank with probability one. This conclusion can be directly employed since the entries of the matrices $\mathbf{B}_q$ and $\mathbf{C}_q$, $q = 1, \ldots, Q$, are independently drawn from a continuous distribution. Therefore, $\mathbf{B} \odot \mathbf{C}$, as well as $(\mathbf{B} \odot \mathbf{C})^{\mathrm{T}}$, is also of full Kruskal-rank with probability 1. This means a submatrix of $\mathbf{B} \odot \mathbf{C}$ consisting of a collection of arbitrarily chosen rows is of full rank with probability one. Note that both $\mathbf{A}_{\mathrm{RL}}$ and $\begin{bmatrix} \mathbf{A}_{\mathrm{RL}} \\ \mathbf{b}^{(k)\mathrm{T}} \end{bmatrix}$ are submatrices of $\mathbf{B} \odot \mathbf{C}$, which are obtained by omitting the rows

$$\begin{bmatrix} \mathbf{h}_{\mathrm{RS}}^{(1,k)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(1,k)*\mathrm{T}} & \cdots & \mathbf{h}_{\mathrm{RS}}^{(Q,k)\mathrm{T}} \otimes \mathbf{h}_{\mathrm{RD}}^{(Q,k)*\mathrm{T}} \end{bmatrix} \tag{D.4}$$

for $k = 1, \ldots, K$ and for $k \neq i$, respectively. Therefore, $\mathbf{A}_{\mathrm{RL}}$ and $\begin{bmatrix} \mathbf{A}_{\mathrm{RL}} \\ \mathbf{b}^{(k)\mathrm{T}} \end{bmatrix}$ are of full rank with probability 1. This completes the proof. □

For the proof of Proposition 3.4, the following lemma[1] is useful. A proof of this lemma has been given in [JStB01], and hence will not be repeated in this thesis.

Lemma D.1. Consider an analytic function $\mathrm{f}(\mathbf{x})$ of several complex variables $\mathbf{x} \in \mathbb{C}^n$. If the function $\mathrm{f}(\cdot)$ is nontrivial in the sense that there exists a $\mathbf{x}_0 \in \mathbb{C}^n$ such that $\mathrm{f}(\mathbf{x}_0) \neq 0$, then the solution set of $\mathrm{f}(\mathbf{x}) = 0$ is of Lebesgue measure zero in $\mathbb{C}^n$.

Proof of Proposition 3.4. Consider the rank of $\mathbf{A}_{\mathrm{IN}}$ first. Let $\mathbf{B}_{\mathrm{DL}}$ be a $K(K-1) \times (2K-1)$ matrix whose columns form a basis of the column space of $\mathbf{A}_{\mathrm{DL}}$. It is clear that the rank of $\mathbf{A}_{\mathrm{IN}}$ is equal to the rank of $\begin{bmatrix} \mathbf{A}_{\mathrm{RL}} & \mathbf{B}_{\mathrm{DL}} \end{bmatrix}$. Therefore, in order to show (3.30), it suffices to show that if $\begin{bmatrix} \mathbf{A}_{\mathrm{RL}} & \mathbf{B}_{\mathrm{DL}} \end{bmatrix}$ is a square matrix, it is almost surely of full rank. That is to say, it needs to be shown that the statement

$$\mathbf{A}_{\mathrm{RL}}\mathbf{z}_1 + \mathbf{B}_{\mathrm{DL}}\mathbf{z}_2 = \mathbf{0} \;\Rightarrow\; \mathbf{z}_1 = \mathbf{z}_2 = \mathbf{0}, \tag{D.5}$$

where $\mathbf{z}_1$ and $\mathbf{z}_2$ are vectors with consistent dimensions, is true with probability 1 if the channel coefficients are independently drawn from a continuous distribution. Equivalently, it will be shown in the following that the negation of (D.5), i.e.,

$$(\mathbf{z}_1 \neq \mathbf{0} \vee \mathbf{z}_2 \neq \mathbf{0}) \;\wedge\; (\mathbf{A}_{\mathrm{RL}}\mathbf{z}_1 + \mathbf{B}_{\mathrm{DL}}\mathbf{z}_2 = \mathbf{0}), \tag{D.6}$$

it true with probability 0 for randomly drawn channel coefficients. Recall that the matrix $\mathbf{A}_{\mathrm{DL}}$ is related to the incidence matrix of a bipartite graph $\mathcal{G}$. For

[1] Alternatively, this lemma can also be expressed using the terminology of algebraic geometry, see Corollary 1.6 and Proposition 1.13 in [Har77].

the convenience of the following discussions, rewrite (3.28) as $\mathbf{A}_{\mathrm{DL}} = \mathrm{diag}\left(\mathbf{h}\right)\boldsymbol{\Psi}_{\mathcal{G}}$, where $\mathbf{h}$ is the $K(K-1)\times 1$ vector

$$\mathbf{h} = \left[h_{\mathrm{DS}}^{(2,1)} \ \cdots \ h_{\mathrm{DS}}^{(K,1)} \ \cdots \ \cdots \ h_{\mathrm{DS}}^{(1,K)} \ \cdots \ h_{\mathrm{DS}}^{(K-1,K)}\right]^{\mathrm{T}}. \tag{D.7}$$

Therefore, $\mathbf{B}_{\mathrm{DL}}$ can also be written as $\mathbf{B}_{\mathrm{DL}} = \mathrm{diag}\left(\mathbf{h}\right)\mathbf{B}_{\mathcal{G}}$, where the columns of $\mathbf{B}_{\mathcal{G}}$ form a basis of the column space of $\boldsymbol{\Psi}_{\mathcal{G}}$. Consider the vector function

$$\begin{aligned}\mathbf{f}(\mathbf{h}) &= \mathbf{A}_{\mathrm{RL}}\mathbf{z}_1 + \mathbf{B}_{\mathrm{DL}}\mathbf{z}_2 \\ &= \mathbf{A}_{\mathrm{RL}}\mathbf{z}_1 + \mathrm{diag}\left(\mathbf{h}\right)\mathbf{B}_{\mathcal{G}}\mathbf{z}_2 \\ &= \mathbf{A}_{\mathrm{RL}}\mathbf{z}_1 + \mathrm{diag}\left(\mathbf{B}_{\mathcal{G}}\mathbf{z}_2\right)\mathbf{h}.\end{aligned} \tag{D.8}$$

In the case of $\left[\mathbf{A}_{\mathrm{RL}} \ \ \mathbf{B}_{\mathrm{DL}}\right]$ being a square matrix, the matrix $\mathbf{A}_{\mathrm{RL}}$ is a tall matrix, and its columns are almost surely linearly independent by Proposition 3.2. The columns of $\mathbf{B}_{\mathcal{G}}$ are also linearly independent by definition. Hence, the function $\mathbf{f}(\cdot)$ is a trivial function of $\mathbf{h}$ only if both $\mathbf{z}_1$ and $\mathbf{z}_2$ are zero vectors. Therefore, given any non-zero $\mathbf{z}_1$ or $\mathbf{z}_2$, the solution set of $\mathbf{f}(\mathbf{h}) = \mathbf{0}$ is of Lebesgue measure zero in $\mathbb{C}^{K(K-1)}$ based on Lemma D.1. In other words, given any non-zero $\mathbf{z}_1$ or $\mathbf{z}_2$, $\mathbf{f}(\mathbf{h}) = \mathbf{0}$ holds with probability 0 for randomly drawn channel coefficients. This proves (3.30).

Following the same line of the above argumentation, (3.31) can be proved as well. This completes the proof. □

Proof of Proposition 3.6. In the first step, it directly follows from the proof of Proposition 3.2 that $\mathbf{A}_{\mathrm{RL}}$ is almost surely of full rank, i.e.,

$$\mathrm{rank}\left(\mathbf{A}_{\mathrm{RL}}\right) = \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2\right\} \tag{D.9}$$

holds with probability one, if the channel coefficients are independently drawn from a continuous distribution.

In the second step, consider the rank of $\left[\mathbf{A}_{\mathrm{DL},1} \ \ \mathbf{A}_{\mathrm{DL},2}\right]$. First consider the case of $K = 3$. In this case, the matrix $\left[\mathbf{A}_{\mathrm{DL},1} \ \ \mathbf{A}_{\mathrm{DL},2}\right]$ specifies the system of linear equations

$$\mathbf{H}_{\mathrm{DS}}^{(2,1)}\mathbf{V}^{(1)} + \mathbf{U}^{(2)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(2,1)} = \mathbf{0} \tag{D.10}$$

$$\mathbf{H}_{\mathrm{DS}}^{(3,1)}\mathbf{V}^{(1)} + \mathbf{U}^{(3)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(3,1)} = \mathbf{0} \tag{D.11}$$

$$\mathbf{H}_{\mathrm{DS}}^{(1,2)}\mathbf{V}^{(2)} + \mathbf{U}^{(1)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(1,2)} = \mathbf{0} \tag{D.12}$$

$$\mathbf{H}_{\mathrm{DS}}^{(3,2)}\mathbf{V}^{(2)} + \mathbf{U}^{(3)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(1,3)} = \mathbf{0} \tag{D.13}$$

$$\mathbf{H}_{\mathrm{DS}}^{(1,3)}\mathbf{V}^{(3)} + \mathbf{U}^{(1)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(1,3)} = \mathbf{0} \tag{D.14}$$

$$\mathbf{H}_{\mathrm{DS}}^{(2,3)}\mathbf{V}^{(3)} + \mathbf{U}^{(2)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(2,3)} = \mathbf{0} \tag{D.15}$$

by definition. Thus the problem of determining the rank of $\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix}$ is equivalent to finding the dimension of the solution space of the system (D.10)–(D.15). Since the $N \times N$ matrices $\mathbf{H}_{\mathrm{DS}}^{(k,j)}$ are almost surely invertible, solving the above system for $\mathbf{V}^{(1)}$ by eliminating the other variables yields

$$\mathbf{B}\mathbf{V}^{(1)} - \mathbf{V}^{(1)}\mathbf{B} = \mathbf{0}, \tag{D.16}$$

where the $N \times N$ matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{H}_{\mathrm{DS}}^{(3,1)-1}\mathbf{H}_{\mathrm{DS}}^{(3,2)}\mathbf{H}_{\mathrm{DS}}^{(1,2)-1}\mathbf{H}_{\mathrm{DS}}^{(1,3)}\mathbf{H}_{\mathrm{DS}}^{(2,3)-1}\mathbf{H}_{\mathrm{DS}}^{(2,1)}. \tag{D.17}$$

It is clear that the dimension of the solution space of the system (D.10)–(D.15) equals the dimension of the solution space of the matrix equation (D.16). Moreover, the matrix equation (D.16) suggests that $\mathbf{V}^{(1)}$ commutes with $\mathbf{B}$. Since $\mathbf{B}$ almost surely has N distinct non-zero eigenvalues, $\mathbf{V}^{(1)}$ can then be represented by a polynomial of degree $N-1$ in $\mathbf{B}$, i.e.,

$$\mathbf{V}^{(1)} = \alpha_0\mathbf{I}_N + \alpha_1\mathbf{B} + \ldots + \alpha_{N-1}\mathbf{B}^{N-1} \tag{D.18}$$

holds, see Theorem 3.2.4.2. in [HJ90]. In other words, the set of all matrices $\mathbf{V}^{(1)}$ satisfying (D.16) must form an N-dimensional space spanned by the set $\left\{\mathbf{I}_N, \mathbf{B}, \ldots, \mathbf{B}^{N-1}\right\}$. Therefore, the solution space of (D.16), as well as the solution space of the system (D.10)–(D.15), is of dimension N. It follows that

$$\operatorname{rank}\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix} = 2KN^2 - N \tag{D.19}$$

holds with probability one for $K = 3$.

Then consider the rank of $\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix}$ for $K > 3$. In this case, the system of linear equations (D.10)–(D.15) is a subsystem specified by $\begin{bmatrix}\mathbf{A}_{\mathrm{DL},1} & \mathbf{A}_{\mathrm{DL},2}\end{bmatrix}$. So $\mathbf{V}^{(1)}$

must be taken from the N-dimensional space spanned by the set $\left\{\mathbf{I}_N, \mathbf{B}, \ldots, \mathbf{B}^{N-1}\right\}$, where $\mathbf{B}$ is introduced in (D.17). Besides, the following system of linear equations

$$\mathbf{H}_{\mathrm{DS}}^{(2,1)}\mathbf{V}^{(1)} + \mathbf{U}^{(2)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(2,1)} = \mathbf{0} \tag{D.20}$$

$$\mathbf{H}_{\mathrm{DS}}^{(2,3)}\mathbf{V}^{(3)} + \mathbf{U}^{(2)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(2,3)} = \mathbf{0} \tag{D.21}$$

$$\mathbf{H}_{\mathrm{DS}}^{(4,3)}\mathbf{V}^{(3)} + \mathbf{U}^{(4)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(4,3)} = \mathbf{0} \tag{D.22}$$

$$\mathbf{H}_{\mathrm{DS}}^{(4,1)}\mathbf{V}^{(1)} + \mathbf{U}^{(4)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(4,1)} = \mathbf{0} \tag{D.23}$$

is another subsystem which $\mathbf{V}^{(1)}$ must satisfy. Solving the system (D.20)–(D.23) for $\mathbf{V}^{(1)}$ yields

$$\mathbf{C}\mathbf{V}^{(1)} - \mathbf{V}^{(1)}\mathbf{C} = \mathbf{0}, \tag{D.24}$$

where the $N \times N$ matrix $\mathbf{C}$ is given by

$$\mathbf{C} = \mathbf{H}_{\mathrm{DS}}^{(4,1)-1}\mathbf{H}_{\mathrm{DS}}^{(4,3)}\mathbf{H}_{\mathrm{DS}}^{(2,3)-1}\mathbf{H}_{\mathrm{DS}}^{(2,1)}. \tag{D.25}$$

Therefore, $\mathbf{V}^{(1)}$ must also commute with $\mathbf{C}$, and hence lie in the N-dimensional space spanned by the set $\left\{\mathbf{I}_N, \mathbf{C}, \ldots, \mathbf{C}^{N-1}\right\}$. Since the entries of $\mathbf{H}_{\mathrm{DS}}^{(k,j)}$ are independently drawn from a continuous distribution, the spaces spanned by the set $\left\{\mathbf{I}_N, \mathbf{B}, \ldots, \mathbf{B}^{N-1}\right\}$ and the set $\left\{\mathbf{I}_N, \mathbf{C}, \ldots, \mathbf{C}^{N-1}\right\}$ can only intersect at the one dimensional space spanned by $\{\mathbf{I}_N\}$. Therefore, $\mathbf{V}^{(1)}$ can only be a scaled identity matrix. Furthermore, it can be verified that

$$\mathbf{V}^{(1)} = \ldots = \mathbf{V}^{(K)} = -\mathbf{U}^{(1)} = \ldots = -\mathbf{U}^{(K)} = \mathbf{I}_N \tag{D.26}$$

is a solution of the system of linear equations specified by $\left[\mathbf{A}_{\mathrm{DL},1} \;\; \mathbf{A}_{\mathrm{DL},2}\right]$. For these reasons, the solution space of the system of linear equations specified by $\left[\mathbf{A}_{\mathrm{DL},1} \;\; \mathbf{A}_{\mathrm{DL},2}\right]$ has only one dimension for $K > 3$. It follows that

$$\operatorname{rank}\left[\mathbf{A}_{\mathrm{DL},1} \;\; \mathbf{A}_{\mathrm{DL},2}\right] = 2KN^2 - 1 \tag{D.27}$$

holds with probability one for $K > 3$.

In the final step, consider the rank of $\mathbf{A}_{\mathrm{IN}} = \left[\mathbf{A}_{\mathrm{RL}} \;\; \mathbf{A}_{\mathrm{DL},1} \;\; \mathbf{A}_{\mathrm{DL},2}\right]$ based on the above results on the ranks of $\mathbf{A}_{\mathrm{RL}}$ and $\left[\mathbf{A}_{\mathrm{DL},1} \;\; \mathbf{A}_{\mathrm{DL},2}\right]$. The proof simply follows the same line of the proof of Proposition 3.4. Therefore,

$$\operatorname{rank}\left(\mathbf{A}_{\mathrm{IN}}\right) = \begin{cases} \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2 + 2KN^2 - N\right\} & \text{for } K = 3 \\ \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2 + 2KN^2 - 1\right\} & \text{for } K > 3 \end{cases} \tag{D.28}$$

holds with probability one if the channel coefficients are independently drawn from a continuous distribution. This completes the proof. □

Proof of Proposition 3.7. For the convenience of the proof, recall the polynomial

$$p_k = \det\left(\sum_{q=1}^{Q} \mathbf{H}_{\mathrm{RD}}^{(q,k)*\mathrm{T}}\mathbf{G}^{(q)}\mathbf{H}_{\mathrm{RS}}^{(q,k)} + \mathbf{H}_{\mathrm{DS}}^{(k,k)}\mathbf{V}^{(k)} + \mathbf{U}^{(k)*\mathrm{T}}\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right) = \det\left(\mathbf{H}_k\right), \tag{D.29}$$

which has been defined in (3.42). In (D.29), the matrix $\mathbf{H}_k$ is introduced to simplify the notation. For a given IN solution $\mathbf{x} \in \mathbb{S}_{\mathrm{IN}}$, the polynomial p_k can be considered as a polynomial $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$ of the channel matrix $\mathbf{H}_{\mathrm{DS}}^{(k,k)}$. If an IN solution $\mathbf{x}$ yields that $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$ is a trivial polynomial, by definition, $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right) = 0$ always holds regardless of the channel realization of $\mathbf{H}_{\mathrm{DS}}^{(k,k)}$. Otherwise, if an IN solution $\mathbf{x}$ yields that $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$ is a non-trivial polynomial in $\mathbf{H}_{\mathrm{DS}}^{(k,k)}$, $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right) = 0$ holds with probability zero for a random channel realization of $\mathbf{H}_{\mathrm{DS}}^{(k,k)}$. Therefore, the key is to find those IN solutions $\mathbf{x} \in \mathbb{S}_{\mathrm{IN}}$ which make $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$ a trivial polynomial.

If the IN solution space $\mathbb{S}_{\mathrm{IN}}$ is one-dimensional, $\mathbb{S}_{\mathrm{IN}}$ must be spanned by the IN solution with $\mathbf{G}^{(q)} = \mathbf{0}_{M_q}$, $\forall q$, and $\mathbf{V}^{(k)} = -\mathbf{U}^{(j)*\mathrm{T}} = \mathbf{I}_N$, $\forall j, k$. Substituting this IN solution in p_k yields that $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$ is a trivial polynomial. This proves the first statement.

Then consider the case where the IN solution space $\mathbb{S}_{\mathrm{IN}}$ has at least two dimensions. Let $[\cdot]_{mn}$ denote the entry in the m-th row and the n-th column of a matrix. Then the term

$$\prod_{n=1}^{N}\left(\left[\mathbf{V}^{(k)}\right]_{nn} + \left[\mathbf{U}^{(k)}\right]_{nn}\right)\left[\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right]_{nn} \tag{D.30}$$

appears in $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right)$, and it is the only term of degree one in $\left[\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right]_{11}$, $\left[\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right]_{22}$, $\cdots$, and $\left[\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right]_{NN}$. To see this, consider $\left[\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right]_{11}$ first. It can be seen from (D.29) that $\left[\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right]_{11}$ only appears in the first row and the first column of $\mathbf{H}_k$ and is of degree one. Let $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\mathrm{DS}}^{(k,k)}\right) = \det\left(\mathbf{H}_k\right)$ be expanded along the first row of $\mathbf{H}_k$ as

$$p_k|_{\mathbf{x}}\left(\mathbf{H}_k\right)) = \alpha_{11}\left[\mathbf{H}_k\right]_{11} - \alpha_{12}\left[\mathbf{H}_k\right]_{12} + \cdots + (-1)^{N+1}\alpha_{1N}\left[\mathbf{H}_k\right]_{1N}, \tag{D.31}$$

where α_{11}, $\ldots$, α_{1N} are the corresponding minors, i.e., the determinants of the submatrices of $\mathbf{H}_k$ obtained by deleting the corresponding row and column. Then only $\alpha_1 [\mathbf{H}_k]_{11}$ on the right hand side of (D.31) is of degree one in $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}$, because the other terms must be of degree two in $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}$. Furthermore, $[\mathbf{H}_k]_{11}$ can be given by

$$[\mathbf{H}_k]_{11} = \left(\left[\mathbf{V}^{(k)}\right]_{11} + \left[\mathbf{U}^{(k)}\right]_{11}\right)\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11} + c, \tag{D.32}$$

where c does not contain $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}$. Therefore, the only term of $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ which is of degree one in $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}$ is

$$\alpha_{11}\left(\left[\mathbf{V}^{(k)}\right]_{11} + \left[\mathbf{U}^{(k)}\right]_{11}\right)\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}. \tag{D.33}$$

Similarly, the minor α_{11} can be expanded to show that the only term of degree one in both $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}$ and $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{22}$ is

$$\beta_{11}\left(\left[\mathbf{V}^{(k)}\right]_{11} + \left[\mathbf{U}^{(k)}\right]_{11}\right)\left(\left[\mathbf{V}^{(k)}\right]_{22} + \left[\mathbf{U}^{(k)}\right]_{22}\right)\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{22}, \tag{D.34}$$

where β_{11} is the determinant of the submatrix of $\mathbf{H}_k$ obtained by deleting the first two rows and the first two columns. Repeating this process yields that the term given in (D.30) is the only term of $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ which is of degree one in $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{11}$, $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{22}$, $\cdots$, and $\left[\mathbf{H}_{\text{DS}}^{(k,k)}\right]_{NN}$. Therefore, a necessary condition for $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ being a trivial polynomial is

$$\prod_{n=1}^{N}\left(\left[\mathbf{V}^{(k)}\right]_{nn} + \left[\mathbf{U}^{(k)}\right]_{nn}\right) = 0. \tag{D.35}$$

However, (D.35) holds with probability zero for a randomly picked IN solution if the IN solution space $\mathbb{S}_{\text{IN}}$ has at least two dimensions. That is to say, a randomly picked IN solution $\mathbf{x} \in \mathbb{S}_{\text{IN}}$ almost surely makes $p_k|_{\mathbf{x}}\left(\mathbf{H}_{\text{DS}}^{(k,k)}\right)$ a non-trivial polynomial in $\mathbf{H}_{\text{DS}}^{(k,k)}$ in this case. This proves the second statement. □

Proof of Proposition 4.1. The proof follows the same line as the proof of Proposition 3.6. In the first step, it is clear that $\mathbf{A}_{\text{RL}}$ is almost surely of full rank, i.e.,

$$\text{rank}\left(\mathbf{A}_{\text{RL}}\right) = \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2\right\} \tag{D.36}$$

holds with probability one, if the channel coefficients are independently drawn from a continuous distribution.

In the second step, consider the rank of $\left[\mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right]$. In the considered fully connected cellular networks, the matrix $\left[\mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right]$ specifies a system of linear equations consisting of the following $K(K-1)$ matrix equations:

$$\mathbf{H}_{\text{BM}}^{(k,j)}\mathbf{V}_{\text{UL}}^{(j)} + \mathbf{U}_{\text{UL}}^{(k)*\text{T}}\mathbf{H}_{\text{BM}}^{(k,j)} = \mathbf{0}, \quad \forall k \neq j, \tag{D.37}$$

where the matrices $\mathbf{V}_{\text{UL}}^{(j)}$ are restricted to diagonal matrices. If the considered cellular network has $K \geq 3$ cells, the matrix $\mathbf{V}_{\text{UL}}^{(1)}$ can be written as

$$\mathbf{V}_{\text{UL}}^{(1)} = \alpha_0 \mathbf{I}_N + \alpha_1 \mathbf{B} + \ldots + \alpha_{N-1}\mathbf{B}^{N-1}, \tag{D.38}$$

where the $N \times N$ matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{H}_{\text{BM}}^{(3,1)-1}\mathbf{H}_{\text{BM}}^{(3,2)}\mathbf{H}_{\text{BM}}^{(1,2)-1}\mathbf{H}_{\text{BM}}^{(1,3)}\mathbf{H}_{\text{BM}}^{(2,3)-1}\mathbf{H}_{\text{BM}}^{(2,1)}, \tag{D.39}$$

see the proof of Proposition 3.6. Since the entries of $\mathbf{B}$ depend on the random channel realization, the coefficients $\alpha_1, \ldots, \alpha_{N-1}$ in (D.38) have to be zero to ensure that $\mathbf{V}_{\text{UL}}^{(1)}$ is a diagonal matrix. Therefore, $\mathbf{V}_{\text{UL}}^{(1)}$ can only be a scaled identity matrix. Furthermore, it can be verified that

$$\mathbf{V}^{(1)} = \ldots = \mathbf{V}^{(K)} = -\mathbf{U}^{(1)} = \ldots = -\mathbf{U}^{(K)} = \mathbf{I}_N \tag{D.40}$$

satisfies all the matrix equations (D.37) specified by $\left[\mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right]$. For these reasons, the solution space of the system of linear equations specified by $\left[\mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right]$ has and only has one dimension for $K \geq 3$. It follows that

$$\text{rank}\left[\mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right] = KN + KN^2 - 1 \tag{D.41}$$

holds with probability one.

In the final step, consider the rank of $\mathbf{A}_{\text{IN}} = \left[\mathbf{A}_{\text{RL}} \;\; \mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right]$ based on the above results on the ranks of $\mathbf{A}_{\text{RL}}$ and $\left[\mathbf{A}_{\text{DL},1} \;\; \mathbf{A}_{\text{DL},2}\right]$. The proof simply follows the same line of the proof of Proposition 3.4. Therefore,

$$\text{rank}\left(\mathbf{A}_{\text{IN}}\right) = \min\left\{K(K-1)N^2, \sum_{q=1}^{Q} M_q^2 + KN + KN^2 - 1\right\} \tag{D.42}$$

holds with probability one if the channel coefficients are independently drawn from a continuous distribution. This completes the proof. $\square$

Proof of Proposition 5.1. The necessity can be simply proved by considering Example 5.1 as a counterexample, where the space $\mathbb{S}^{\{1\}}_{\text{intra+EC}}$ is only one-dimensional and every IN solution is invalid with respect to every node pair in subnetwork 1.

The sufficiency can be proved using mathematical induction. First consider a single subnetwork, say the q-th subnetwork, which is a fully connected ad-hoc network. Due to the results from Subsection 3.4.1, a randomly picked solution in $\mathbb{S}^{\{q\}}_{\text{intra+EC}}$ is almost surely valid with respect to every node pair in the q-th subnetwork, if $\dim \mathbb{S}^{\{q\}}_{\text{intra+EC}} \geq 2$ holds.

Then consider a subset $\{1, \ldots, q-1\}$ of subnetworks. Suppose that $\dim \mathbb{S}^{\Phi}_{\text{intra+EC}} \geq 2$ holds for every subset $\Phi \subseteq \{1, \ldots, q-1\}$ of subnetworks, and that a randomly picked solution in $\mathbb{S}^{\{1,\ldots,q-1\}}_{\text{intra+EC}}$ is almost surely valid with respect to every node pair in $\{1, \ldots, q-1\}$. Furthermore, let the q-th subnetwork be taken into consideration, where $\dim \mathbb{S}^{\{q\}}_{\text{intra+EC}} \geq 2$ and $\dim \mathbb{S}^{\{1,\ldots,q\}}_{\text{intra+EC}} \geq 2$ hold. Then, a randomly picked solution $\mathbf{x}$ in $\mathbb{S}^{\{1,\ldots,q\}}_{\text{intra+EC}}$ can be considered as being formed by a randomly picked solution $\mathbf{x}_1$ in $\mathbb{S}^{\{1,\ldots,q-1\}}_{\text{intra+EC}}$ and a randomly picked solution $\mathbf{x}_2$ in $\mathbb{S}^{\{q\}}_{\text{intra+EC}}$. Since $\mathbf{x}_1$ is almost surely valid with respect to every node pair in the subset $\{1, \ldots, q-1\}$ of subnetworks due to the assumption and $\mathbf{x}_2$ is almost surely valid with respect to every node pair in the q-th subnetwork due to the results from Subsection 3.4.1, $\mathbf{x}$ is then almost surely valid with respect to every node pair in the subset $\{1, \ldots, q\}$ of subnetworks. This completes the proof. □

Proof of Proposition 5.2. The proposition can be proved using mathematical induction. If Φ contains a single subnetwork, say $\Phi = \{q\}$, it directly follows from (5.32) and (5.33) that

$$M^2_{q,\min} \geq K_q(K_q - 3) + N^{\{q\}}_{\text{EC}} + 2 \tag{D.43}$$

holds.

If Φ contains more than one subnetwork, one may assume that Φ consists of a subset Φ' of subnetworks and the q-th subnetwork, where any subnetwork in the subset Φ' has an index smaller than q. Furthermore, suppose

$$\sum_{r\in\Phi'} M^2_{r,\min} \geq \sum_{r\in\Phi'} K_r(K_r - 3) + N^{\Phi'}_{\text{EC}} + 2 \tag{D.44}$$

and

$$M^2_{q,\min} = K_q(K_q - 3) + \max\left\{N^{\{q\}}_{\text{EC}} + 2,\ N^{\{1,\ldots,q\}}_{\text{EC}} - N^{\{1,\ldots,q-1\}}_{\text{EC}}\right\} \tag{D.45}$$

hold. It remains to prove that

$$\sum_{r\in\Phi'} M_{r,\min}^2 + M_{q,\min}^2 \geq \sum_{r\in\Phi'} K_r(K_r-3) + K_q(K_q-3) + N_{\mathrm{EC}}^{\Phi} + 2 \tag{D.46}$$

holds. Comparing the right hand side of the sum of (D.44) and (D.45) with the right hand side of (D.46), it suffices to prove that

$$N_{\mathrm{EC}}^{\{1,\ldots,q\}} - N_{\mathrm{EC}}^{\{1,\ldots,q-1\}} - N_{\mathrm{EC}}^{\{q\}} \geq N_{\mathrm{EC}}^{\Phi} - N_{\mathrm{EC}}^{\Phi'} - N_{\mathrm{EC}}^{\{q\}} \tag{D.47}$$

holds. The left and the right hand sides of (D.47) represent the numbers of linearly independent external constraints between the subset $\{1,\ldots,q-1\}$ and the q-th subnetwork and that between the subset Φ' and the q-th subnetwork, respectively. Since $\Phi' \subseteq \{1,\ldots,q-1\}$, every external constraint between the q-th subnetwork and the subset Φ' must be an external constraint between the q-th subnetwork and the subset $\{1,\ldots,q-1\}$. Therefore, (D.47) holds. This completes the proof. □

Appendix E.

Glossary of abbreviations and symbols

Abbreviations

AF	amplify-and-forward
AWGN	additive white Gaussian noise
BC	broadcast channel
BS	base station
CSI	channel state information
DoF	degree of freedom
FDD	frequency division duplexing
FDMA	frequency division multiple access
i.i.d.	independently identically distributed
IA	interference alignment
IC	interference channel
ILM	interference leakage minimization
IN	interference-nulling
LTE-A	Long Term Evolution Advanced
MAC	multiple access channel
MIMO	multiple-input-multiple-output
MISO	multiple-input-single-output
MS	mobile station
MSE	mean square error
NP	non-deterministic polynomial time
OFDM	orthogonal frequency division multiplexing
PSNR	pseudo signal-to-noise ratio
SIC	successive interference cancellation
SIMO	single-input-multiple-output
SISO	single-input-single-output
SNR	signal-to-noise ratio
SINR	signal-to-interference-plus-noise ratio
SVD	singular value decomposition
TDD	time division duplexing
TDMA	time division multiple access
ZF	zero-forcing

Symbols

a, A, α	a scalar
$\mathbf{a}$	a column vector
$\mathbf{i}_k$	the index vector with the k-th entry being one and the others zero
$\mathbf{A}$	a matrix
$\mathbf{A}^*$	conjugate matrix
$\mathbf{A}^{\mathrm{T}}$	transposed matrix
$\mathbf{A}^{*\mathrm{T}}$	conjugate transposed matrix
$[\mathbf{A}]_{mn}$	the entry in the m-th row and the n-th column of $\mathbf{A}$
$\mathbf{0}$	the all zero matrix
$\mathbf{I}_N$	the $N \times N$ identity matrix
$\mathcal{E}$	the edge set of a graph
$\mathcal{G}$	a graph
$\mathcal{V}$	the vertex set of a graph
Φ	a subset of subnetworks
$\mathbb{S}$	a linear space/a non-linear algebraic set
$\mathbb{C}$	the complex field
$\det(\cdot)$	determinant
$\mathrm{diag}(\cdot)$	a diagonal matrix/the diagonal entries of a matrix
dim	dimension of a linear space
$\mathrm{E}\{\cdot\}$	expectation
$\max\{\cdot\}$	maximum
$\min\{\cdot\}$	minimum
$\mathrm{null}(\cdot)$	null space
$\mathrm{rank}(\cdot)$	rank
$\mathrm{tr}(\cdot)$	trace
$\mathrm{vec}(\cdot)$	vectorization
$\sum$	sum
$\prod$	product/Cartesian product
$\bigcup$	union
$\otimes$	Kronecker product
$\odot$	Khatri-Rao product

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Thesen

- The multiuser interference is a major performance-limiting factor in current wireless radio communication systems, due to the scarceness of spectrum.
- IA is able to achieve the DoFs of many multiuser interference networks, leading to outstanding performances in the high-SNR regime.
- Relay-aided IA requires few resource extensions and few antennas at the source and destination nodes; many relay-aided IA problems have closed-form solutions.
- To achieve relay-aided IA, the transmit filters, the receive filters, and the relay processing filters shall be cooperatively designed to satisfy all the IN conditions while not violating any of the validity conditions.
- The existence of such an IA solution requires that the invalid IN solutions form either linear hyperplanes or negligibly small non-linear subsets of the IN solution space.
- Given any valid IN solution, the achievable sum rate can be maximized under a total sum transmit power constraint or under individual sum power constraints.
- In the cellular networks, relay-aided IA includes inter-cell IN and intra-cell interference management exploiting beamforming techniques such as ZF and MMSE.
- The uplink-downlink duality of relay-aided IA implies that both the inter-cell IN solutions and the beamforming matrices designed for intra-cell interference management in the uplink and the downlink are dual.
- In the partially connected ad-hoc networks, relay-aided IA can be achieved with partial channel knowledge, which includes the intra-subnetwork CSI, the network topology, and the side information.

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Publikationsliste

[1] H. Al-Shatri, X. Li, R. S. Ganesan, A. Klein, and T. Weber, "Closed-form solutions for minimizing sum MSE in multiuser relay networks," in *Proc. IEEE 77th Vehicular Technology Conference (VTC'13-Spring)*, Dresden, Jun. 2013, pp. 1–5, doi: 10.1109/VTCSpring.2013.6692476.

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